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Exercises for the course in

Convex Analysis

Problem sheet 1

Aufgabe 1:
Show Proposition 2: If V is a linear subspace (affine subspace, convex set, respectively) and $U \subseteq V$ then $\text{lin}(U)$ ($\text{aff}(U)$, $\text{co}(U)$, respectively) is a subset of V.

Furthermore show that $\text{lin}(U)$ ($\text{aff}(U)$, $\text{co}(U)$, respectively) is the intersection of all linear subspaces (affine subspaces, convex sets, respectively) V with $U \subseteq V$.

(And consequently lin (aff, co) is a linear subspace (affine subspace, convex set))

Aufgabe 2:
Show that if V is an affine subspace and $v_0 \in V$ then, $V_0 := V - v_0 = V + \{-v_0\}$ is a (linear) subspace. Show further that V_0 is independent of the choice of $v_0 \in V$.

Aufgabe 3:
Show Observation 5: Images and preimages of affine subspaces (convex sets, respectively) under an affine map are affine subspaces (convex sets, respectively).

Aufgabe 4:
Show Observation 6: 1) If $C \subseteq X_1 \times \ldots \times X_k$ is convex then so is its projection onto X_i for every $i = 1, \ldots, k$.

2) $C_1 \times \ldots \times C_k \subseteq X_1 \times \ldots \times X_k$ is convex if and only if $C_i \subseteq X_i$ is convex for every $i = 1, \ldots, k$.

Aufgabe 5:
Show Observation 7: 1) $C_1, C_2 \subseteq X$ convex and $\alpha_1, \alpha_2 \in \mathbb{R}$ then $\alpha_1 C_1 + \alpha_2 C_2$ is convex.

2) If $C_1, \ldots, C_k \subseteq X$ are convex then

$$\text{co}(C_1 \cup \ldots \cup C_k) = \left\{ \sum_{i=1}^{k} \lambda_i x_i \colon \lambda_i \geq 0,\ x_i \in C_i\ \forall i = 1, \ldots, k,\ \sum_{i=1}^{k} \lambda_i = 1 \right\}$$

You can hand in your solutions by 15 Oct. 2018 during the lecture