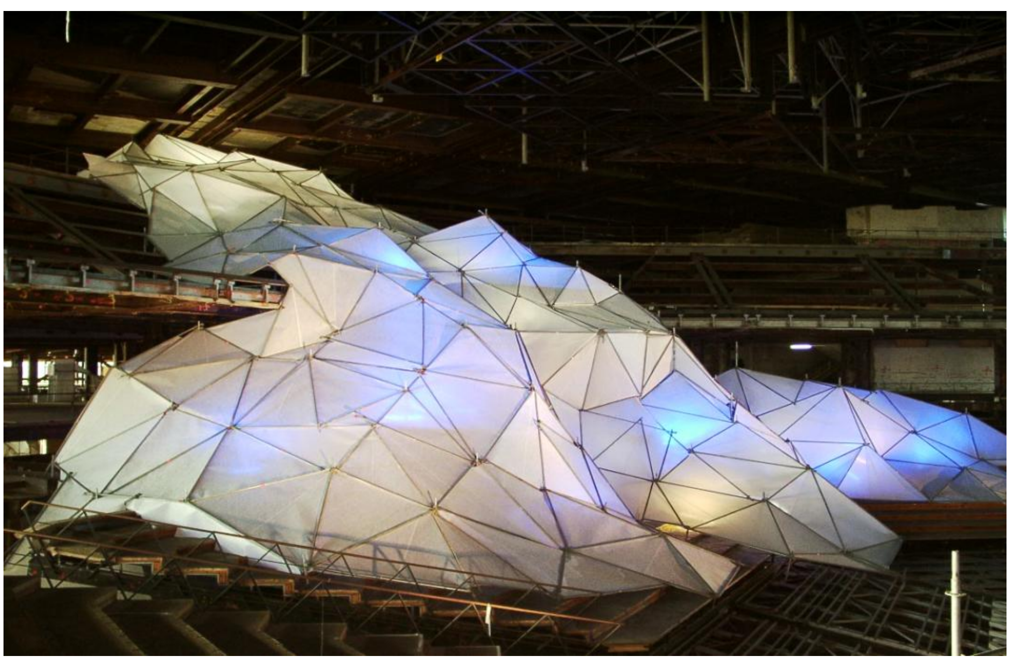
WORKSHOP OF THE WORKING GROUP ON *VARIATIONAL METHODS ON GRAPHS AND NETWORKS*

WITHIN THE COST ACTION CA 18232
MATHEMATICAL MODELS FOR INTERACTING DYNAMICS ON NETWORKS
https://mat-dyn-net.eu/

February 17, 2021
2:30 pm CET Ernesto Estrada (Zaragoza)
3:30 pm CET Gissell Estrada-Rodriguez (UPMC, Paris)
4:15 pm CET Matthias Keller (Potsdam)
5:15 pm CET Christian Rose (Potsdam)

February 18, 2021
2:30 pm CET Peter Stollmann (Chemnitz)
3:30 pm CET Marcel Schmidt (Jena/Leipzig)
4:15 pm CET Yehuda Pinchover (Technion, Haifa)
5:15 pm CET Matthias Hoffmann (Lisbon)
6:00 pm CET Andrea Serio (Stockholm)

https://fernuni-hagen.zoom.us/j/82430736046?pwd=eVVmcUtmRm1ybmJvVkVQRlJ3bExmZz09
Meeting ID: 824 3073 6046 Passcode: rgfW6+7H



Ernesto Estrada (Zaragoza)

Hubs-biased Laplacian operators and hubs-biased dynamics on graphs/networks

I will start by motivating the topic by considering some dynamical processes on a graph, such as diffusive and synchronization ones. Then, I will introduce the notion of degree-biased dynamics and will motivate it with examples. At this point I will introduce the hubs-biased Laplacian operators for graphs and will present some of their properties, mainly spectral ones. I will use these operators in hubs-biased diffusion and synchronization on networks, such as in activation in brain networks, propagation flight delays in airports and sync of autonomous robots. Finally, I will introduce and briefly study the hubs-biased resistance metrics on graphs. I will dedicate a couple of slides to mention some future directions of this topic.

References:

Estrada E. ‘Hubs-repelling’Laplacian and related diffusion on graphs/networks. Linear Algebra and its Applications, 2020 596, 256-280.

Gambuzza LV, Frasca M, Estrada E. Hubs-attracting Laplacian and Related Synchronization on Networks. SIAM Journal on Applied Dynamical Systems. 2020, 19, 1057-79.

Gissell Estrada-Rodriguez (UPMC, Paris)

Metaplex networks: influence of the exo-endo structure of complex systems on diffusion

In this talk I will introduce the concept of metaplexes and the dynamical systems on them. A metaplex combines the internal structure of the entities of a complex system with the discrete interconnectivity of these entities in a global topology. We focus here on the study of diffusive processes on metaplexes, both model and real-world examples: macaque visual cortex metaplex and landscape metaplex. We provide theoretical and computational evidence pointing out the role of the endo- and exo-structure of the metaplexes in their global dynamics, including the role played by the size of the nodes, the location of sinks and sources, and the strength and range of the coupling between nodes.

