TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2006

Programme

Collection of abstracts

List of participants

Chemnitz, September 25 - 27, 2006

Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- finite element methods for Maxwell equations
- finite elements for advanced problems in solid mechanics
- (non-symmetric) saddle point problems
- inverse problems for PDEs

Invited Speakers:

Carsten Carstensen (Berlin)

Roland Griesse (Linz)

Ralf Hiptmair (Zürich)

Andy Wathen (Oxford)

Scientific Committee:

Th. Apel (München), G. Haase (Graz), B. Heinrich (Chemnitz), M. Jung (Dresden),
U. Langer (Linz), A. Meyer (Chemnitz), O. Steinbach (Graz)

WWW: `http://www.tu-chemnitz.de/mathematik/fem-symp/`

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Chemnitz, September 25 - 27, 2006

Programme for Monday, September 25, 2006

Start at 09:00

Chairman: A. Meyer

9:00 A. Meyer
Welcome

9:05 C. Carstensen
Convergence of Adaptive Finite Element Methods

9:55 Ch. Kreuzer
Convergence of Adaptive Finite Element Methods for nonlinear PDEs

10:20 A. Schroeder
A posteriori error estimates for contact problems

Tea and coffee break

Chairman: B. Heinrich

11:10 R. Schneider
Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

11:35 M. Grajewski
Towards r-h-adaptivity in FEM

12:00 V. Garanzha
Discrete curvatures and optimal quasi-isometric manifold parametrizations

Lunch

Chairman: C. Carstensen

13:45 R. Hiptmair
Edge elements and coercivity

14:35 D. Teleaga
Towards a fully space-time adaptive finite element method for magnetoquasistatics

14:55 G. Wimmer

Calculation of Transient Magnetic Fields Using Space-Time Adaptive Methods

15:20 E. Creusé

Discrete compactness of the approximation of Maxwell's system by a discontinuous Galerkin method

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Tea and coffee break

Chairman: R. Hiptmair

16:15 T. Hohage

Hardy space infinite elements for scattering and resonance problems

16:40 R. Klose

Pole condition: Numerical solution of Helmholtz-type scattering problems with far field evaluation

17:05 M. Roland

Simulations of the Turbulent Channel Flow at $Re_\tau = 180$ with Finite Element Variational Multiscale Methods

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short break

Chairman: Ch. Wieners

17:45 M. Braack

A stabilized finite element method for Navier-Stokes on anisotropic meshes

18:10 G. Matthies

A unified convergence analysis for the local projection stabilisation applied to the Oseen problem

18:35 Th. Apel

Non-conforming finite elements of arbitrary order for the Stokes problem on anisotropic meshes

---

20:00 Conference dinner

Programme for Tuesday, September 26, 2006

Start at 08:30

Chairman: M. Jung

08:30 A. Wathen
Preconditioning mixed finite elements for incompressible flow

09:20 U. Langer
A Boundary Element Based Finite Element Method on Polyhedral Meshes

09:45 S. Beuchler
Properties of sparse shape functions for p-FEM on triangles and tetrahedra

---

Tea and coffee break

Chairman: Th. Apel

10:40 P. Knobloch
Are stabilized methods a reliable tool for suppressing spurious oscillations?

11:05 S. Franz
Superconvergence analysis of Galerkin and Streamline Diffusion FEM for a singularly perturbed convection-diffusion problem with characteristic layers

11:30 M. Vlasak
Numerical solution of unstationary nonlinear convection-diffusion problems by higher order finite elements methods

11:55 T. Linß
Maximum-norm error analysis of a non-monotone FEM for a singularly perturbed reaction-diffusion problem in 1D

Lunch

13:50 Excursion

Programme for Wednesday, September 27, 2006

Start at 08:30

Chairman: A. Wathen

08:30 R. Griesse
Finite Elements for Magnetohydrodynamics and its Optimal Control

09:20 A. Rösch
On the finite element approximation of elliptic optimal control problems with Neumann boundary control

09:45 G. Winkler
Optimal Control in 3D Non-Convex Domains

10:05 K. Krumbiegel
A new iterative concept for solving linear-quadratic optimal control problems

