Sampling numbers of smoothness classes via ℓ^1-minimization

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Using compressed sensing techniques, we prove new upper bounds for general (nonlinear) sampling numbers of smoothness classes. In relevant cases such as mixed and isotropic weighted Wiener classes or Sobolev spaces with mixed smoothness, sampling numbers in L^2 can be upper bounded by best n-term widths in L^∞. We describe a recovery procedure based on ℓ^1-minimization (basis pursuit denoising) which uses only m function values. With this method, a significant gain in the rate of convergence compared to recently developed linear recovery methods is achieved. Surprisingly, our approach allows to recover mixed smoothness Sobolev functions from S^r_pW on the d-torus with a better rate than the decay of the corresponding linear widths when $1 < p < 2$ and d is large. This is joint work with Tino Ullrich and Felix Voigtlaender.