Super-resolution of points and curves.
The term super-resolution is used in many different contexts but in general asks for resolving fine details from low-frequency measurements. Algebraic techniques have proven useful in this context with different imaging tasks such as spike reconstruction (single molecule microscopy), phase retrieval (X-ray crystallography), and contour reconstruction (natural images). The available data typically consists of a blurred version of the specimen or equivalently trigonometric moments of low to moderate order and one asks for the reconstruction of fine details modeled by zero- or positive-dimensional algebraic varieties. Often, such reconstruction problems have a generically unique solution when the number of data is larger than the degrees of freedom in the model. Beyond that, we concentrate on simple a-priori conditions to guarantee that the reconstruction problem is well or only mildly ill conditioned. For the reconstruction of points on the complex torus, popular results ask the order of the moments to be larger than the inverse minimal distance of the points. Moreover, simple and efficient eigenvalue based methods achieve this stability numerically in specific settings. Recently, the situations involving clustered points, points with multiplicities, and positive-dimensional algebraic varieties are starting to gain interest, and these shall be discussed within this talk.