Artificial Intelligence & Neuro Cognitive Systems
Fakultät für Informatik

Velocity Kinematics - Examples

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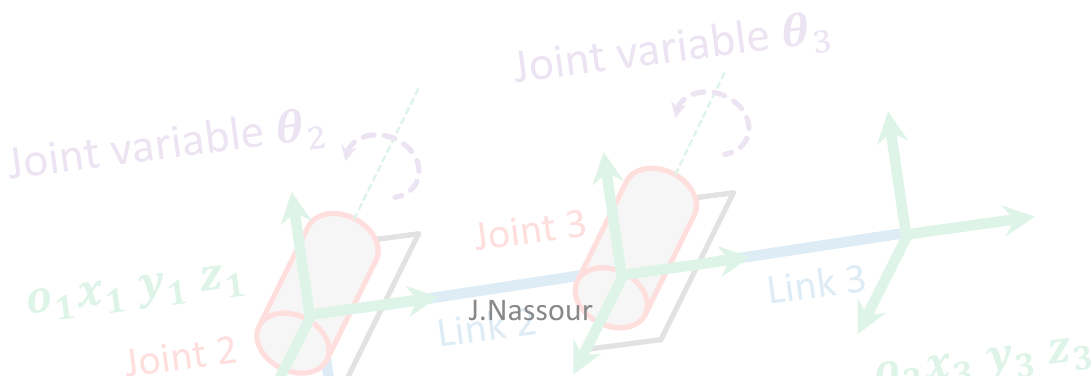
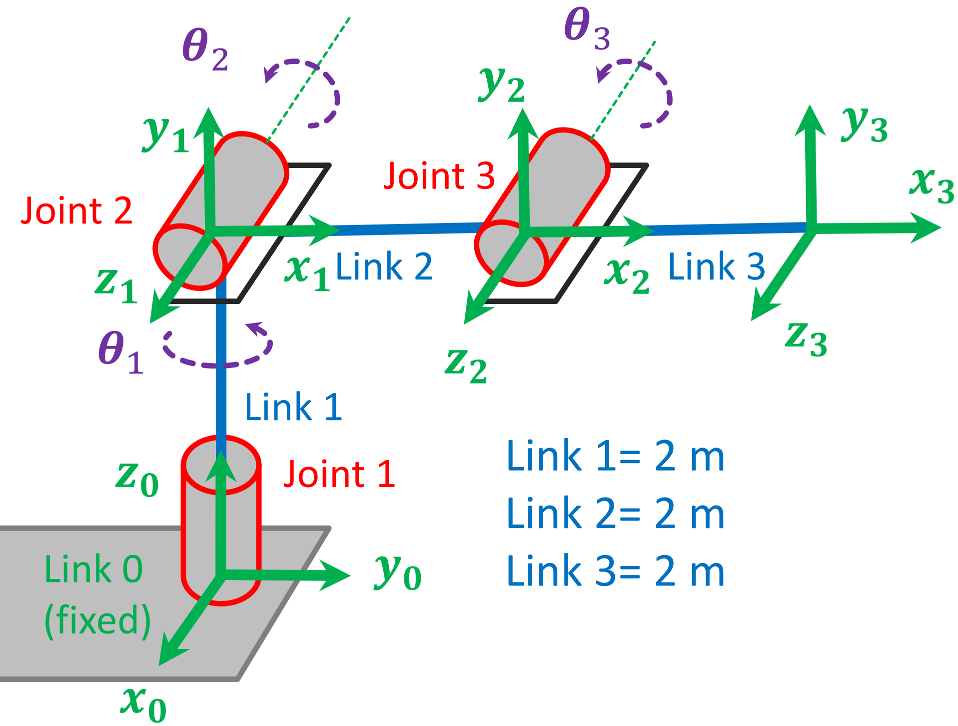


