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Introduction to Discrete Mathematics
Exercise 3

1. Suppose, the random number X is either 0 or 1. Prove

$$VX = EX \cdot E(1 - X).$$

2. The fields of a 4×7 chessboard are colored arbitrarily with colors black and white. Prove the existance of a rectangle with uniquely colored corners! Is this true even for a 4×6 chessboard?

3. Prove Markov's Inequality:
If the random variable X takes only nonnegative values, then

$$P(X \geq t) \leq E(X)/t.$$

Use Markov's inequality to prove Chebyshev's Inequality:
For a random variable X and a real number $\alpha \geq 0$ the following holds:

$$p(|X - EX| \geq \alpha) \leq \frac{VX}{\alpha^2}$$

4. Using the previous results, find a bound for the probability of a permutation of n elements to have $k + 1$ fix points. (All permutaitions shall have equal probability.)

5. Show Ramsey's Theorem: Let k and l be integers with $k, l \geq 2$. Then there is a smallest integer $R(k, l)$ (called Ramsey number) such that the following holds: On a meeting of $n \geq R(k, l)$ persons there are always either k persons such that each of this k persons knows all of that k persons, or l persons such that each of this l persons knows no other of that l persons.

   Hint: For all integers k, l prove $R(k, 2) = k$ and $R(2, l) = l$ Prove $R(k, l) \leq R(k - 1, l) + R(k, l - 1)$ Use induction to prove Ramsey's Theorem.

   Prove additionally $R(k, l) \leq \binom{k+l-2}{k-1}$.