

# The Willmore Energy Landscape of Spheres

*Jona Seidel*

TU Darmstadt

A geometric energy landscape is usually investigated locally through its critical points and their Morse indices. Understanding its global structure is challenging and requires both analytic and topological approaches. We present a special case where such global analysis is feasible: the Willmore energy of immersed 2-spheres in Euclidean 3-space. We show that the subset of immersions with energy at most  $12\pi$  consists of four path components. As a consequence, we obtain insight into the singular behavior of the Willmore flow and a relation between self-intersections and the Willmore energy beyond the Li-Yau inequality. To prove these results, we glue together different instances of the Willmore flow and devise an invariant for triple-point-free immersed spheres.

arXiv:2506.23359 and arXiv:2506.21130