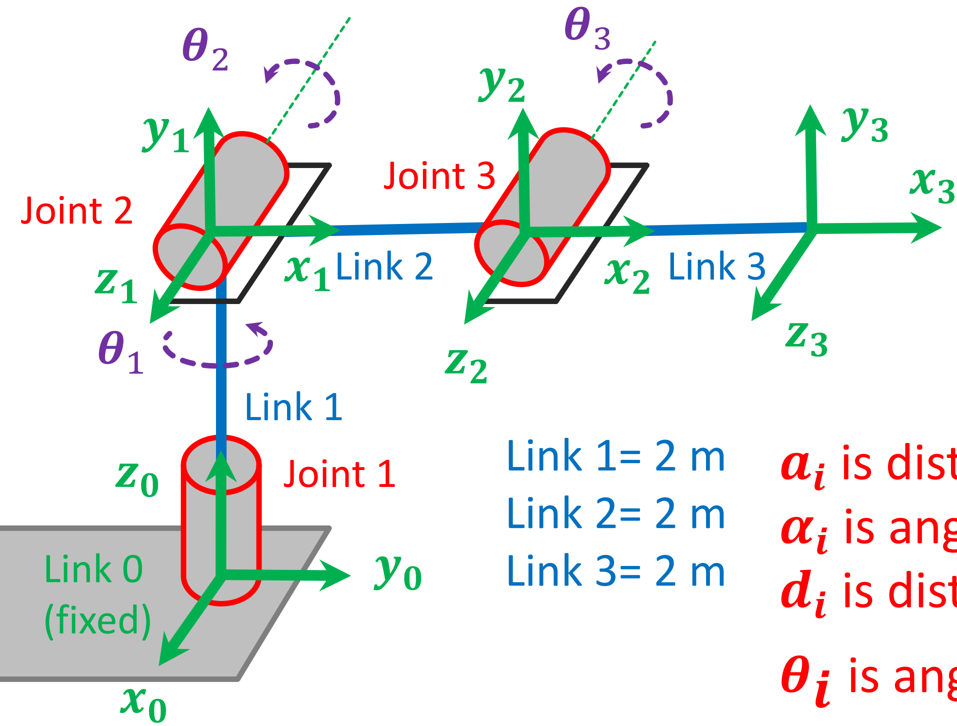
Elbow Manipulator



Link 1= 2 m
Link 2= 2 m
Link 3= 2 m

Work out the linear velocity Jacobian at the end-effector then discuss the singular configurations.

Elbow Manipulator



Find DH parameters for this robot. Identify the joint variables.

Link 1= 2 m
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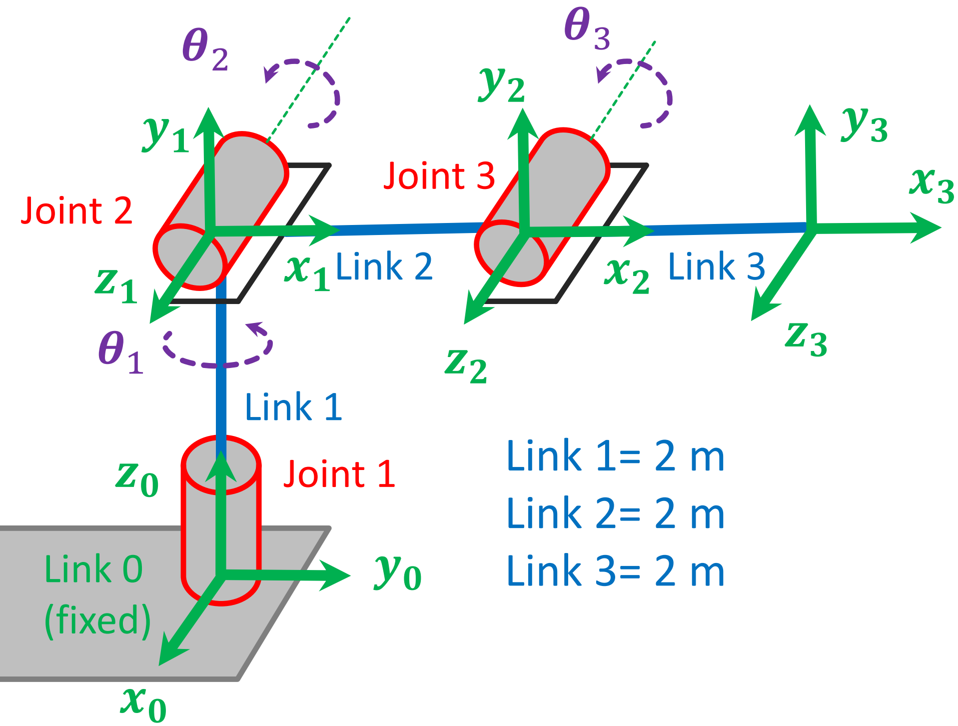
a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

