DFG-Sonderforschungsbereich 393
"Parallele Numerische Simulation für Physik und Kontinuumsmechanik"

and

Fakultät für Mathematik, Technische Universität Chemnitz

NUMERISCHE
PARALLELE
SIMULATION
SFB 393

Chemnitz FEM-Symposium 2004

Programme

Collection of abstracts

and

List of participants

Chemnitz, September 20 - 22, 2004

Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- error estimation,
- fem with stochastic data,
- singular complement methods,
- sparse approximation, H-matrices.

Invited Speakers:

Patrick Ciarlet (Paris)

Wolfgang Hackbusch (Leipzig)

Hermann G. Matthies (Braunschweig)

Rolf Rannacher (Heidelberg)

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Programme for Monday, September 20, 2004

Start at 10:00

Chairman: A. Meyer

10:00 – 10:05	A. Meyer Welcome
10:05 – 10:50	R. Rannacher Adaptive finite elements for eigenvalue problems
11:00 – 11:15	S. Oestmann A residual error estimator for the 3-d FEM-BEM
11:20 – 11:35	R. Schneider Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

short break

Chairman: H. Blum

11:50 – 12:05	J. Kienesberger Computational plasticity: An efficient solution algorithm and its first implementation for h and p elastoplastic interface adaptivity
12:10 – 12:35	Y. Kondratyuk Uniformly Convergent Adaptive Finite Element Algorithm for a Singularly Perturbed Reaction-Diffusion Equation
12:40 – 13:05	M. Grajewski Concepts of patchwise mesh refinement in the context of DWR

Lunch

Chairman: R. Rannacher

14:30 – 14:45	J. Schöberl Primal-Dual a posteriori error estimates for high order finite elements (Part I)
14:50 – 15:05	A. Becirovic Primal-Dual a posteriori error estimates for high order finite elements (Part II)
15:10 – 15:35	F.T. Suttmeier Consistent error estimation of FE-approximations of variational inequalities

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Tea and coffee break

Chairman: P. Benner

16:00 – 16:45	W. Hackbusch Introduction to Hierarchical Matrices
17:00 – 17:25	B.N. Khoromskij Data-Sparse Schur Complement Domain Decomposition
17:30 – 17:45	U. Baur Factorized solution of Lyapunov equations using H-matrix arithmetic
17:50 – 18:15	D. Kressner Arnoldi-type Algorithms for Solving Algebraic Riccati Equations

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Dinner

20:00	Wine reception

Programme for Tuesday, September 21, 2004

Start at 09:00

Chairman: W. Hackbusch

09:00	–	09:45	H.G. Matthies Numerical simulation of stochastic elliptic partial differential equations
10:00	–	10:25	S. Kube The Stochastic Finite Element Method (SFEM) for Advection-Diffusion Equations
10:30	–	10:55	J. Gwinner FEM-Discretization for unilateral problems with random data

short break

Chairman: H.G. Matthies

11:15	–	11:40	M. Braack A multiscale method towards turbulent flow based on local projection stabilization
11:45	–	12:10	G. Lube Residual-based stabilized higher-order FEM for advection-dominated problems
12:15	–	12:40	M. Köster An Optimal-Order Multigrid Method for Quadratic Conforming Finite Elements

short break

Chairman: G. Lube

13:00 – 13:25	G. Matthies Inf-sup stable non-conforming finite elements of arbitrary order on triangles
13:30 – 13:55	S. Beuchler An inexact Domain Decomposition preconditioner for p-FEM in 2D using methods of multi-resolution analysis

Lunch

15:00 Excursion

Dinner

Programme for Wednesday, September 22, 2004

Start at 09:00

Chairman: Th. Apel

09:00 – 09:45	P. Ciarlet	The Singular Complement Method for solving Maxwell equations
10:00 – 10:25	S. Labrunie	Poisson's and Maxwell's equations in axisymmetric domains: the Fourier-Singular Complement Method
10:30 – 10:55	C. Pester	Finite element methods for the computation of 3D corner singularities