Tea and coffee break

Chairman: U. Langer

11:00 C. Wieners
SQP-methods for incremental plasticity

11:25 P. Steinhorst
FEM for problems with piezoelectric material

11:50 A. Semenov
Vector potential formulation for 3D nonlinear finite element analysis of fully coupled electro-mechanical problems

12:10 M. Müller
A inf-sup stable local grid refinement with hanging nodes

12:35 A. Meyer
Closing

Lunch

Non-conforming finite elements of arbitrary order for the Stokes problem on anisotropic meshes

Thomas Apel[1] Gunar Matthies[2]

Non-conforming finite elements of arbitrary order for the Stokes problem on anisotropic meshes

Anisotropic meshes are characterized by elements with large or even asymptotically unbounded aspect ratio. Such meshes are known to be particularly effective for the resolution of directional features of the solution, like edge singularities and boundary layers.

We consider here the numerical solution of the Stokes problem in two-dimensional domains by non-conforming finite elements of higher order. The pressure is approximated by discontinuous, piecewise polynomials of order $r-1$. For approximating the velocity we discuss four non-conforming spaces of approximation order r.

For the stability of finite element methods for solving the Stokes problem it is necessary that the discrete spaces fulfil an inf-sup condition. All of the considered families fulfill this condition but only two of them have an inf-sup constant which is independent of the aspect ratio of the meshes. For these two families we show optimal error estimates on anisotropic meshes. The proof is restricted to rectangular triangulations with special properties.

References:

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Properties of sparse shape functions for p-FEM on triangles and tetrahedra

Sven Beuchler[1]

In this talk, the second order boundary value problem $-\nabla \cdot (\mathcal{A}(x, y)\nabla u) = f$ is discretized by the Finite Element Method using piecewise polynomial functions of degree p on a triangular/tetrahedral mesh.

On the reference element, we define several interior ansatz functions based on integrated Jacobi polynomials. If $\mathcal{A}$ is a constant function on each element and each triangle has straight edges, the element stiffness matrix has not more than $\mathcal{O}(p^d)$, $d = 2, 3$ nonzero matrix entries.

We investigate the growth of the condition number of the element stiffness matrix with respect to the weight of the integrated Jacobi polynomials.

This is a joint work with V. Pillwein (SFB F013, Linz) and J. Schöberl.

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A stabilized finite element method for Navier-Stokes on anisotropic meshes

Malte Braack[1] T. Richter[2]

The development of stabilized finite elements for fluid mechanics on anisotropic meshes is of major importance for computations of high speed flows with boundary layers. However, residual based stabilization techniques may be less suitable on anisotropic meshes, because the separation into the different spatial coordinates is not possible. In this work, we extend the local projection stabilization to anisotropic meshes. It turns out that an appropriate modification of the isotropic case for Stokes [3] and Navier-Stokes [4] leads to an optimal a priori estimate also on anisotropic meshes. The interpolation operator used is based on the work of Apel [1] and Becker [2]. The work being presented is the natural extension of [5, 6].

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Convergence of Adaptive Finite Element Methods

Carsten Carstensen[1]

Typical adaptive mesh-refining algorithms for first-oder (conforming) finite element methods consist of a sequence of the following steps:

$$\text{SOLVE} \Rightarrow \text{ESTIMATE} \Rightarrow \text{MARK} \Rightarrow \text{COARSEN/REFINE}$$

Unlike uniform mesh-refinements, the goal of adaptive finite element methods (AFEM) is to omit some basis functions in oder to save degrees of freedom and so reduce computational costs. Thus, the sequence of generated subspaces in an AFEM is on purpose *not* necessarily dense and hence the question of strong convergence has a priori *no* trivial affirmative answer.

This presentation gives a survey conditions on known convergence results for a class of adaptive finite element methods applied to a linear elliptic benchmark problem, non-standard finite element methods, to convex minimization problems such as an optimal design task.

