TU Chemnitz
C. Helmberg

Chemnitz, December 12, 2017
due on January 9, 2018

Nonlinear Optimization
Exercises 6

1. (4) Show for the Line-Search Newton-CG method that the generated step directions are descent directions.

2. (4) Prove: In the Trust-Region Steihaug-CG method the length of the step is nondecreasing. In the notation of the algorithm, $0 = \|p_0\| < \cdots < \|p_j\| < \cdots < \|p\| \leq \Delta$.

3. (5) Nonlinear Systems of Equations: Let $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ be of the form

$$F(x) = \begin{pmatrix} x^T A_1 x + b_1^T x + c_1 \\ x^T A_2 x + b_2^T x + c_2 \end{pmatrix}.$$

with $A_i \in \mathbb{R}^{2\times 2}$ symmetric, $b_i \in \mathbb{R}^2$, $c_i \in \mathbb{R}$ for $i \in \{1,2\}$ and Jacobian $J(x)$. Denote the set of degenerate (non regular) points by $D = \{x \in \mathbb{R}^2 \colon J(x) \text{ singular}\}$. If possible, give a choice of coefficientes A_i, b_i, c_i so that
(a) $D = \emptyset$ (b) $D = \mathbb{R}^2$ (c) $|D| = 1$
(d) $D = \{x\colon a_1^T x = 0\} \cup \{x\colon a_2^T x = 0\}$ for given $a_1, a_2 \in \mathbb{R}^2$.

4. (4) Consider the optimization problem

$$\begin{array}{ll} \min & c^T x \\ \text{s.t.} & \frac{1}{2} x^T A x \leq b_1 \\ & a^T x \leq b_2 \\ & x \in \mathbb{R}^3 \end{array} \quad \text{where } c = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},\ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ a = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix},\ b = \begin{pmatrix} \frac{1}{2} \\ -1 \end{pmatrix}.$$

Give the set of optimal solutions. Describe the set of limiting directions for the point $\bar{x} = (1,0,0)$, the linearised tangent cone $F_1(\bar{x})$, and its consequences on the existence of Lagrange multipliers. Provide a description of the feasible set, so that Lagrange-multipliers exist for all optimal solutions.

5. (3) A mathematical program with equilibrium constraints (MPEC) is an optimization problem of the form

$$\begin{array}{lll} \min & f(x) & \\ \text{s.t.} & g(x) \leq 0,\ h(x) = 0, & \\ & G(x) \geq 0,\ H(x) \geq 0,\ G(x)^T H(x) = 0 & \end{array} \quad \text{with} \quad \begin{array}{l} f\colon \mathbb{R}^n \to \mathbb{R} \\ g\colon \mathbb{R}^n \to \mathbb{R}^m,\ h\colon \mathbb{R}^n \to \mathbb{R}^p \\ G, H\colon \mathbb{R}^n \to \mathbb{R}^M \end{array}$$

Prove that for an MPEC there is no feasilbe point in which LICQ holds.