Yuri Malykhin: Widths and rigidity.

It is known that any orthonormal system $f_1, \dots, f_N$ cannot be well-approximated in L_2 by low dimensional spaces. In terms of Kolmogorov widths:

$$d_n(f_1, .., f_N, L_2) = (1 - n/N)^{1/2}.$$

The situation is different in weaker metrics. It is known, for example, that the Walsh system may be approximated with an error $N^{-\varepsilon}$ by linear spaces of dimension $N^{1-\varepsilon}$ in L_p for all $p < 2$ (and $\varepsilon = \varepsilon(p)$). But the Rademacher system remains rigid in L_1.

We will give some sufficient conditions for rigidity in L_p for $p = 0$, $p = 1$ and $1 < p < 2$. They are based on "average widths" of finite-dimensional random vectors. Also we will discuss some positive results on approximation of trigonometric and Walsh functions.