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Discrete compactness of the approximation of Maxwell's system by a discontinuous Galerkin method

Emmanuel Creusé[1] Serge Nicaise[2]

We are interested in the discrete compactness property for a discontinuous Galerkin approximation of Maxwell's system on quite general tetrahedral meshes, which has already been studied for standard Galerkin approximation for a quite large family of edge elements on two and three dimensional domains. We here concentrate on the interior penalty method. The success of DG methods is today well recognized due to its flexibility in the choice of the approximation space, and is so well suited for $h-p$ adaptivity. Our proof of the discrete compactness property is based on the same property than the one for the standard Galerkin approximation, and the use of a decomposition of the discontinuous approximation space into a continuous one and its orthogonal for an appropriate inner product. The discrete Friedrichs inequality follows from this discrete compactness property and a contradiction argument. The convergence of the discrete eigenvalues to the continuous ones is deduced using the theory of collectively compact operators, which requires pointwise convergence of the sequence of the discrete operators. In our case, the collectively compact property is deduced from the discrete compactness property and the pointwise convergence is obtained by introducing mixed formulations and using a variant of the second Strang lemma. We restrict ourselves to the h-version of the method, without estimating the dependence of the constant with respect to the polynomial's degree. Numerical experiments are also presented. Since the null space of the operator is relatively large, we have used a discrete regularization method that allows us to work in the setting of positive definite matrices for the standard edge elements.

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Superconvergence analysis of Galerkin and Streamline Diffusion FEM for a singularly perturbed convection-diffusion problem with characteristic layers

Sebastian Franz[1] Torsten Linß[2]

For the model convection-diffusion equation

$$Lu := -\varepsilon\Delta u - bu_x + cu = f \text{ in } \Omega = (0,1)^2,$$

subject to Dirichlet boundary conditions

$$u = 0 \text{ on } \Gamma = \partial\Omega$$

with $b \geq \beta > 0$, we analyse the superconvergence properties of the Galerkin FEM and of the streamline-diffusion finite element method (SDFEM) using bilinear functions. The presence of the small perturbation parameter ε with $0 < \varepsilon \ll 1$ gives rise to an exponential layer of width $\mathcal{O}(\varepsilon)$ near the outflow boundary at $x = 0$ and to two parabolic layers of width $\mathcal{O}(\sqrt{\varepsilon})$ near the characteristic boundaries at $y = 0$ and $y = 1$. To resolve the layers we use appropriate Shishkin meshes. For the SDFEM we give an optimal choice for the stabilisation parameter δ, that ensures maximal stability in the induced streamline-diffusion norm without loosing the superconvergence property

$$|||u^N - u^I|||_{SD} \leq CN^{-2}\ln^2 N.$$

In the characteristic (or parabolic) boundary layer we are able to show that δ can be chosen of order $\delta = C\varepsilon^{-1/4}N^{-2}$ which is confirmed by numerical results. Using the superconvergence property we construct an enhanced numerical solution by postprocessing. The resulting numerical solution converges with almost second order accuracy.

References:

[1] `http://www.math.tu-dresden.de/~sfranz/papers.html`

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Discrete curvatures and optimal quasi-isometric manifold parameterizations

Vladimir Garanzha[1]

Optimal grid generation and adaptation can be considered in the framework of manifold parameterization.

Consider the problem of untangling and optimization of 2D unstructured grids via node movement. The basic idea is to have unified treatment for any spatial dimensions and for different types of elements. The concept of cell shape/size optimization is quite clear. For each grid cell the mapping of canonical target cell onto real cell is considered. Certain distortion measure of this mapping is minimized using variational methods.

One can construct polyhedral manifold by gluing all target elements together (generally it cannot be embedded as a surface in 3D space). Grid optimization in 2D is just a flattening of this manifold: one-to-one mapping of this manifold onto plane with minimal distortion. For general non-simplicial grids the result of gluing is not polyhedral manifold, but it is still Alexandrov manifold (manifold of bounded curvature) [4]. The "optimal" parameterization is the one which provides minimal distortion of the flattening.

The problem of adaptation of hybrid/unstructured grids is equivalent to the problem of finding bilipschitz mapping of above polyhedral manifold onto another manifold where (intrinsic) adaptation metric is defined.

Problem of existence of bilipschitz mappings between Alexandrov manifolds is considered in [4], [7]. The distortion of parameterization is estimated via positive and negative instrinsic curvatures of manifold [4], [7]. The case of arbitrary genus is considered in recent paper by Yu.D. Burago [6].

