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Geodesically complete Riemannian metrics on the space of embedded curves.

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We consider a certain family of Riemannian metrics that has proven to be very useful in the numerical optimization of the so-called tangent-point energy. The latter is a geometric functional for curves whose finiteness implies that the curve must be embedded. Motivated by numerical evidence, we prove that these Riemannian metrics have the following properties:

1. bounded subsets are weakly relatively compact with respect to the “right” topology
2. geodesically completeness: One can travel in every direction with constant speed and never hit the boundary i.e., one cannot reach a non-embedded curve in finite time.
3. completeness: The distance induced by the metric turns the space into a complete metric space.
4. existence of length minimizers: Every two points in the same connected component can be joined by a length minimizing geodesic.

For finite-dimensional Riemannian manifolds the Hopf–Rinow Theorem shows that statements 1.) – 3.) are equivalent to each other, and that each of 1.), 2.), 3.) implies 4.). However, our setting is infinite-dimensional, so we have to show each of them “by hand” from some energy principles.
Last, but not least, we will see a couple of numerical simulations of length-minimizing geodesics in the spaces of knots and links.