A unified convergence analysis for the local projection stabilisation applied to the Oseen problem

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The discretisation of the Oseen problem by finite element methods suffers in general from two reasons. First, the discrete inf-sup or Babuvska-Brezzi condition is violated. Second, spurious oscillations occur due to the dominating convection. One way to overcome both difficulties is the use of local projection techniques.

Originally proposed by Becker and Braack for the Stokes problem, it was extended by them to the transport equation. A convergence analysis for first and second order discretisations on quadrilaterals was recently given by Braack and Burman.

We will consider the local projection method for a large class of equal-order approximations of the Oseen problem. When defining the local projection in the right way, we can show that the stabilised method converges for arbitrary polynomial degree with optimal order. This result holds true on simplices, quadrilaterals, and hexahedra.

On simplices, the spectral equivalence of the stabilising terms of the local projection method and the subgrid modeling introduced by Guermond will be shown. This makes the close relation between both methods visible.

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