In [3] it was conjectured that instead of positive curvature parameterization distortion should be estimated via certain constant from isoparametric inequalities - called "depth of pockets" in [3]. Numerical experiments and grid refinement studies results are in good agreement with suggested estimates.

In 3D the numerical treatment is similar to 2D, but theoretical understanding is still lacking. The polyhedral manifolds are defined similarly by gluing simplices, but theoretical results about existence of global bilipschitz parameterizations are not available. The concept of discrete curvatures in multidimensional case is still not very well defined. In practice manifolds are constructed by gluing together target hexahedra (as a rule chosen as cubes), prisms, pyramids and simplices.

Within above concept there is no difference between structured (block structured) mapped grids and unstructured/hybrid grids. As a result of this unified treatment the same functional [2], [3] satisfying polyconvexity conditions [1] is used for any dimensions and any types of elements [5]. The minimization problem for this functional is well posed. Recently the rigorous convergence results for the iterative minimization method were

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obtained [8].

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Towards r-h-adaptivity in FEM

Matthias Grajewski[1] Stefan Turek[2]

Error control and adaptive algorithms are essential ingredients for accurate and fast FEM simulation. To obtain adapted computational grids, in many FEM packages h-adaptivity, i.e. the selective refinement of single elements, is employed. However, this kind of adaptivity bears disadvantages. Element-based h-adaptivity leads to highly unstructured grids which decrease the numerical efficiency of an FEM code as these grids require many unaligned and costly memory accesses during the program run. In contrast to this, r-adaptivity preserves the topology of the grid and thus is a natural candidate for an alternative adaptivity technique which may overcome the aforementioned difficulties. Moreover, in contrast to h-adaptivity, r-adaptivity allows to adjust the grid to curves in the computational domain like i.e. interfaces with superior accuracy, as in contrast to h-adaptivity the orientation of the grid cells can be aligned. Besides of the presentation of a new deformation technique, the emphasis in the talk is put on the application of the deformation method as tool for grid adaptation in the context of (goal-oriented) error control. We present prototypical test problems as well as comparisons with other mesh adaptation techniques. Lastly, we sketch suitable combinations of both r- and h-adaptivity techniques which feature the advantages of both types of adaptation techniques.

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Finite Elements for Magnetohydrodynamics and its Optimal Control

Roland Griesse[1] Marco Discacciati[2]

Magnetohydrodynamics, or MHD, deals with the mutual interaction of electrically conducting fluids and magnetic fields. In particular, the magnetic fields interact with the electric currents in the fluid and exert a Lorentz force. This feature renders it so phenomenally attractive for exploitation especially in processes involving liquid metals, and in crystal growth.

We consider the problem of stationary incompressible MHD, and a stable and conforming discretization by finite elements. In addition, an optimal control problem, its necessary optimality conditions and numerical methods for its solution will be presented.

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Edge elements and coercivity

Ralf Hiptmair[1]

Many variational problems from computational electromagnetism are set in Sobolev spaces of differential 1-forms and feature non-elliptic sesquilinear forms. In order to establish a rigorous convergence theory of Galerkin discretizations the inherent coercivity of the variational problems has to be exploited. This can be accomplished when using finite elements or boundary elements that can be viewed as discrete differential forms. Key is the commuting diagram property of nodal interpolation operators that underlies discrete counterparts of Hodge-type decompositions.

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Hardy space infinite elements for scattering and resonance problems

Thorsten Hohage[1] Lothar Nannen[2]

We study the solution of time-harmonic wave equations in unbounded domains. The unbounded domain is split into a bounded interior domain and an exterior domain, which is the complement of a ball. We propose a new class of tensor product infinite elements for the exterior domain which lead to super-algebraic convergence with respect to the number of degrees of freedom. The radial tensor product factors of the local element matrices have a simple tridiagonal structure.

To derive these infinite elements, we use a Möbius-Laplace transform along a family of rays connecting the coupling boundary to infinity. By virtue of the pole condition, functions satisfying a radiation condition are mapped to Hardy-space functions, i.e. L^2-boundary values of holomorphic functions on the unit disc. This leads to a complex symmetric variational formulation, which is discretized by a Galerkin method using trigonometric polynomials of finite degree.