i	a_i	α_i	d_i	θ_i
1				
2				
3				

Elbow Manipulator



Link 1= 2 m
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Link 3= 2 m

Find the A matrices

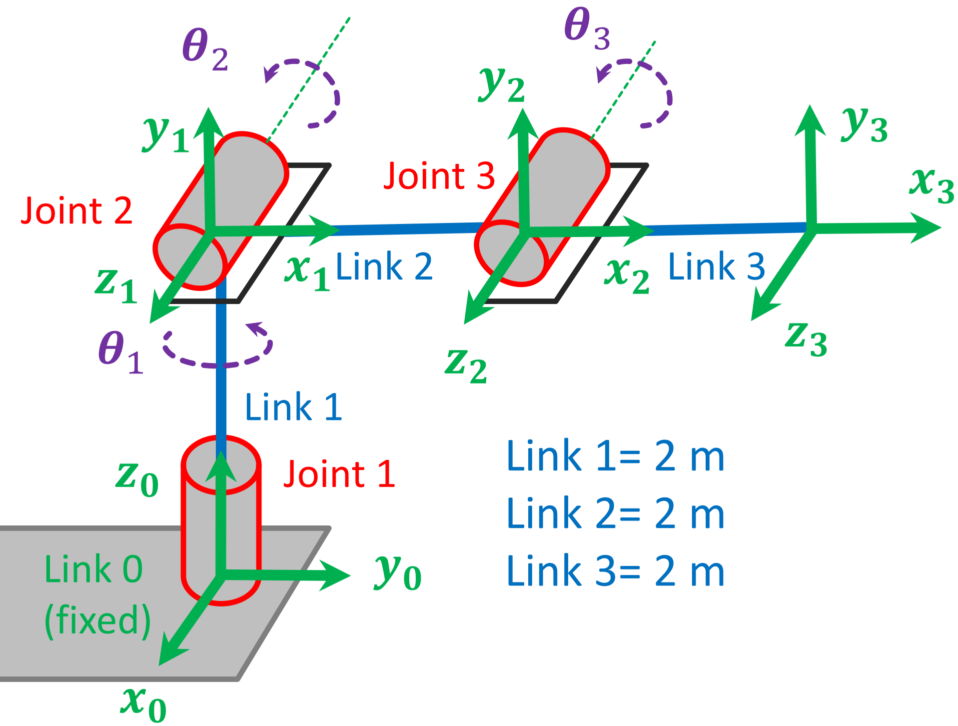
Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	a_i	α_i	d_i	θ_i
1				
2				
3				

Elbow Manipulator

Find the T matrices



Link 1= 2 m

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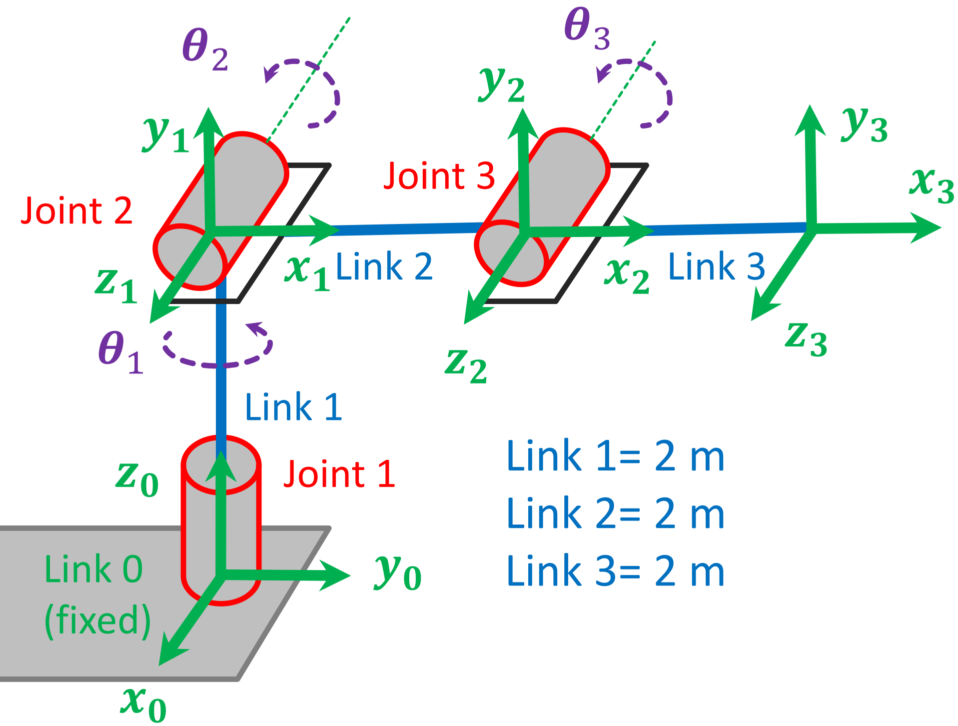
Link 3= 2 m

