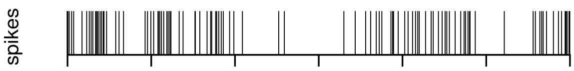
Model neurons

Poisson neurons

Suggested reading:

Chapter 1.4 in Dayan, P. & Abbott, L., Theoretical Neuroscience, MIT Press, 2001.

Model neurons: Poisson neurons



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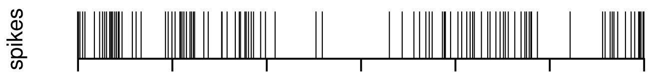
Motivation

In the cortex, the timing of successive action potentials is highly irregular. This irregularity might arise from stochastic forces. If so, the irregular interspike interval reflects a random process and implies that an instantaneous estimate of the spike rate can be obtained by averaging the pooled responses of many individual neurons.
In keeping with this theory, one would expect that the precise timing of individual spikes conveys little information.

We assume that the generation of each spike depends only on an underlying continuous/analog driving signal, r(t), that we will refer to as the instantaneous firing rate. It follows that the generation of each spike is independent of all the other spikes, hence we refer to this as the *independent spike hypothesis*.

If the *independent spike hypothesis* were true, then the spike train would be completely described a particular kind of random process called a Poisson process.

Probability of a spike sequence



Assumption: The relationship between spikes and stimulus is stochastic

Probability that a spike occurs within an interval Δt :

$$P = p[t]\Delta t$$

Spike train with a probability density $p[t_1, t_2, ..., t_n]$

→ requires to determine all probability densities

Simplification:
An action potential is independent of the presence of other spikes.

Homogeneous Poisson process

Assume the average firing rate of a cell is constant: $r(t) = r$

→ Every sequence of *n* spikes over a fixed time interval has an equal probability. (Example)

Thus, the spike train with a probability $P[t_1, t_2, ..., t_n]$

can be expressed by a probability function that considers only the number of spikes

$$P_T[n]$$

within a duration *T*

Divide the time *T* into *M* bins of size $\Delta t = T / M$

We assume that Δt is small enough such that we never get two spikes within any one bin.

Homogeneous Poisson process

$P_T[n]$ is the product of three factors:

- the probability of generating *n* spikes within *M* bins
- the probability of not generating spikes in the remaining bins
- a combinatorial factor equal to the number of ways of putting *n* spikes into *M* bins

The firing rate *r* determines the probability of firing a single spike in a small interval around the time *t*. The probability of a single spike occurring in one specific bin is

$$r\Delta t$$

The probability of *n* spikes appearing in *n* specific bins is

$$\boxed{(r\Delta t)^n}$$

The probability of not having a spike in a given bin is

$$1 - r\Delta t$$

Homogeneous Poisson process

The probability of having the remaining *M-n* bins without any spikes in them is $\boxed{(1-r\Delta t)^{M-n}}$

The number of ways of putting *n* spikes into *M* bins is given by the binomial coefficient

$$\boxed{\frac{M!}{(M-n)!n!}}$$

n balls from a ballet with *M* balls

As a result we get

$$P_T[n] = \lim_{\Delta t \to 0} \frac{M!}{(M-n)!n!}(r\Delta t)^n (1-r\Delta t)^{M-n}$$

For the limit $\Delta t \to 0$ *M* grows without bound (*n* is fixed)

Homogeneous Poisson process

$$P_T[n] = \lim_{\Delta t \to 0} \frac{M!}{(M-n)!n!}(r\Delta t)^n (1-r\Delta t)^{M-n}$$

For the limit $\Delta t \to 0$ *M* grows without bound (*n* is fixed)

$$M - n \approx M = T/\Delta t$$

$$\lim_{\Delta t \to 0} (1-r\Delta t)^{M-n} \approx \lim_{\Delta t \to 0} (1-r\Delta t)^{M} = \lim_{\varepsilon \to 0} \left((1+\varepsilon)^{\frac{1}{\varepsilon}} \right)^{-rT} = e^{-rT}$$

using $\varepsilon = -r\Delta t$

because by definition $\lim_{\varepsilon \to 0} \left((1+\varepsilon)^{\frac{1}{\varepsilon}} \right) = e$