Hardy space infinite elements are particularly attractive for computing resonances since, as opposed to classical infinite elements, boundary elements and local transparent boundary conditions they preserve the eigenvalue structure of the problem. We demonstrate their performance in a number of numerical experiments.

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Pole condition: Numerical solution of Helmholtz-type scattering problems with far field evaluation

Roland Klose[1] Frank Schmidt[2] Achim Schädle[3]

We consider electromagnetic scattering problems, modeled by the Helmholtz equation on unbounded domains. A central task is the numerical solution of the exterior problem and its coupling to the interior problem. In this talk we present a numerical realization of the pole-condition method, a new approach to the solution of the exterior problem. The method provides a representation formula for the far field and is applicable to certain types of inhomogeneous exterior domains. The pole condition is coupled with a finite element method for the interior domain. Numerical examples illustrate the convergence of the method.

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[2] F. Schmidt: Solution of Interior-Exterior Helmholtz-Type Problems Based on the Pole Condition Concept – Theory and Algorithms. *Habilitation thesis, Free University, Berlin (2002).*

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Are stabilized methods a reliable tool for suppressing spurious oscillations?

Petr Knobloch[1] Volker John[2]

We consider the application of the finite element method to the numerical solution of the scalar convection–diffusion equation

$$-\varepsilon\,\Delta u + \boldsymbol{b}\cdot\nabla u = f \quad \text{in } \Omega, \qquad u = u_b \quad \text{on } \Gamma^D, \qquad \varepsilon\,\frac{\partial u}{\partial \boldsymbol{n}} = g \quad \text{on } \Gamma^N. \tag{1}$$

Here Ω is a bounded two–dimensional domain with a polygonal boundary $\partial\Omega$, Γ^D and Γ^N are disjoint and relatively open subsets of $\partial\Omega$ satisfying $\text{meas}_1(\Gamma^D) > 0$ and $\overline{\Gamma^D \cup \Gamma^N} = \partial\Omega$, $\boldsymbol{n}$ is the outward unit normal vector to $\partial\Omega$, f is a given outer source of the unknown scalar quantity u, $\varepsilon > 0$ is the constant diffusivity, $\boldsymbol{b}$ is the flow velocity, and u_b, g are given functions.

It is well known that the classical Galerkin finite element discretization of (1) is inappropriate in the convection–dominated regime (i.e., if $\varepsilon \ll \|\boldsymbol{b}\|$) since the discrete solution is typically globally polluted by spurious oscillations. During the last three decades, an extensive research has been devoted to the development of methods which diminish spurious oscillations in the discrete solutions of (1) and the aim of the talk is to review some of the most important approaches and to compare them by means of both numerical tests and theoretical considerations. In particular, we shall discuss the quality of the discrete solutions (spurious oscillations, smearing of inner and boundary layers), dependence of the methods on parameters, triangulations and data, and also the cost needed to compute the discrete solutions.

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Convergence of Adaptive Finite Element Methods for nonlinear PDEs

Christian Kreuzer[1] Lars Diening[2]

We consider the homogeneous Dirichlet Problem for the p-Laplacian, $p \in (1, \infty)$, embedded in the more general theory of Orlicz-spaces. We propose an adaptive algorithm with continuous piecewise affine finite elements and prove an error reduction rate of approximate solutions to the exact one. We improve the a posteriori estimations for quasi-norms and generalize the error reduction property of the linear case to an energy reduction property in the nonlinear case. For adaptive refinement we use a marking strategy incorporating error estimators and oscillation. Thus we obtain a reduction of energy differences. Since these are proportional to the error measured in quasi-norms we get a strict error reduction in each step. This in turn implies convergence.

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A new iterative concept for solving linear-quadratic optimal control problems

Klaus Krumbiegel[1]

We consider a linear-quadratic optimal control problem governed by an elliptic partial differential equation with pointwise control constraints, where the PDEs are solved by a finite element method. Such problems are usually treated by multilevel iterative methods. We present a new error estimation technique for a current iterate with respect to the solution of the discretized problem. These error estimates can be used as stopping criteria for iterative methods. The presented theory is illustrated by numerical examples, where a primal-dual active set strategy and a CG-algorithm as iterative methods are used. The final aim is to find a balance between the different errors (including the discretization error) of the over-all solution process.

