Technische Universität Chemnitz
M. Winter

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Graph Theory

Exercise 5

Terms

- straight line embedding, convex embedding,
- polyhedron, polyhedral graph, theorem of Steinitz,
- forbidden minors, theorem of Kuratowski,
- (feasible) coloring, list coloring, edge coloring,
- chromatic number $\chi(G)$, chromatic index $\chi'(G)$, list coloring number $\mathrm{Ch}(G)$,
- Brooks' theorem, Vizing's theorem, class 1/2 graphs, 4-coloring theorem,
- Ramsey numbers, König's Lemma.

Tasks

1. Model *Sudoku* as a graph-coloring problem.
2. *Art gallery problem.* Consider the floor plan of an art gallery (a polygon). The management wants to place a minimal amount of 360°-cameras to monitor the whole room. How many cameras are (at least, at most) necessary if the room has n corners.
3. Show that a cubic Hamiltonian graph can be edge colored using at most three colors. Show that if the edge coloring is unique (up to permuting color classes), then this graph has exactly three Hamiltonian cycles.
4. Let $G = (V, E)$ be a graph, and $k_1, k_2 \in \mathbb{N}$. Show that $\chi(G) \leq k_1 k_2$ if and only if G can be written as $G_1 \cup G_2$, with $G_i = (V, E_i)$ and $\chi(G_i) \leq k_i$.
5. Proof that an infinite graph with only finite degrees contains an infinite path.
6. Given the 4-color theorem for finite planar graphs, prove that every infinite planar graph with only finite degrees can be colored with 4-colors as well.

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7. Consider n football teams. A contest is held in which any two teams play against each other. The games happen one after the other, and it is undesirable to let one team play two times in a row (they need some time to recover). For what n is it possible to find such a sequence of games?