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short break

Chairman: B. Heinrich

11.15 – 11:40	M. Maischak	Numerical simulation of electrostatic spray painting
11:45 – 12:10	G. Of	Efficient Iterative Solvers for Boundary Element Tearing and Interconnecting Methods
12:15 – 12:40	L. Angermann	Analysis of a Petrov-Galerkin FEM for a problem arising in option valuation
12:45 – 13:10	I. Greff	Some box schemes on rectangular meshes
13:15 – 13:25	R. Iankov	Application of generelized finite element formulation to metal forming technological processes
13:30 – 13:35	A. Meyer	Closing

Lunch

Analysis of a Petrov-Galerkin FEM for a problem arising in option valuation

Lutz Angermann[1]

An analysis of a numerical method for a degenerate PDE, called the Black-Scholes equation, governing option pricing is given. The method is based on the vertical method of lines, where the spatial discretization is formulated as a Petrov-Galerkin finite element method with each basis function of the trial space being determined by local two-point boundary value problems.

The stability and an error bound of the solution of the fully discretized system are established.

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Factorized solution of Lyapunov equations using $\mathcal{H}$-matrix arithmetic

Ulrike Baur[1] Peter Benner[2]

We investigate the numerical solution of the Lyapunov equation

$$AX + XA^T + BB^T = 0 \tag{1}$$

for given $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$.
We assume that A is stable and $n \gg m$, where A comes from a FEM or BEM discretization of some elliptic differential operator. The standard methods for (1) are too expensive in terms of computation time ($\mathcal{O}(n^3)$) and memory requirements ($\mathcal{O}(n^2)$).
We discuss approaches based on an $\mathcal{H}$-matrix implementation of the sign function method which allows to reduce the computational cost to
$\mathcal{O}(k^2\, n\log(n)^3)$ and the memory requirements to $\mathcal{O}(k\, n\log(n))$. Here k denotes the maximal block rank in the $\mathcal{H}$-matrix approximation.
Rather than approximating X itself by an $\mathcal{H}$-matrix, we propose to compute a factorized solution. Often the solution factors are what is really needed for applications in control and systems theory, in particular in model reduction algorithms.
The proposed method is based on a partitioned Newton iteration for the matrix sign function, where one part of the iteration uses formatted arithmetic for the hierarchical matrices while the other part converges to an approximate full-rank factor of the solution.

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Primal-Dual a posteriori error estimates for high order finite elements (Part II)

Almedin Becirovic[1]

[1] Johannes Kepler University Linz, Computational Mathematics, Altenbergerstrasse 69, Austria, ab@jku.at

Multi-level pre-conditioners for the p-Version of the Finite Element Method

Sven Beuchler[1]

In this talk, we discretize a uniformly elliptic second order boundary value problem in 2D by the p-version of the finite element method. An inexact Dirichlet-Dirichlet domain decomposition pre-conditioner for the system of linear algebraic equations is investigated. The ingredients of such a pre-conditioner are an pre-conditioner for the Schur complement, an pre-conditioner for the sub-domains and an extension operator operating from the edges of the elements into their interior. Using methods of multi-resolution analysis, we propose a new method in order to compute the extension efficiently. This type of extension is optimal, i.e. the $H^1(\Omega)$-norm of the extended function is bounded by the $H^{0.5}(\partial\Omega)$-norm of the given function. Numerical experiments show the optimal performance of the presented extension.

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A multiscale method towards turbulent flow based on local projection stabilization

Malte Braack[1] Erik Burman[2]

We propose to apply the recently introduced local projection stabilization to the numerical computation of the Navier-Stokes equation at high Reynolds number. The discretization is done by nested finite element spaces. We show how this method may be cast in the framework of variational multiscale methods. We indicate what modelling assumptions must be made to use the method for large eddy simulations. Using a priori error estimation techniques we prove the convergence of the method in the case of a linearized model problem. The a priori estimates are independent of the local Peclet number and give a sufficient condition for the size of the subgrid viscosity parameter in order to insure optimality of the approximation when the exact solution is smooth.