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A Boundary Element Based Finite Element Method on Polyhedral Meshes

Ulrich Langer[1] David Pusch[2]

We derive and analyze new boundary element based finite element discretizations of diffusion-type equations on polyhedral meshes. These approximations leads to large-scale sparse linear systems which can efficiently be solved by Algebraic Multigrid Methods.

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Maximum-norm error analysis of a non-monotone FEM for a singularly perturbed reaction-diffusion problem in 1D

Torsten Linß[1]

We consider a non-monotone FEM discretization of a singularly perturbed one-dimensional reaction-diffusion problem whos solution exhibits strong layers. The method is shown to be maximum-norm stable although it is not inverse monotone. Both a priori and a posteriori error bounds in the maximum norm are derived. The a priori result allows to deduce immediately the uniform convergence of various layer-adapted meshes proposed in the literature, while the a posteriori results may be used for adaptive regridding. Numerical experiments complement our theoretical results.

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A unified convergence analysis for the local projection stabilisation applied to the Oseen problem

Gunar Matthies[1] Piotr Skrzypacz[2] Lutz Tobiska[3]

The discretisation of the Oseen problem by finite element methods suffers in general from two reasons. First, the discrete inf-sup or Babuvska-Brezzi condition is violated. Second, spurious oscillations occur due to the dominating convection. One way to overcome both difficulties is the use of local projection techniques.

Originally proposed by Becker and Braack for the Stokes problem, it was extended by them to the transport equation. A convergence analysis for first and second order discretisations on quadrilaterals was recently given by Braack and Burman.

We will consider the local projection method for a large class of equal-order approximations of the Oseen problem. When defining the local projection in the right way, we can show that the stabilised method converges for arbitrary polynomial degree with optimal order. This result holds true on simplices, quadrilaterals, and hexahedra.

On simplices, the spectral equivalence of the stabilising terms of the local projection method and the subgrid modeling introduced by Guermond will be shown. This makes the close relation between both methods visible.

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A inf-sup stable local grid refinement with hanging nodes

Markus Müller[1]

An inf-sup stable local grid refinement with hanging nodes for a spherical 3D convection code is introduced. The discretization uses continuous bilinear elements for velocity and pressure with different mesh-resolution. A proof of inf-sup stability is given using a macro-element technique.

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On the finite element approximation of elliptic optimal control problems with Neumann boundary control

Arnd Rösch[1] Mariano Mateos[2]

A Neumann boundary control problem for a linear-quadratic elliptic optimal control problem in a convex and polygonal domain is investigated. The main goal is to show an optimal approximation order for discretized problems after a postprocessing process. It turns out that two saturation processes occur: The regularity of the boundary data of the adjoint is limited if the largest angle of the polygon is at least $2\pi/3$. For piecewise linear finite elements, the theory cannot deliver optimal approximation rates for convex domains. We will derive error estimates of order h^σ with $\sigma \in [3/2, 2]$ depending on the lagest angle and properties of the finite elements. Moreover, we will investigate also the case of domains with a reentrant corner. Here, we obtain error estimates of order h^σ with $\sigma \in [1, 3/2]$ Finally, numerical tests illustrates the theoretical results.

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Simulations of the Turbulent Channel Flow at $Re_\tau = 180$ with Finite Element Variational Multiscale Methods

Michael Roland[1] Prof. Volker John[2]

Projection based variational multiscale (VMS) methods coupled with higher order finite element methods are studied in simulations of the turbulent channel flow problem at $Re_\tau = 180$. For comparison, the Smagorinsky LES model with van Driest damping is included into the study. The simulations are performed on rather coarse grids. The evaluation of the results concentrates on the mean velocity profile.

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Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

Rene Schneider[1] Peter Jimack[2]

Efficient numerical approximation of solution features like boundary or interior layers by means of the finite element method requires the use of layer adapted meshes. Anisotropic meshes, like for example Shishkin meshes, allow the most efficient approximation of these highly anisotropic solution features. However, application of this approach relies on empha priori analysis on the thickness, position and stretching direction of the layers. If it is impossible to obtain this information empha priori, as it is often the case for problems with interior layers of unknown position for example, automatic mesh adaption based on empha posteriori error estimates or error indicators is essential in order to obtain efficient numerical approximations.