Elbow Manipulator



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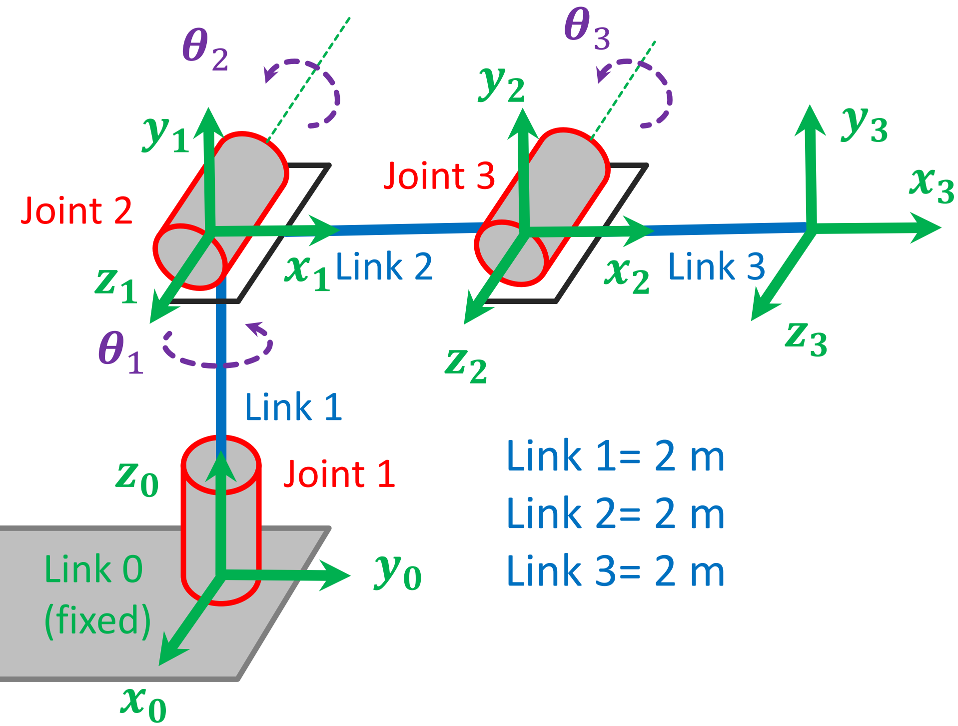
Find the linear velocity Jacobian.

For Prismatic joint:
$\mathcal{J}_{v_i} = z_{i-1}^0$
For Revolute joint:
$\mathcal{J}_{v_i} = z_{i-1}^0 \times (o_n - o_{i-1})$

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - b_y a_z \\ a_z b_x - b_z a_x \\ a_x b_y - b_x a_y \end{pmatrix}$$

