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Introduction to Discrete Mathematics
Exercise 7

1. A set S of vertices of a graph G is called *independent* if the $G[S]$ (maximal subgraph of G with vertex sut S, subgraph of G *induced* by S) is edgeless (i.e. G contains no edge connecting vertices of S) $\alpha(G) = \max\{|U| : U \text{ independent}\}$ is called *independence number* of G. Calculate $\alpha(G)$ for cycles and paths G on n vertices!

2. Let G be a graph without vertices of odd degree. Show that G we can replace all edges of G by directed edges in such a way, that in the resulting digraph D for all vertices u the equation $d_D^-(u) = d_D^+(u)$ holds!

3. Let $G = (V, E)$ be a connected graph. Define $r(u) = \max\{d(u, v) : v \neq u\}$ for all $u \in V$. The graph parameter $r(G) = \min\{r(u) : u \in V\}$ is called *radius* of G, the graph parameter $Z(G) = \{u \in V : r(u) = r(G)\}$ is called *center* of G. Show that the center of G either is a vertex or consists of two neighboring vertices!

4. Let $d_1 \geq \ldots \geq d_n > 0$ be a sequence of natural numbers. Show, that $(d_1, \ldots, d_n)$ is the degree sequence of a tree, if and only if $\sum_{i=1}^{n} d_i = 2n - 2$ holds!

5. A graph on 10 vertices is given by the following adjacency lists:

1: 6,5,3,2	4: 2,3,5	7: 10	10: 7
2: 1,3,4	5: 4,3,1,6	8: 9	
3: 1,5,4,2	6: 1,9,5	9: 8,6	

   Run a BFS and a DFS algorithm starting with $v_0 = 1$ using the given order of adjacencies and figure out the resulting edge sets and vertex numberings!