Homogeneous Poisson process

$$P_T[n] = \lim_{\Delta t \to 0} \frac{M!}{(M-n)!n!}(r\Delta t)^n (1 - r\Delta t)^{M-n}$$

$$\lim_{\Delta t \to 0} (1 - r\Delta t)^{M-n} \approx e^{-rT}$$

For large *M*:

$$\frac{M!}{(M-n)!} \approx M^n = (T/\Delta t)^n$$

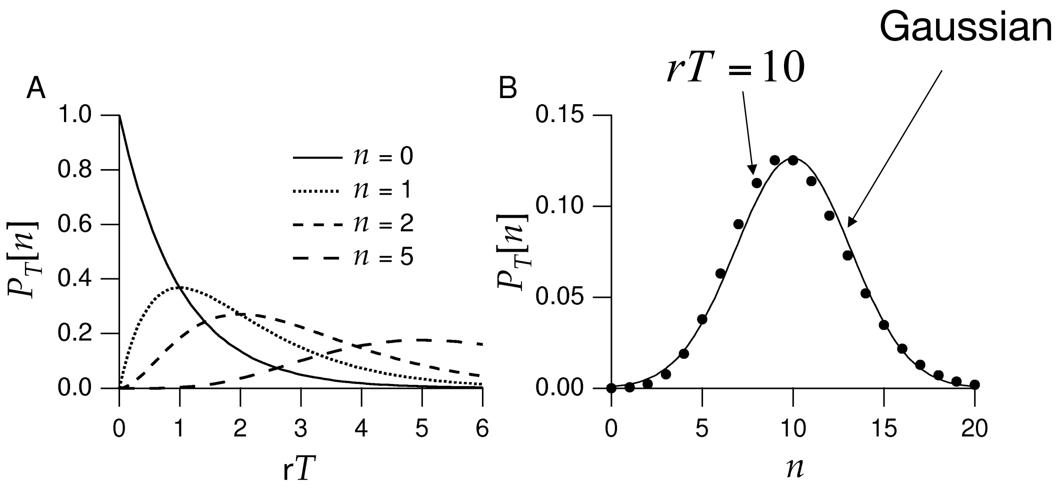
this leads to

$$\boxed{P_T[n] \approx \frac{(T/\Delta t)^n}{n!}(r\Delta t)^n e^{-rT}}$$

Poisson distribution

As a result we get the Poisson distribution

$$P_T[n] = \frac{(rT)^n}{n!} e^{-rT}$$



Poisson spike generator

Simple version:

For designing a spike generator within a computer program, we can use the fact that the probability of firing a spike within a short interval is

$$r\Delta t$$

As long as the rate varies slowly with respect the the time interval we can still use this approach. The rate function $r(t)$ is sampled with a sampling interval of Δt to produce a discrete-time sequence $r[i]$.

The program can simply progress in time through small time steps Δt and generate, at each time step, a random variable x_{rand} between 0 and 1 and compare this with the probability of firing a spike.

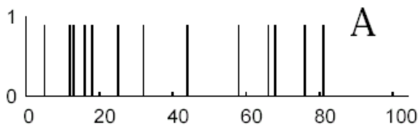
$$r_i \Delta t \begin{cases} > x_{rand} & \text{fire a spike} \\ \leq x_{rand} & \text{nothing} \end{cases}$$

This method is appropriate for small steps Δt, e.g. 1ms and each spike is assigned a discrete time bin, not a continuous time value.

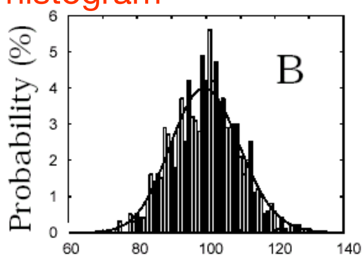
Example:

The instantaneous firing rate was chosen to be r=100 spikes/second, and the time binsize was chosen to be T=1msec.

Poisson spike train



Spike count histogram



Discussion

The poisson model provides a good description of some data, especially considering its simplicity.

However, it does not provide the proper mechanistic explanation of neuronal response variability. Spike generation, by itself, is highly reliable and deterministic, as has been demonstrated by countless numbers of *in vitro* studies.
The noise in *in vivo* neural responses is believed to result from the fact that synapses are very unreliable, not the spike generator!