Are stabilized methods a reliable tool for suppressing spurious oscillations?

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We consider the application of the finite element method to the numerical solution of the scalar convection–diffusion equation

$$-\varepsilon\,\Delta u + \boldsymbol{b}\cdot\nabla u = f \quad \text{in } \Omega, \qquad u = u_b \quad \text{on } \Gamma^D, \qquad \varepsilon\,\frac{\partial u}{\partial \boldsymbol{n}} = g \quad \text{on } \Gamma^N. \tag{1}$$

Here Ω is a bounded two–dimensional domain with a polygonal boundary $\partial\Omega$, Γ^D and Γ^N are disjoint and relatively open subsets of $\partial\Omega$ satisfying $\text{meas}_1(\Gamma^D) > 0$ and $\overline{\Gamma^D \cup \Gamma^N} = \partial\Omega$, $\boldsymbol{n}$ is the outward unit normal vector to $\partial\Omega$, f is a given outer source of the unknown scalar quantity u, $\varepsilon > 0$ is the constant diffusivity, $\boldsymbol{b}$ is the flow velocity, and u_b, g are given functions.

It is well known that the classical Galerkin finite element discretization of (1) is inappropriate in the convection–dominated regime (i.e., if $\varepsilon \ll \|\boldsymbol{b}\|$) since the discrete solution is typically globally polluted by spurious oscillations. During the last three decades, an extensive research has been devoted to the development of methods which diminish spurious oscillations in the discrete solutions of (1) and the aim of the talk is to review some of the most important approaches and to compare them by means of both numerical tests and theoretical considerations. In particular, we shall discuss the quality of the discrete solutions (spurious oscillations, smearing of inner and boundary layers), dependence of the methods on parameters, triangulations and data, and also the cost needed to compute the discrete solutions.

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