Elbow Manipulator

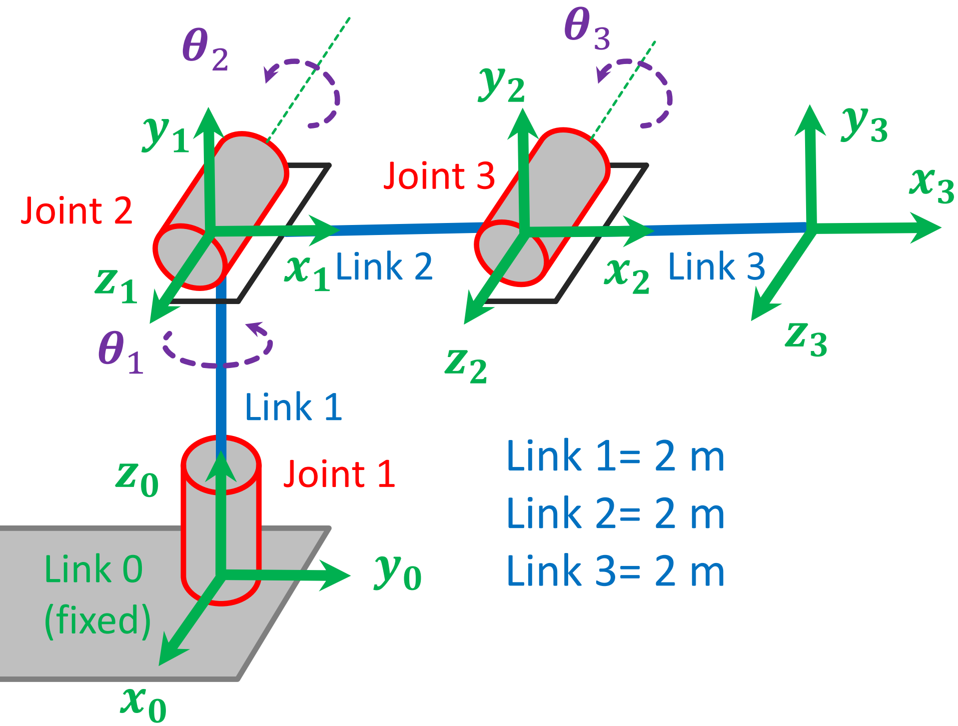


Link 1= 2 m
Link 2= 2 m
Link 3= 2 m

Work out the linear velocity Jacobian at the end-effector then discuss the singular configurations.

$$J = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} & -a_2 s_2 c_1 - a_3 s_{23} c_1 & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_2 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{bmatrix}$$

Elbow Manipulator



Link 1= 2 m

Link 2= 2 m

Link 3= 2 m

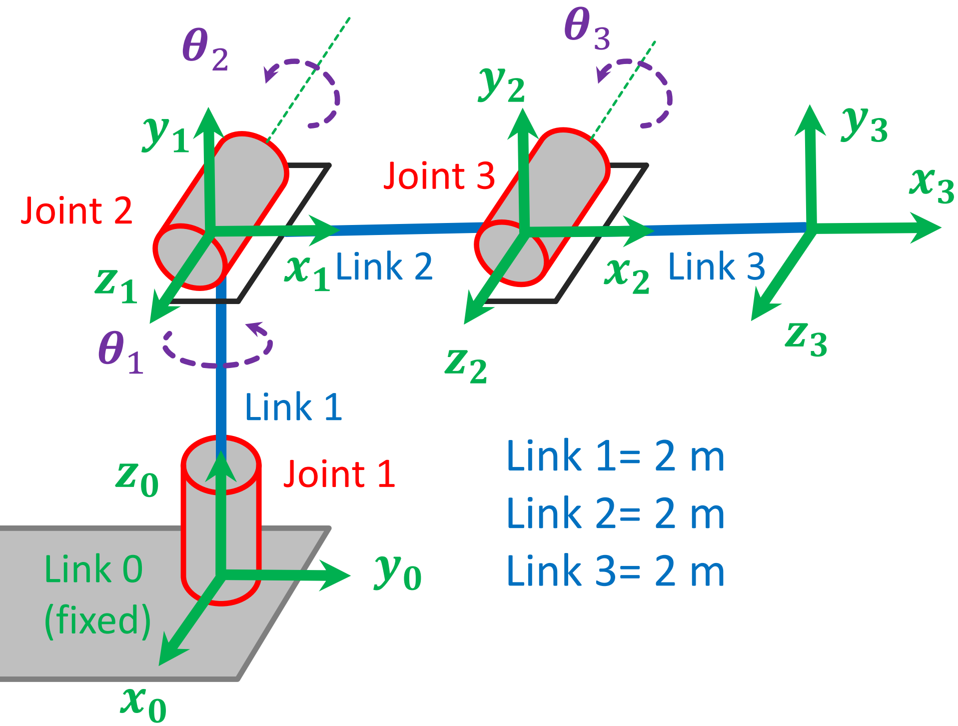
Work out the linear velocity Jacobian at the end-effector then discuss the singular configurations.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$

