Model neurons

Electrical circuits

Suggested reading:

Model neurons: Electrical circuits

Contents:
- Ohm's law
- Capacitor
- Kirchhoff's law
Ohm’s law

Biophysical models of single cells involve equivalent circuits composed of resistors, capacitors, and voltage and current sources.

A resistor satisfies Ohm’s law, which states that the voltage $V_R = V_1 - V_2$ across a resistance $R$ carrying a current $I_R$ is $V_R = I_R R$.

Resistance is measured in ohms ($\Omega$) defined as the resistance through which one ampere of current causes a voltage drop of one volt ($1\ V = 1\ A \times 1\ \Omega$).

Capacitor

A capacitor stores charge across an insulating medium, and the voltage across it $V_C = V_1 - V_2$ is related to the charge it stores $Q_C$ by $CV_C = Q_C$ where $C$ is the capacitance.

Electrical current cannot cross the insulating medium, but charges can be redistributed on each side of the capacitor, which leads to the flow of current.

We can take a time derivative of both sides of the eq. above and use the fact that current is equal to the rate of change of charge, $I_C = \frac{dQ_C}{dt}$, to obtain the basic voltage-current relationship for a capacitor.
Capacitor

Basic voltage-current relationship for a capacitor:

\[ C \frac{dV_C}{dt} = I_C \]

Capacitance is measured in units of farads (F) defined as the capacitance for which one ampere of current causes a voltage change of one volt per second (1 F \times 1 V/s = 1 A).

Kirchhoff’s law

Kirchoff’s laws state that:

• voltage differences around any closed loop in a circuit must sum to zero
• the sum of all the currents entering any point in a circuit must be zero

Applying the second of these rules to the circuit on the left, we find that \( I_1 = I_2 \). Ohm’s law tells us that \( V_1 - V_2 = I_1 R_1 \) and \( V_2 = I_2 R_2 \). Solving these gives \( V_1 = I_1 (R_1 + R_2) \), which tells us that resistors arranged in series add, and \( V_2 = \frac{V_1 R_2}{R_1 + R_2} \), which is why this circuit is called a voltage divider.
Kirchhoff’s law

For the circuit on the left, Kirchoff’s and Ohm’s laws tells us that

\[ I_e = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}. \]

This indicates how resistors add in parallel, \( V = I_e \frac{R_1 R_2}{R_1 + R_2} \).

Parallel resistor circuit. \( I_e \) represents an external current source.

Kirchhoff’s law

Kirchoff’s laws require that \( I_C + I_R = 0 \):

\[ C \frac{dV}{dt} = I_C = -I_R = -\frac{V}{R} \]

Solving this, gives

\[ V(t) = V(0) \exp\left(-\frac{t}{RC}\right) \]

showing the exponential decay (with time constant \( \tau = RC \)) of the initial voltage \( V(0) \) as the charge on the capacitor leaks out through the resistor.

RC circuits: Current \( I_C = -I_R \) flows in the resistor-capacitor circuit as the stored charge is released.
Kirchhoff’s law

Two extra components needed to build a simple model neuron, the voltage source $E$ and the current source $I_e$. Using Kirchhoff’s laws, $I_e - I_C - I_R = 0$, and the equation for the voltage $V$ is

$$C \frac{dV}{dt} = \frac{E - V}{R} + I_e$$

If $I_e$ is constant, the solution of this equation is

$$V(t) = V_\infty + (V(0) - V_\infty) \exp(-t/\tau)$$

where $V_\infty = E + R I_e$ and $\tau = RC$. This shows exponential relaxation from the initial potential $V(0)$ to the equilibrium potential $V_\infty$ at a rate governed by the time constant $\tau$ of the circuit.