Discrete compactness of the approximation of Maxwell's system by a discontinuous Galerkin method

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We are interested in the discrete compactness property for a discontinuous Galerkin approximation of Maxwell's system on quite general tetrahedral meshes, which has already been studied for standard Galerkin approximation for a quite large family of edge elements on two and three dimensional domains. We here concentrate on the interior penalty method. The success of DG methods is today well recognized due to its flexibility in the choice of the approximation space, and is so well suited for $h-p$ adaptivity. Our proof of the discrete compactness property is based on the same property than the one for the standard Galerkin approximation, and the use of a decomposition of the discontinuous approximation space into a continuous one and its orthogonal for an appropriate inner product. The discrete Friedrichs inequality follows from this discrete compactness property and a contradiction argument. The convergence of the discrete eigenvalues to the continuous ones is deduced using the theory of collectively compact operators, which requires pointwise convergence of the sequence of the discrete operators. In our case, the collectively compact property is deduced from the discrete compactness property and the pointwise convergence is obtained by introducing mixed formulations and using a variant of the second Strang lemma. We restrict ourselves to the h-version of the method, without estimating the dependence of the constant with respect to the polynomial's degree. Numerical experiments are also presented. Since the null space of the operator is relatively large, we have used a discrete regularization method that allows us to work in the setting of positive definite matrices for the standard edge elements.

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