Vector potential formulation for 3D nonlinear finite element analysis of fully coupled electro-mechanical problems

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Ferroelectroelastic materials are widely used to design various types of smart systems, memory devices, MEMS, etc. These materials exhibit strong coupling between mechanical and electric fields and also manifest nonlinear behavior when they are subjected to high electromechanical loading. Using the standard formulation with scalar potential ($\phi : \mathbf{E} = -\nabla\phi \rightarrow \nabla \times \mathbf{E} \equiv 0$) as electric nodal variables in the nonlinear finite element analysis leads to a low convergence of iteration procedures. Therefore the formulation with vector potential ($\psi : \mathbf{D} = -\nabla \times \psi \rightarrow \nabla \cdot \mathbf{D} \equiv 0$) as electric nodal variables is developed for the analysis of such problems.

In coupled electromechanical finite element formulations, the electric vector potential ensures the positive definiteness of the stiffness matrix, in contrast to formulations based on the scalar electric potential. Solutions of boundary value problems using the scalar potential formulation lie on a saddle point, while solutions for the vector potential formulation exist at a global minimum in the space of the nodal degrees of freedom. This difference favors the electric vector potential especially for the solution of nonlinear electromechanical problems.

The solution of the boundary value problem for the vector potential involving the "curl-curl" operator is non-unique in the three-dimensional case. A Coulomb gauge condition in combination with a discrete set of Dirichlet boundary conditions enforces the uniqueness of vector potential solutions. Based on a spectral analysis of the stiffness matrix, the Coulomb gauge is compared with other gauge conditions. A penalized version of the weak vector potential formulation with the Coulomb gauge is proposed, implemented in the finite element program PANTOCRATOR and tested for some numerical examples in electrostatics, piezoelectricity and ferroelectroelasticity.

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