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The Singular Complement Method for solving Maxwell equations

Patrick Ciarlet[1]

The *Singular Complement Method* (SCM) was originally developed to solve Maxwell equations in bounded non-convex domains, with a piecewise smooth boundary. When the domain is either convex or with a smooth boundary, all components of the electromagnetic field belong to $H^1(\Omega)$. This feature guarantees the convergence of Finite Element techniques, based on the continuous P_1 or P_2 Lagrange Finite Element approximation. But when there are *geometrical singularities*, such as reentrant corners in 2D, or reentrant edges and/or non-convex vertices in 3D, the components of the electromagnetic field do not belong in general to $H^1(\Omega)$. What is more, the subspace of all $\mathbf{H}^1(\Omega)$-smooth fields is closed in the space of admissible electromagnetic fields. Since the continuous Lagrange FE approximation always belongs to the regular subspace, this prevents convergence.
To address this difficulty, one possibility is to enrich the space of test-fields by *singular fields*, which reflect accurately the behaviour of the electromagnetic field in the neighborhood of the geometrical singularities. Interestingly, this method works for both the static and time-dependent Maxwell equations.
Since the singular behavior of the electromagnetic field is related to the behavior of singular solutions of the Poisson problem, we will first recall some results concerning this scalar problem. Then, we shall explain how they can be used to build the SCM for solving Maxwell equations. Numerical results will be presented, and comparisons with other numerical approximations schemes for the electromagnetic field will be discussed.

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Concepts of patchwise mesh refinement in the context of DWR

M. Grajewski[1] S. Turek[2]

The performance of the new FEM-package FEAST in connection with the hierachical solver concept ScaRC as generalised multigrid/domain decomposition approach is based upon special data structures requiring meshes consisting of (many) generalized tensor product grids (makros). This has impact on the adaptive refinement procedure of such grids, as the special grid structure and the underlying data structures do not allow the elementwise refinement commonly used in existing adaptive codes. We introduce concepts of corresponding grid refinement strategies which allow adaptive refinement preserving the high speed of FEAST. The refinement strategies rely on three main techniques:

- adjust the coarse grid level by varying the number of makros
- refine the grid adaptively in a patchwise manner allowing hanging makro nodes
- apply local grid deformation if the error source is highly located.

The criteria for mesh refinement stem from error estimation by the DWR (dual weighted residual based)-method. We address the problem of the reliability of such estimation by numerical examples and present approaches to guarantee reliable goal-oriented error estimation.

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Some box schemes on rectangular meshes

I. Greff[1]

We are interested in the design of numerical schemes which combine the advantages of the finite volume and of the finite element mixed methods. These schemes generalize Keller's box scheme on 2D domain, [3]. Recently, some work has been done on box schemes on triangular meshes [1, 2]. Here we introduce different box schemes for the 2D Poisson problem on a rectangular domain with rectangular meshes. The box scheme is defined by the following formulation: find $(p_h, u_h) \in X_{1,h} \times M_{1,h}$ such that

$$\begin{array}{l} (\nabla \cdot p_h + f, v_h)_{0,h} = 0 \ , \quad \forall v_h \in M_{2,h} \\ (p_h - \nabla u_h, q_h)_{0,h} = 0 \ , \quad \forall q_h \in X_{2,h} \ . \end{array} \tag{2}$$

This formulation is of mixed Petrov-Galerkin type in the sense that the trial and test spaces are different. The trial spaces $X_{1,h}$, $M_{1,h}$ are non-conforming, whereas the test spaces $X_{2,h}$, $M_{2,h}$ are of discontinuous Galerkin type. The fundamental difficulty is to find trial spaces and test spaces which satisfy the relation

$$\dim X_{1,h} + \dim M_{1,h} = \dim X_{2,h} + \dim M_{2,h}.$$

We present several box schemes using the lowest order Raviart-Thomas space on rectangles to approximate the flux p and the non-conforming Q^1 space or the Rannacher-Turek space [4] to approximate u. We prove existence and uniqueness of the solution of problem (1), its equivalence to a non-conforming scheme in u_h and a local reconstruction formula of p_h (in function of u_h and the data f) and give *a priori* error estimates. The method is demonstrated by some numerical results.