Elbow Manipulator



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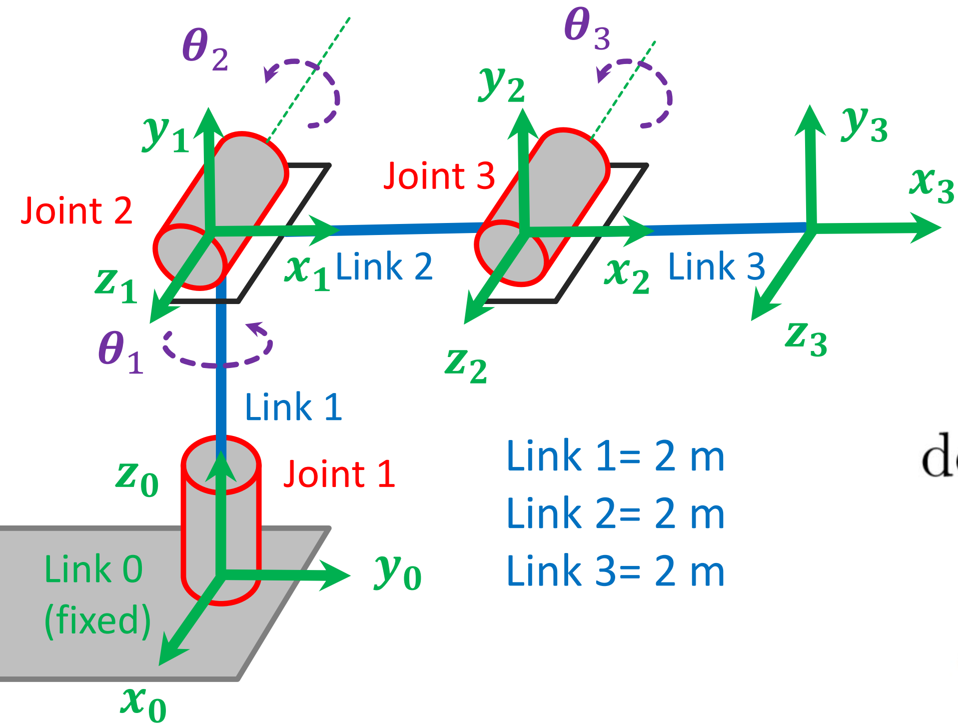
Work out the linear velocity Jacobian at the end-effector then discuss the singular configurations.

$$J = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} & -a_2 s_2 c_1 - a_3 s_{23} c_1 & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_2 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{bmatrix}$$

