Want to solve cubic NLS on a fractal? Maybe not by fixed point if you start below Sobolev...

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Abstract:
Taking a course on (partial) different equations, chances are that Duhamel and the fixed point theorem become one of your best allies to solve (non-linear) problems. Like the cubic non-linear Schrödinger equation (NLS).

A non-linear PDE with origins in quantum mechanics, it plays a prominent role in the modeling of dispersive wave phenomena, as for instance Bose-Einstein condensation. Dispersion is affected by the nature of the underlying geometry of the space it takes place, and in this talk we want to address the basic question of existence of solutions to the cubic NLS modeling dispersion on a compact fractal set.

We will call back our ally the fixed point theorem and realize with a mixture of surprise (?) and perplexity (?) that the method is of no help to prove existence of solutions with regularity below the threshold for the Sobolev embedding. A completely different picture than the torus or the sphere! In the end, excitement will fill the stage (what's next?).

Results are joint work with Gigliola Staffilani (MIT).