References

[1] B. Courbet, J-P. Croisille, Finite volume-box schemes on triangular meshes, M2AN, 32,5,631-649, (1998).

[2] J-P. Croisille, I. Greff, Some box-schemes for elliptic problems, Numer. Meth. Partial Diff. Equations,18, 355-373, (2002).

[3] H.B. Keller, A new difference scheme for parabolic problems, Num. sol. of PDE, II, B. Hubbard ed., Acad. press, 1971, 327-350.

[4] R. Rannacher, S. Turek, Simple Non-conforming Quadrilateral Stokes Element, Numer. Meth. Partial Diff. Equations, 8, 97-111, (1992).

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FEM-Discretization for unilateral problems with random data

J. Gwinner[1]

As a model problem we treat the following unilateral problem: Find a function u in a bounded domain $D \subset \mathcal{R}^d$ $(d = 2, 3)$ with its boundary ∂D such that

$$\begin{cases} -\nabla.(S\,\nabla u) + u = Rf & \text{in } D \\ p = -S\,\nabla u \cdot \nu & \text{on } \partial D \\ u \leq Qg, p \geq 0, (u - Qg)p = 0 & \text{on } \partial D \end{cases}$$

Here f and g are given data in the domain, respectively on its boundary, randomness enters in the problem by the given real-valued random variables Q, R and S. In contrast to earlier work (appeared in Stochastic Analysis and Applications, vol. 19, 2000) we admit a random obstacle in decoupled form $Q\,g$. These problems lead to a class of random variational inequalities for which a theory of combined probabilistic – deterministic discretization is developped that includes nonconforming approximation of the unilateral constraints. Without any regularity assumptions on the solution, norm convergence of the full approximation procedure is established. In the application to the model problem, Galerkin discretization is realized by finite element approximation.

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Introduction to Hierarchical Matrices

Wolfgang Hackbusch[1]

We introduce the construction of hierarchical matrices, the performance of matrix operations and their cost and recompression techniques. We give applications to FEM, BEM, functions of matrices and problems to high spatial dimension.

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Application of the generelized plain finite element formulation to metal forming technological processes.

Dr. R. Iankov[1]

A generalized plain strain finite element formulation is presented for numerical simulation of metal forming technological processes. A multi-pass profile rolling of wire and rod rolling are considered. The thermo coupled rigid-plastic and elastic-plastic material models are assumed for describing the plastic deformation. The process of plastic deformation, strain field distribution and temperature field due to the different profile of groves are investigated. The coupled effect like grain size distribution is included into the model. The boundary value problems for wire drawing and rod rolling simulation are solved based on MSC.MARC software. The user subroutine technique is developed for extension the material model and including the some coupled effects. The results of numerical simulation are presented.

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Data-Sparse Schur Complement Domain Decomposition

B.N. Khoromskij[1] W. Hackbusch[2] R. Kriemann[3]

A class of hierarchical matrices ($\mathcal{H}$-matrices) allows the data-sparse approximation to integral and more general nonlocal operators (say, the Poincaré-Steklov operators) with almost linear cost. We consider the $\mathcal{H}$-matrix-based approximation to the Schur complement on the interface [2] corresponding to the FEM discretisation of an elliptic operator $\mathcal{L}$ with jumping coefficients in $\mathbb{R}^d$. As with the standard Schur complement domain decomposition methods, we split the elliptic inverse $\mathcal{L}^{-1}$ as a sum of local inverses associated with subdomains (this can be implemented in parallel), and the corresponding Poincaré-Steklov operator on the interface.