$$\det J = a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23})$$

Elbow Manipulator

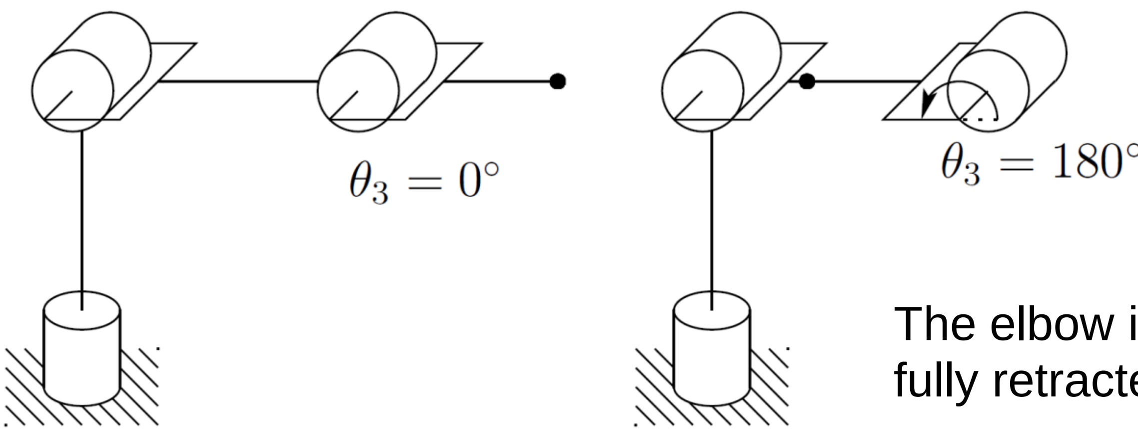


Link 1= 2 m
Link 2= 2 m
Link 3= 2 m

Singularities …

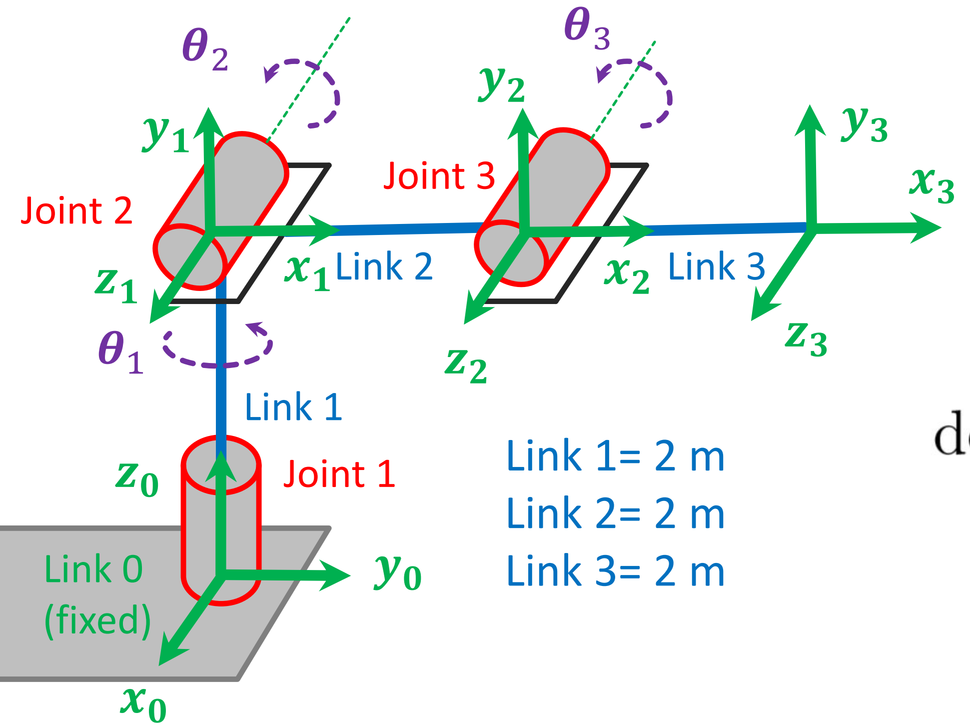
$$\det J = a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23})$$

