

# Permeable Subsets of the $\mathbb{R}^d$ : Connecting Stochastic Differential Equations with the Analysis of Lipschitz Functions

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We present the notion of permeability of a set in  $\mathbb{R}^d$ . This quite intuitive concept originally emerged from the study of stochastic differential equations with discontinuous coefficients and immediately turned to be studied in itself.

Permeability of a set  $\Theta$  roughly means that any two points  $x$  and  $y$  in  $\mathbb{R}^d$  can be connected by a 'short curve' which avoids hitting  $\Theta$  'too often'. Further, permeability allows to conclude global Lipschitz continuity of a continuous function, that is 'intrinsically Lipschitz' with exception set  $\Theta$ , leading to a servicable notion of 'piecewise Lipschitz continuity' in  $\mathbb{R}^d$ .

We give a number of examples, non-examples, properties and conditions, in particular for iterated function systems, and we illustrate that the concept of permeability leads to a rich theory in  $\mathbb{R}^d$  and also in metric spaces.