Using the hierarchical formats based on either standard or weakened admissibility criteria (cf. [1]) we elaborate the *approximate Schur complement inverse* in an explicit form that is proved to have a linear-logarithmic cost $O(N_\Gamma \log^q N_\Gamma)$, where N_Γ is the number of degrees of freedom on the interface. The $\mathcal{H}$-matrix-based preconditioner can be also applied.

Numerical tests confirm the almost linear cost of our *parallel direct Schur complement method*. In particular, we consider examples with brick and mortar structure of the coefficients.

References

[1] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Hierarchical Matrices Based on Weak Admissibility Criterion.* Preprint MPI MIS no. 2, Leipzig, 2003; Computing (to appear).

[2] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Direct Schur Complement Method by Domain Decomposition based on Hierarchical Matrix Approximation.* Preprint MPI MIS no. 25, Leipzig, 2004 (submitted).

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Computational plasticity: An efficient solution algorithm and its first implementation for h and p elastoplastic interface adaptivity

Johanna Kienesberger[1] Jan Valdman[2]

One time step of the primal formulation in elastoplasticity leads to a convex but nonsmooth minimization problem with unknown displacement and plastic strain. Our algorithm is based on a Schur-Complement system for the displacement which is solved by a multigrid preconditioned conjugate gradient method.

Motivated by the work of Prof. Rank's group (TU Munich) initial steps towards hp adaptivity were taken. First results using high order fem methods with a fixed polynomial degree as well as a simple approach on elastoplastic interface adaptivity in terms of h refinement will be presented.

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An Optimal-Order Multigrid Method for Quadratic Conforming Finite Elements

Köster, Michael[1] Turek, Stefan[2]

Quadratic and higher order finite elements are interesting candidates for the numerical solution of PDE's due to their improved approximation properties in comparison to linear/bilinear approaches. The linear systems that arise from the discretization of the underlying differential equation are very often solved by iterative solvers like CG-, BiCGStab, GMRES or others. Multigrid solvers are rarely used, which might be caused by the high effort that is associated with the appropriate numerical realization of smoothers and intergrid transfer operators.

In this talk we discuss the numerical analysis of the quadratic conforming finite element Q_2 in a multigrid solver. Numerical tests indicate that – if the problem is smooth enough and the correct grid transfer operator is provided – this element provides much better convergence rates than the use of linear/bilinear finite element spaces like Q_1: If m denotes the number of smoothing steps, the convergence rates behave like $O(\frac{1}{m^2})$ in contrast to $O(\frac{1}{m})$ for first order FEM.

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Uniformly Convergent Adaptive Finite Element Algorithm for a Singularly Perturbed Reaction-Diffusion Equation

Yaroslav Kondratyuk[1] Rob Stevenson[2]

For solving elliptic boundary value problems with a solution that has singularities, adaptive finite element methods are able to reduce the computational cost enormously compared to non-adaptive methods. Although adaptive finite element methods indeed exhibit such a reduction, their convergence is often not proven. In this talk we present an adaptive finite element algorithm for a singularly perturbed reaction-diffusion equation that, in the energy norm, converges uniformly in the size of the reaction term. In particular, the analysis includes also the inexact solution of the arising Galerkin systems by an iterative solver. Preconditioning is based on the transformation to a wavelet basis. The number of arithmetic operations is of the order of the number of triangles in the final triangulation. Finally, we report on numerical experiments and analyse the computational complexity of the algorithm.

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Arnoldi-type Algorithms for Solving Algebraic Riccati Equations

D. Kressner[1] P. Benner[2]

Several optimal and robust control problems for instationary partial differential equations require the solution of operator Riccati equations. The usual computational approaches to address such equations firstly use a Galerkin-type spatial semi-discretization method leading to finite-dimensional, algebraic Riccati equations with highly structured coefficient matrices. The exact nature of this structure depends on the employed discretization technique. Finite element methods typically yield large and sparse coefficient matrices. Other discretization techniques may lead to other types of structures, such as hierarchical matrices. We will present new variants of Arnoldi's method to solve these nonlinear matrix equations by respecting the underlying structure. Combined with recently developed restarting strategies, these methods are inexpensive in terms of memory and runtime requirements; they thus represent viable alternatives to existing approaches. Numerical examples will be given to compare the discussed methods.