Historically the majority of work on automatic mesh adaption used locally uniform refinement, splitting each element into smaller elements of similar shape. This procedure is clearly not suitable to produce anisotropically refined meshes. The resulting meshes are over-refined in at least one spatial direction, rendering the approach far less efficient than that of the anisotropic meshes based on empha priori analysis.

Automatic anisotropic mesh adaption is an area of active research. Here we present a new approach to this problem, based upon using not only an empha posteriori error estimate to guide the mesh refinement, but its sensitivities with respect to the positions of the nodes in the mesh as well. Once this sensitivity information is available, techniques from mathematical optimisation can be used to minimise the estimated error by moving the positions of the nodes in the mesh appropriately.

The basic idea of minimising an error estimate is of course not new, but the approach taken to realise it is. The discrete adjoint technique is utilised to evaluate the sensitivities of an error estimate, reducing the cost of this evaluation to solving one additional equation system. This approach is crucial to make gradient based optimisation techniques, such as BFGS-type schemes, applicable.

References:

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A posteriori error estimates for contact problems

Andreas Schroeder[1]

A general concept for obtaining a posteriori error estimates for contact problems is presented. The approach consists in treating appropiate saddle point formulations and making use of a posteriori error estimates for variational equations. According to this concept, a residual based error estimator is developed for Signorini-type problems, obstacle problems and contact problems with friction. Eventually, several numerical results confirm the reliability of the estimates and their applicability in respect of *h*- and *hp*-adaptive finite element methods.

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Vector potential formulation for 3D nonlinear finite element analysis of fully coupled electro-mechanical problems

Artem Semenov[1] A. Liskowsky[2] H. Balke[3]

Ferroelectroelastic materials are widely used to design various types of smart systems, memory devices, MEMS, etc. These materials exhibit strong coupling between mechanical and electric fields and also manifest nonlinear behavior when they are subjected to high electromechanical loading. Using the standard formulation with scalar potential ($\phi:\ \mathbf{E} = -\nabla\phi \rightarrow \nabla \times \mathbf{E} \equiv 0$) as electric nodal variables in the nonlinear finite element analysis leads to a low convergence of iteration procedures. Therefore the formulation with vector potential ($\psi:\ \mathbf{D} = -\nabla \times \psi \rightarrow \nabla \cdot \mathbf{D} \equiv 0$) as electric nodal variables is developed for the analysis of such problems.

In coupled electromechanical finite element formulations, the electric vector potential ensures the positive definiteness of the stiffness matrix, in contrast to formulations based on the scalar electric potential. Solutions of boundary value problems using the scalar potential formulation lie on a saddle point, while solutions for the vector potential formulation exist at a global minimum in the space of the nodal degrees of freedom. This difference favors the electric vector potential especially for the solution of nonlinear electromechanical problems.

The solution of the boundary value problem for the vector potential involving the "curl-curl" operator is non-unique in the three-dimensional case. A Coulomb gauge condition in combination with a discrete set of Dirichlet boundary conditions enforces the uniqueness of vector potential solutions. Based on a spectral analysis of the stiffness matrix, the Coulomb gauge is compared with other gauge conditions. A penalized version of the weak vector potential formulation with the Coulomb gauge is proposed, implemented in the finite element program PANTOCRATOR and tested for some numerical examples in electrostatics, piezoelectricity and ferroelectroelasticity.

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FEM for problems with piezoelectric material

Peter Steinhorst[1] Arnd Meyer[2]

Piezoelectricity describes the coupling of deformation and electric field in special materials. The talk introduces a numerical approach to simulate piezoelectric material behaviour by using Finite Elements. We use the method of adaptive mixed FEM for handling the resulting coupled differential equations.

We give an introduction to the linear model, followed by a briefly description of the used solver (Bramble–Pasciak–CG) with preconditioner and ideas for error estimation needed by the refinement strategy. First computational results using an experimental program will be shown. Beside simple test examples, we investigate a special problem including a crack in some details. Known analytical solutions in special cases allow a partial validation of the FEM program. Finally, we discuss occuring numerical instabilities.

