Chemnitz University of Technology
Dr. F. Göring

Chemnitz, january 19th 2004
due: january 26th 2004

Introduction to Discrete Mathematics
Exercise 13

1. Solve the balancing problem, if it is known that the wrong coin is heavier.

2. Let n and q be non negative integers and let $q \geq 2$. Prove that there is a complete (n, q)-tree if and only if $n - 1$ is a multiple of $q - 1$.

3. Let $x^* \in S = \{1, \dots, n\}$ be unknown. We are only able to test $x^* < i$ $(i = 1, \dots, n)$ with answers yes or no. Prove that $L = \lceil \lg n \rceil$ is the optimal running time of an algorithm searching for x!

4. Let $(p_1, \dots, p_n)$ be a distribution and $q \geq 2$. Prove, that $\overline{L}(p_1, \dots, p_n) \leq \overline{L}(\frac{1}{n}, \dots, \frac{1}{n})$ holds.

5. Prove that every permutation $a_1, a_2, \dots, a_n$ of the numbers $1, \dots, n$ can be ordered to $1, \dots, n$ by gradual exchanging neighbouring elements.

   Example: $3123 \rightarrow 1324 \rightarrow 1234$

   Find the minimum number of exchanges to order an arbitrary such permutation!