Matthias Keller (Potsdam)

From Hardy to Rellich inequalities and Agmon estimates on graphs

We present how the supersolution construction can be used to obtain optimal Hardy weights on graphs. In particular, one obtains the optimal constants and asymptotics for the Hardy weights on the Euclidean lattice. We proceed to discuss Rellich inequalities and Agmon estimates for (generalized) eigenfunctions. This includes joint work with Florian Fischer, Yehuda Pinchover and Felix Pogorzelski.

Christian Rose (Potsdam)

Neumann heat kernel estimates for integral Ricci curvature conditions

On Riemannian manifolds, in contrast to Dirichlet heat kernel bounds, Neumann heat kernel estimates depend on the geometry of the boundary if the considered domain is not a geodesic ball. In this talk I will present new Neumann heat kernel estimates for subsets whose boundaries satisfy only certain curvature restrictions in ambient manifolds with integral Ricci curvature conditions. New Sobolev extension operators whose norms depend quantitatively only on our curvature restrictions rather than a fixed atlas will be presented and how to use them to pass the heat kernel estimate from the ambient manifold to our subsets. This is joint work with Olaf Post and Xavier Ramos Olivé.

Peter Stollmann (Chemnitz)

A new uncertainty principle at low energies

This talk is about a new *uncertainty principle at low energies* for graphs and more general spaces and its very simple proof. Based on earlier joint works with Anne Boutet de Monvel, Daniel Lenz, Marcel Schmidt and Gunter Stolz and on work in progress with Daniel Lenz, Marcel Schmidt, Gunter Stolz and Martin Tautenhahn.

Marcel Schmidt (Jena/Leipzig)

Poincaré inequalities on graphs

[Abstract TBA]

Yehuda Pinchover (Technion, Haifa)

On the equivalence of heat kernels of second-order parabolic operators

Let P be a second-order, symmetric, and nonnegative elliptic operator with real coefficients defined on noncompact Riemannian manifold M, and let V be a real valued function which belongs to the class of small perturbation potentials with respect to the heat kernel of P in M. We prove that under some further assumptions (satisfying by a large classes of P and M) the positive minimal heat kernels of $P-V$ and of P on M are equivalent. Moreover, the parabolic Martin boundary is stable under such perturbations, and the cones of all nonnegative solutions of the corresponding parabolic equations are affine homeomorphic. This is a joint work with D. Ganguly.

Matthias Hoffmann (Lisbon)

An existence theory for nonlinear equations on metric graphs via energy methods

We will develop a general existence theory for constrained minimization problems for functionals defined on function spaces on metric measure spaces $(\mathcal{M}, d, \mu)$. We apply this theory to functionals defined on metric graphs $\mathcal{G}$, in particular L^2 constrained minimization problems for functionals of the form

$$E(u) = \frac{1}{2}a(u,u) - \frac{1}{q}\int_{\mathcal{K}} |u|^q \,\mathrm{d}x,$$

where $q > 2$, $a(\cdot,\cdot)$ is a suitable symmetric sesquilinear form on some function space on $\mathcal{G}$ and $\mathcal{K} \subseteq \mathcal{G}$ is given. We show how the existence of solutions can be obtained via decomposition methods using spectral properties of the operator A associated with the form $a(\cdot,\cdot)$ and discuss the spectral quantities involved. An example that we consider is the higher-order variant of the stationary NLS (nonlinear Schrödinger) energy functional with potential $V \in L^2 + L^\infty(\mathcal{G})$

$$E^{(k)}(u) = \frac{1}{2}\int_{\mathcal{G}} |u^{(k)}|^2 + V(x)|u|^2 \,\mathrm{d}x - \frac{1}{p}\int_{\mathcal{K}} |u|^q \,\mathrm{d}x$$

defined on a class of higher-order Sobolev spaces $H^k(\mathcal{G})$ that we introduce. When $\mathcal{K}$ is a bounded subgraph, one has localized nonlinearities, which we treat as a special case. When $k = 1$ we also consider metric graphs with infinite edge set as well as magnetic potentials. Then the operator A associated to the linear form is a Schrödinger operator, and in the L^2-subcritical case $2 < q < 6$, we obtain generalizations of existence results for the NLS functional as for instance obtained by Adami, Serra and Tilli [JFA 271 (2016), 201-223], and Cacciapuoti, Finco and Noja [Nonlinearity 30 (2017), 3271-3303], among others.

Andrea Serio (Stockholm)

On extremal eigenvalues of the graph Laplacian

Certain upper and lower eigenvalue bounds can be attained at the same time by degenerate eigenvalues of the graph Laplacian. We present the results of a recent work where the graphs exhibiting such eigenvalues are characterized.