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The Stochastic Finite Element Method (SFEM) for Advection-Diffusion Equations

Susanna Kube[1]

In subsurface flow simulation, transport processes are modeled by the advection-diffusion equation. In general, one assumes complete knowledge of the problem data, such as boundary conditions, initial data, coefficients, and source terms. However, in many cases the available information is limited, i.e. the data suffer from uncertainties. To reflect the propagation of uncertainties to the simulation output, the advection-diffusion equation is considered as a stochastic partial differential equation in which the functions are stochastic processes. They are assumed to depend on a small number of independent random variables. To solve the equation, the stochastic finite element method is applied. Several methods exist for a finite dimensional approximation of the random space. I follow the approach proposed by Babuska who uses global polynomials in the random variables. These polynomials are required to fulfill an orthogonality relation in order to compute the matrices efficiently. It turns out that this approach is especially useful for the model problem where the diffusion-dispersion tensor depends quadratically on the velocity field which is chosen as stochastic input. I concentrate on advection-dominated problems and show how a stochastic velocity and a stochastic diffusion influence the solution.

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Solving Poisson's and Maxwell's equations by the Fourier–Singular Complement Method

Simon Labrunie[1] Patrick Ciarlet[2] Beate Jung[3]

The Singular Complement Method (SCM) was developed [1,2] as an alternative to the "usual" edge element and finite volume methods, in order to solve Maxwell's equations in singular domains. It can also serve as an accelerator of convergence for the usual $\mathbb{P}_1$ element method for Poisson's equation, as well as for the Lamé and Stokes problems. The SCM is based on a splitting of the "natural" spaces of solutions into a *regular* part (which has the regularity expected for a smooth or convex domain) and a *singular* part (which needs some special representation). It incorporates the well-known computation of *singularity coefficients* (a.k.a. "stress intensity factors" in mechanics).
However, the SCM had so far been implemented only in two-dimensional situations. This limitation stemmed from the difficulties of practical description of the singular spaces in general three-dimensional geometries.
We show how the SCM can be extended to some simple, but genuinely three-dimensional situations: prismatic or axisymmetric domains with arbitrary data. In this case, the splitting w.r.t. regularity can be combined by a Fourier expansion in the longitudinal or azimuthal coordinate.
This Fourier–Singular Complement Method (FSCM) achieves a good compromise between simplicity and efficiency: it has the optimal convergence rate for $\mathbb{P}_1$ element methods with an L^2 data [3], and the least computational cost among the optimally convergent methods. It can be easily extended to the time-dependent Maxwell equations [4].

References:

[1] **F. Assous, P. Ciarlet Jr., J. Segré**, Numerical solution to the time-dependent Maxwell equations in two-dimensional singular domains : the singular complement method. *J. Comput. Phys.* **161** (2000) 218–249.
[2] **C. Hazard, S. Lohrengel**, A singular field method for Maxwell's equations: numerical aspects in two dimensions. *SIAM J. Appl. Math.* **40** (2002) 1021–1040.
[3] **P. Ciarlet Jr., B. Jung, S. Kaddouri, S. Labrunie, J. Zou**, The Fourier–Singular Complement method for Poisson's equation. Part I: prismatic domains. *Submitted to Numer. Math.* Available online at: `http://www.iecn.u-nancy.fr/~labrunie/pubs/prisma.pdf` Part II: axisymmetric domains. *Submitted to Numer. Math.*
Available online at: `http://www.iecn.u-nancy.fr/Preprint/publis/Textes/2004-18.pdf` Part III: numerical implementation. *In preparation.*
[4] **S. Labrunie**, La méthode du complément singulier avec Fourier pour les équations de Maxwell en domaine axisymétrique, *Submitted to Compte Rendu Mathématique.* Available online at: `http://www.iecn.u-nancy.fr/~labrunie/pubs/cras04.ps`

