Mathieu Dolbeault: Optimal L^2 approximation through weighted least-squares.

We investigate the problem of approximating a function u in L^2 with a linear space of functions of dimension n, using only evaluations of u at m chosen points, with m of the order of n. A first approach, based on weighted least-squares at i.i.d random points, provides a near-best approximation of u, but requires m of order $n \log(n)$. To reduce the sample size while preserving the quality of approximation, we will need a recent result on sums of rank-one matrices, which answers to the Kadison–Singer conjecture formulated in 1959.