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due: january 5th 2003

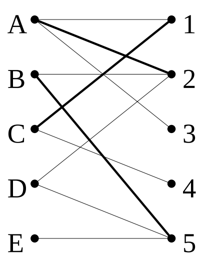
Introduction to Discrete Mathematics
Exercise 10

1. Let B be the independence matrix of $G = (V, E)$ formed of the elements of the field with elements $0, 1$ (addition $0 + 0 = 1 + 1 = 0$, $0 + 1 = 1 + 0 = 1$; multiplication $0 = 0 \cdot 0 = 0 \cdot 1 = 1 \cdot 0$, $1 = 1 \cdot 1$).

   Show that $A \subset E$ is independent in the graphic matroid if and only if the representing columns of B are independent. (The sum over a nonempty subset of this columns never is the zero column.)

2. For the following bipartite graph we start with the bold drawn matching X. Present the next two auxiliary graphs D_X according to the next two iterations in the matroid cut algorithm of Edmonds. Here matroid 1 is represented at the part A-E and matroid 2 is represented at the part 1-5. Find R!

   Hint: A subset of edges is independent in one of the matroids, if no vertex of its part is incident with more than one edge of the subset.



3. Find a maximum matching (and prove maximality) of the bipartite graph on $7 + 7$ vertices given by the following adajacency matrix:

	1	2	3	4	5	6	7
1	1	1	0	1	0	1	0
2	0	1	1	0	0	0	0
3	0	1	0	0	0	1	0
4	0	1	1	0	0	1	0
5	1	0	1	1	1	0	1
6	0	0	1	0	0	1	0
7	0	1	0	0	1	0	1

4. How can one apply shortest path algorithms for directed graphs also to undirected graphs with non negative edge weights? Is this possible for arbitrary edge weights, too?

5. For all $i \in \{2 \ldots 7\}$ calculate the shortest $1i$-paths of the directed graph given by the following matrix of edge weights. Here missing entries indicate missing edges.

	1	2	3	4	5	6	7
1			4	10	3		
2			1	3	2	11	
3		9		8	3	2	1
4		4	5		8	6	3
5	1		1	2		3	1
6		1	1	3	2		
7	2	4	3			2	