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Residual-based stabilized higher-order FEM for advection-dominated problems

Lube, Gert[1] Rapin, Gerd[2]

The numerical solution of the nonstationary, incompressible and non-isothermal Navier-Stokes problem with standard turbulence models can be splitted into linearized auxiliary problems of Oseen and of advection-diffusion-reaction type. An important ingredient of this approach is a proper stabilization of the latter model. Here we present the numerical analysis of a *hp*-version of stabilized Galerkin methods of streamline-diffusion type (SUPG) together with shock-capturing stabilization in crosswind directions. The analysis is supported by some numerical experiments.

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Numerical simulation of electrostatic spray painting

Matthias Maischak[1]

We investigate the electrostatic subproblem of the electrostatic spray painting process. Between the target and at least one electrode a high voltage is applied. Due to the small diameter of the tip of the electrode a hugh electrical field strength leads to the existence of a charge cloud around the electrod, i.e. to the corona emission of electrons from the metal surface. This charges are transported in a convection process along the electrical field lines. The boundary condition for the charge transport problem is given indirectly by the Peek field strength, which means that the charge density on the emitting surface takes a value such that the electrical field strength on the surface equals the Peek field strength, which is determined by the geometry. We solve this nonlinear coupled problem iteratively by modeling the electrostatic problem by Poissons equation discretized by FEM or a FEM-BEM coupling procedure and the charge transport problem by a nonlinear convection equation discretized by a Least-Squares approach or the Method of Characteristics. We will present numbers for both cases and compare.

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Numerische Simulation von stochastischen elliptischen partiellen Differentialgleichungen

Hermann G. Matthies[1]

Erst werden Verfahren zur Ermittlung von Statistiken der Lösung betrachtet; diese können zum einen direkt als hoch-dimensionale Integrale angesehen werden, und die Auswertung durch "dünne" (Smolyak-)Quadratur, Monte Carlo, und Quasi Monte Carlo wird angerissen.

Als alternativer Weg werden Galerkin-Verfahren aufgezeigt, um die Lösung als Element in einem stochastischen Ansatz-Raum zu erhalten. Hierbei sind Gleichungssysteme in Tensorprodukt-Räumen zu lösen, was auf sehr strukturierte riesige Gleichungssysteme führt. Es werden verschieden Ansätze zur Lösung betrachtet.

Alternativ koennen die Koeffizienten im Ansatz auch "direkt" berechnet werden durch orthogonale Projektion. Dies ist wieder etwas ähnlich zu den originalen Monte Carlo Ideen.

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Inf-sup stable non-conforming finite elements of arbitrary order on triangles

Gunar Matthies[1] Lutz Tobiska[2]

First we introduce a family of scalar non-conforming finite elements of arbitrary order $k \geq 1$ with respect to the H^1-norm on triangles and show that the local function space is unisolvent with respect to the used nodal functionals.

After recalling a general convergence result for non-conforming discretisations of the Stokes problem we consider consistency and stability and prove that the vector-valued versions of the defined scalar finite element generates together with a discontinuous pressure approximation of order $k-1$ an inf-sup stable finite element pair of order k for the Stokes problem in the energy norm. For $k = 1$ the well-known Crouzeix-Raviart element is recovered.

We present numerical results which confirm our theoretical predictions and compare the new finite element pairs with conforming finite element pairs.

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A residual error estimator for the 3-d FEM-BEM coupling

Stephan Oestmann[1] Matthias Maischak[2] Ernst P. Stephan[3]

We consider the 3-dimensional finite element and boundary element coupling method for an elasticity problem. We present an a posteriori residual error estimator, which we use to steer an adaptive refinement algorithm. Due to the singular behaviour of the appearing boundary integral operators, we use modified representations of some operators to compute the error terms in a more convenient way. For the adaptive refinement of a hexahedral mesh we allow hanging nodes to enable better local refinement. Numerical examples show the efficient use of nonconforming meshes and the reliability of the residual error estimator.

