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Exercises for the course in
Convex Analysis
Problem sheet 2

Aufgabe 1:
Prove: For $A\colon X \to Y$ affine and $D \subseteq Y$ convex with $A^{-1}(\operatorname{ri} D) \neq$ it holds

$$\operatorname{ri} A^{-1}(D) = A^{-1}(\operatorname{ri} D).$$

Aufgabe 2:
Prove that $\operatorname{ri} C_1 \cap \operatorname{ri} C_2 \neq \emptyset \Leftrightarrow 0 \in \operatorname{ri}(C_1 - C_2)$.

Aufgabe 3:
Show that for $x \in X$, $C \subseteq X$ with C closed and convex the following infimum is actually attained at some $y \in C$:

$$\inf \left\{ \frac{1}{2} \|x - y\|^2 \colon y \in C \right\}.$$

Aufgabe 4:
Let $C \subseteq X$ be convex and $x \in \operatorname{rbd} C$ (so $C \neq \operatorname{aff} C$). Prove that there exists a hyperplane which supports C at x *non-trivially*.

Aufgabe 5:
A function $f\colon I \to \mathbb{R}$ on an Intervall I is called convex, if

$$\forall\, a, b \in I, \alpha \in [0,1]\colon f(\alpha a + (1-\alpha)b) \leq \alpha f(a) + (1-\alpha) f(b)$$

Show that $f\colon I \to \mathbb{R}$ is convex if and only if for every $x_0 \in I$ the slope function

$$x \mapsto s_{x_0}(x) := \frac{f(x) - f(x_0)}{x - x_0}$$

is monotonically increasing on $I \setminus \{x_0\}$.

You can hand in your solutions by 29 Oct. 2018 during the lecture