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A brief introduction to Markov-type inequalities with mixed norms.

Markov-type inequalities provide upper bounds on the norm of the (higher order) derivative on an algebraic polynomial in terms of the norm of the polynomial itself. Such an inequality is

$$\|f^{(\nu)}\|_\beta \leq C_n^\nu(\alpha,\beta)\|f\|_\alpha \quad \text{for all } f \in \mathcal{P}_n.$$

Here $\mathcal{P}_n$ denotes the space of algebraic polynomials with complex coefficients of degree at most n, and $\|\cdot\|_\alpha$ is one of the Laguerre, Gegegenbauer, or Hermite norms with some weight α, and $C_n^{(\nu)}(\alpha,\beta)$ is a constant depending on n, ν, α, and β, but not on f. Finding the smallest such constant that the above inequality holds for every polynomial f of degree at most n was the main achievement of the dissertation “Best constants in Markov-type inequalities with mixed weights” (Langenau, 2016).

In the work, asymptotically exact values for the constants were given as n goes to infinity. This heavily depended on the parameter difference $\omega = \beta - \alpha - \nu$ ($\omega = \beta - \alpha$ in the Hermite case). However, a small gap for $\omega \in [-1/2, 0)$ remained.

In the talk, we will give a brief overview over how this was achieved and have a deeper look into some special cases, concerning ω in this gap and present some more recent results that will provide insights into this situation and also partly proving a conjecture raised by the dissertation.