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Efficient Iterative Solvers for Boundary Element Tearing and Interconnecting Methods

G. Of[1] O. Steinbach[2]

The Boundary Element Tearing and Interconnecting (BETI) methods have recently been introduced as boundary element counterparts of the well-established Finite Element Tearing and Interconnecting (FETI) methods. As domain decomposition methods, the BETI methods are efficient parallel solvers for large scale boundary element equations. Several systems of linear equations for the BETI formulation, namely a Schur complement system, a saddle point problem and a twofold saddle point problem, are discussed and compared to each other. Efficient preconditioners are used for the local boundary integral operators and the realization of BETI preconditioners is discussed. Sparse approximations of the occurring boundary integral operators are realized by the use of the Fast Multipole Method.

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Finite element methods for the computation of 3D corner singularities

Cornelia Pester[1]

For the analysis of the stress distribution at the top of a polyhedral corner or at a crack tip, one typically expands the displacement in terms of the form $kr^{\alpha}u$, where r is the distance to the tip, u is a function of the spherical angles and k is the stress intensity coefficient. The exponent α and the function u do not depend on the loading but only on the geometry and the material parameters. They form an eigenpair of a quadratic operator eigenvalue problem.

Since the eigenvalue problem can in general not be solved analytically, the finite element method is used to solve it approximately. This method is flexible enough, such that also anisotropic or composite materials can be treated. By the finite element approximation, the operator eigenvalue problem is transformed into a quadratic matrix eigenvalue problem with a special structure. It can be reformulated as an eigenvalue problem of a Hamiltonian, a skew-Hamiltonian or a symplectic matrix. There are special Arnoldi and Lanczos algorithms which exploit the structure of the underlying eigenvalue problem. In this talk, we introduce different methods of finite element discretization and present recent numerical results.

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Adaptive finite elements for eigenvalue problems

Rolf Rannacher[1] Vincent Heuveline[2]

We discuss the numerical solution of eigenvalue problems such as arising in hydrodynamic stability theory. The goals are reliability and efficiency of the computation. To this end, a posteriori error estimates are derived for discrete eigenvalues and eigenfunctions. The key to these error estimates is the embedding of the eigenvalue approximation into an abstract framework of Galerkin methods for nonlinear variational problems. The theoretical results are illustrated by several examples.

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Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

Rene Schneider[1] Peter Jimack[2]

The benefits of anisotropic meshes have long been established for a variety of problems, such as those featuring sharp layers in the solution for example. For many such problems a priori knowledge is available as to how to design a mesh to efficiently deliver a good solution to the problem, e.g. meshes of Shishkin type. Unfortunately, for many practical problems a priori analysis can only deliver limited information and therefore anisotropic mesh adaption based on suitable a posteriori error estimates is ultimately necessary.

Historically, the majority of work on automatic mesh adaption has focused on locally uniform h-refinement which is clearly inappropriate for producing anisotropic meshes. The development of efficient and reliable methods for generating suitable anisotropy in the mesh has therefore become an important topic of research in recent years.

In this talk we will present some provisional results based upon a new approach to this problem which aims to minimise an a posteriori error estimate for a quantity of interest by moving the positions of the nodes of a mesh with fixed connectivity (r-refinement). Utilisation of the discrete adjoint method for sensitivity analysis combined with hierarchical approaches allow a relatively cheap implementation of the method and indeed makes it feasible for problems of practical interest. While this method is not intended to be a substitute for well established h-refinement approaches it promises to be a valuable enhancement for them.

Issues of reliability and performance of the new approach will be discussed for a number of model problems.

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Primal-Dual a posteriori error estimates for high order finite elements (Part I)

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Consistent error estimation of FE-approximations

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