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Towards a fully space-time adaptive finite element method for magnetoquasistatics

Delia Teleaga[1] Jens Lang[2]

This paper reports on our current work on fully space-time adaptive magnetic field computation.

We describe a Whitney finite element method (WFEM) for solving a magnetoquasistatic formulation of Maxwell's equations on unstructured 3D tetrahedral grids, using the software package KARDOS. High order spatial discretization is achieved by employing the hierarchical tetrahedral H(curl)-conforming element proposed by Ainsworth and Coyle. For the time discretization we use linearly implicit one-step Rosenbrock methods up to 4th order accurate in time. To control the adaptive mesh refinement we extend the hierarchical error estimator proposed by Beck, Hiptmair and Wohlmuth to Rosenbrock methods.

Finally we present numerical results for the eddy current benchmark problem TEAM 7.

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Numerical solution of unstationary nonlinear convection-diffusion problems by higher order finite elements methods

Miloslav Vlasak[1] Vit Dolejsi[2]

We deal with the numerical solution of a scalar nonstationary nonlinear convection-diffusion equation. We employ a combination of the discontinuous Galerkin finite element method for the space semi-discretization and the k-step backward difference formula for the time discretization. The diffusive and stabilization terms are treated implicitly whereas the nonlinear convective term is treated by an higher order explicit extrapolation, which leads to the necessity to solve only linear algebraic problem at each time step. We analyse this scheme and derive a priori asymptotic error estimations in the discrete $L^\infty(L^2)$–norm and $L^2(H^1)$–seminorm with respect to the mesh size h and time step τ for $k = 2, 3$. Several numerical examples verifying the theoretical results are presented.

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Preconditioning mixed finite elements for incompressible flow

Andy Wathen[1] Howard Elman[2] David Silvester[3]

The Stokes and incompressible Navier-Stokes problems present interesting and important examples with repectively symmetric and non-symmetric saddle-point structure. In this talk we will discuss block preconditioning and iterative solution of the discrete linear(ised) systems which arise.

In particular we will describe and demonstrate general preconditioned iterative approaches which yield highly effective and efficient solvers for large scale applications.

References:

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SQP-methods for incremental plasticity

Christian Wieners[1]

The standard procedure in computational plasticity reformulates the incremental step into a minimization problem or an equivalent nonlinear variational problem, where the nonlinearity results from the projection onto the set of admissible stresses. Numerically, the incremental problem is solved by a semi-smooth Newton method, where the consistent tangent is chosen from the multi-valued derivative of the projection.

This standard procedure is compared with an realization of the SQP method, where the Newton method is replace by a sequence of quadratic minimization problems with linearized constraints (which are solved approximately by a small number of semi-smooth Newton steps). We show that this optimization approach is more robust and more efficient in difficult cases, e. g., near to the limit load in perfect plasticity.

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Calculation of Transient Magnetic Fields Using Space-Time Adaptive Methods

Georg Wimmer[1] Thorsten Steinmetz[2] Daniel Weida[3]

The discretization of transient magneto-dynamic field problems with geometric discretization schemes like the Finite Integration Technique or the Finite-Element Method based on Whitney form functions results in nonlinear differential-algebraic systems of equations of index 1. The efficient transient computation of magnetic fields in induced eddy current layers as well as in regions of ferromagnetic saturation that may appear or vanish depending on the external current excitation requires the adaptation of the finite element mesh at each time step. Hence, a combination of error controlled spatial adaptivity and an error controlled implicit Runge-Kutta scheme is used to reduce the number of unknowns for the algebraic problems effectively and to avoid unnecessary fine grid resolutions both in space and time. Prolongation and restriction operators are introduced to map the solution of the last time step to the actual time step.

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Optimal Control in 3D Non-Convex Domains

Gunter Winkler[1]

Solutions of partial differential equations in non-convex domains can have corner and edge singularities. The talk shows the influence on the rate of convergence for a simple optimal control problem. A finite element method with a piecewise linear approximation of the state and a piecewise constant approximation of the control is used. Results on quasi uniform and a-priori graded meshes are shown.

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