

Planar hypohamiltonian graphs

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A graph is called *hypohamiltonian* if it is not hamiltonian but, when omitting an arbitrary vertex, it becomes hamiltonian. The smallest hypohamiltonian graph is the famous Petersen graph (found by Kempe in 1886) on 10 vertices. In 1963, Sousselier posed a problem of recreational nature, and thus began the study of hypohamiltonian graphs. Many authors followed, in particular Thomassen with a series of very interesting papers written in the Seventies and Eighties. For more details, see the survey by Holton and Sheehan [4].

Among the work concerning hypohamiltonian graphs, Chvátal [1] asked in 1973 if there existed hypohamiltonian graphs with the additional requirement of planarity, while Grünbaum conjectured that there are no such graphs. An infinite family of such graphs was subsequently found by Thomassen [5], the smallest among them having 105 vertices. In 1979, Hatzel [3] improved this lower bound to 57 vertices. Many years later, in 2007, Zamfirescu and Zamfirescu [8] found a planar hypohamiltonian graph on 48 vertices, and only very recently Araya and Wiener [7] constructed the currently smallest known example, which has 42 vertices. All these graphs, extremal in the sense of minimal order, were constructed by applying Grinberg's hamiltonicity criterion for planar graphs [2]. This leads to the natural question whether one might construct even smaller planar hypohamiltonian graphs with Grinberg's criterion.

In this talk we shall investigate the pivotal role of Grinberg's criterion in the context of planar hypohamiltonian graphs, and present a result answering (partially) the above question in the negative. We will also discuss the recent constructions in [7], which by applying a method of Thomassen [6] and results from [3] and [8] settle the open question whether there exists an N such that there is a planar hypohamiltonian graph of every order $n \geq N$.

References

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