

Geometric Finite Elements

Hanne Hardering

TU Dresden

Constrained minimization problems where solutions take values in a Riemannian manifold motivate the development of geometric finite element methods. These methods are designed to be independent of embeddings, invariant under manifold isometries, and reduce to standard finite elements when the manifold is \mathbb{R}^n .

Classical Galerkin-based numerical analysis for second-order elliptic problems relies heavily on functional analysis, from the definition of errors in Sobolev norms, to the concept of Galerkin orthogonality, to technical results such as inverse estimates via norm equivalence, and the use of Sobolev- and Hölder inequalities in a priori estimates. However, all of these concepts make use of the vector space structure of the target. In the manifold setting, these notions must be reformulated: addition is replaced by retractions or integration, Sobolev distances by the Riemannian distance map and higher order generalizations, and norms by nonlinear counterparts compatible with scaling and embedding.

This talk outlines an intrinsic framework for the numerical analysis of geometric finite elements and shows how standard convergence results for second-order problems can be recovered in this nonlinear context.