TU Chemnitz
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Nonlinear Optimization
Exercises 5

1. (2) Suppose the values of a function f contain roundoff errors u. Show that a suitable value for the perturbation ϵ in the central-difference formula is $\epsilon = u^{1/3}$ (up to some function dependent constants).

2. (3) Given the function $f : \mathbb{R}^8 \to \mathbb{R}^6$

$$f(x_1, \ldots, x_8) = \begin{pmatrix} x_1^2 e^{x_3^2} \cos x_2 \\ x_1^2 e^{x_3^2} \sin x_4 \\ e^{x_3^2} \cos x_2 + \sqrt{x_7} \\ e^{x_3^2} \sin x_4 / x_5 \\ x_6^3 x_7^{\frac{1}{2}} x_8^2 \\ x_6^3 x_8^2 / x_5 \end{pmatrix},$$

how many function evaluations are needed in order to compute an approximate Jacobian by finite differences? (Hint: build the graph and find its chromatic number, i.e., a coloring with minimum number of colors.)

3. (3) Prove the formula

$$\frac{\partial^2 f}{\partial x_i \partial x_j}(x) = \frac{f(x + \varepsilon e_i + \varepsilon e_j) - f(x + \varepsilon e_i) - f(x + \varepsilon e_j) + f(x)}{\varepsilon^2} + O(\varepsilon).$$

4. (6) Develop a forward mode for calculating $p^T \nabla^2 f q$ for given directions p, q. For this purpose introduce for each node i the values $d_i^p = \nabla x_i^T p$, $d_i^q = \nabla x_i^T q$, and $d_i^{pq} = p^T \nabla^2 x_i q$ and show how to compute them for +, *, and a unary function $g : \mathbb{R} \to \mathbb{R}$ where first and second derivative g' and g'' are supposed to be available.

5. (6) Show that in the proof of Theorem VI.1 (inexact Newton method) linear / superlinear / quadratic convergence of the gradients to zero is equivalent to linear / superlinear / quadratic convergence of the points to x^* with respect to the $[\nabla^2 f(x^*)]^2$-norm.