$$s_3 = 0, \quad \textit{that is, } \theta_3 = 0 \textit{ or } \pi$$



The elbow is fully extended or fully retracted

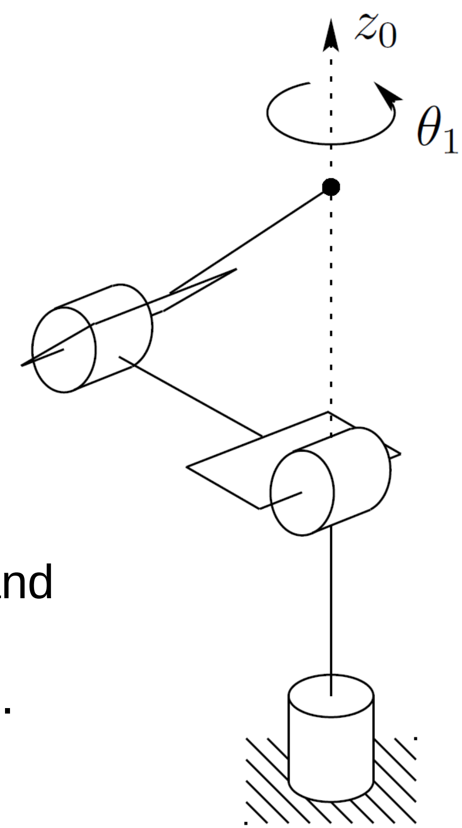
Elbow Manipulator



Singularities …

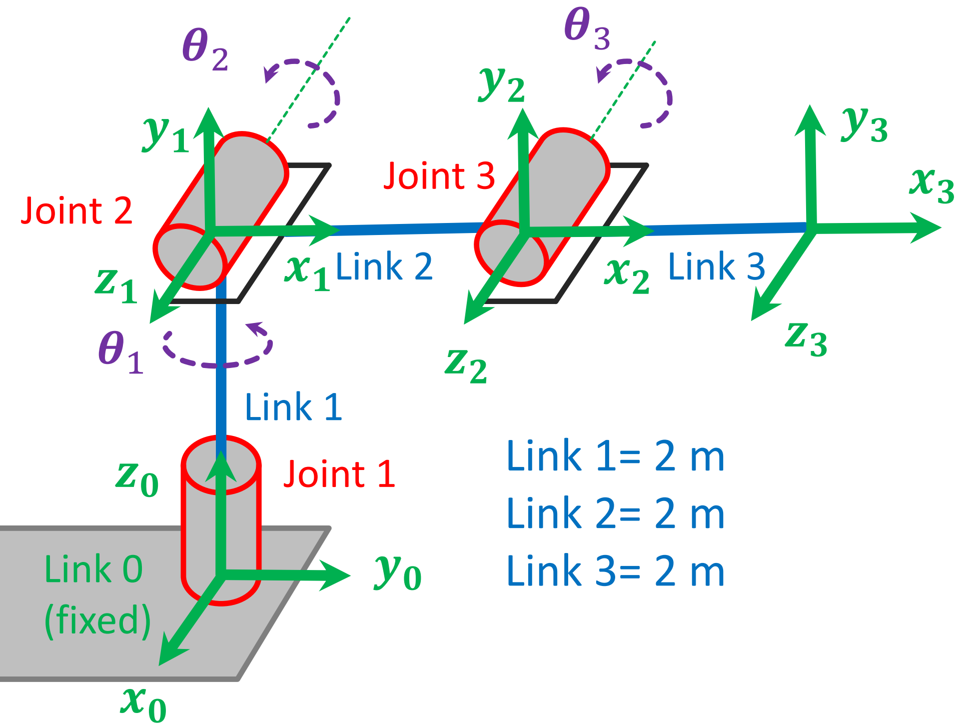
$$\det J = a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23})$$

$$a_2 c_2 + a_3 c_{23} = 0$$



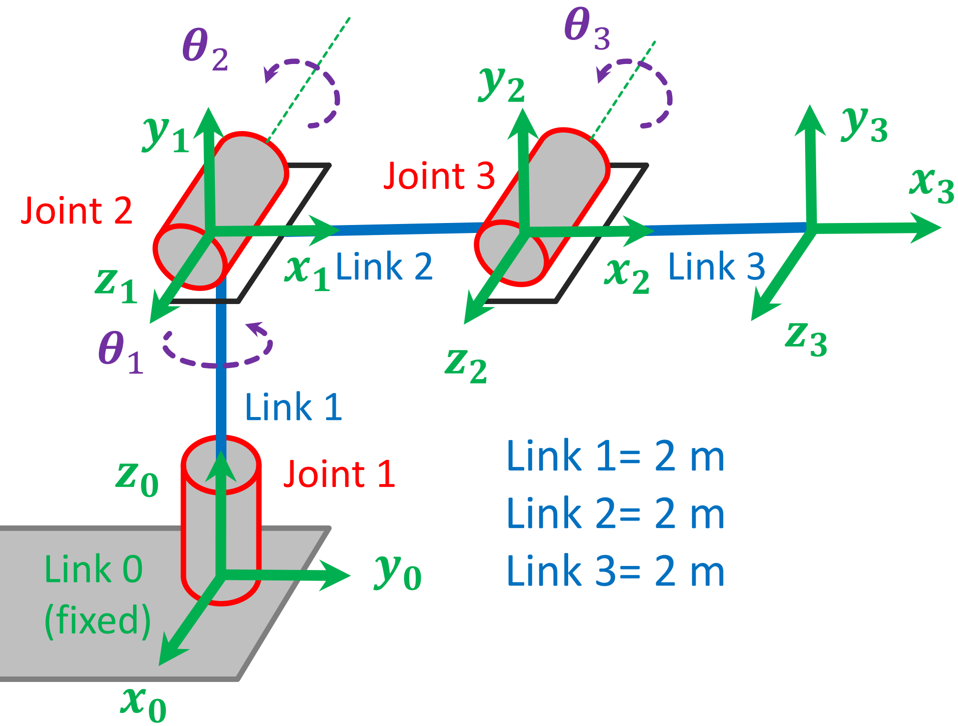
The wrist center intersects the axis of the base rotation, Z_0.
There are infinitely many singular configurations and infinitely many solutions to the inverse position kinematics when the wrist center is along this axis.

Elbow Manipulator



Work out the angular velocity Jacobian at the end-effector.

Elbow Manipulator



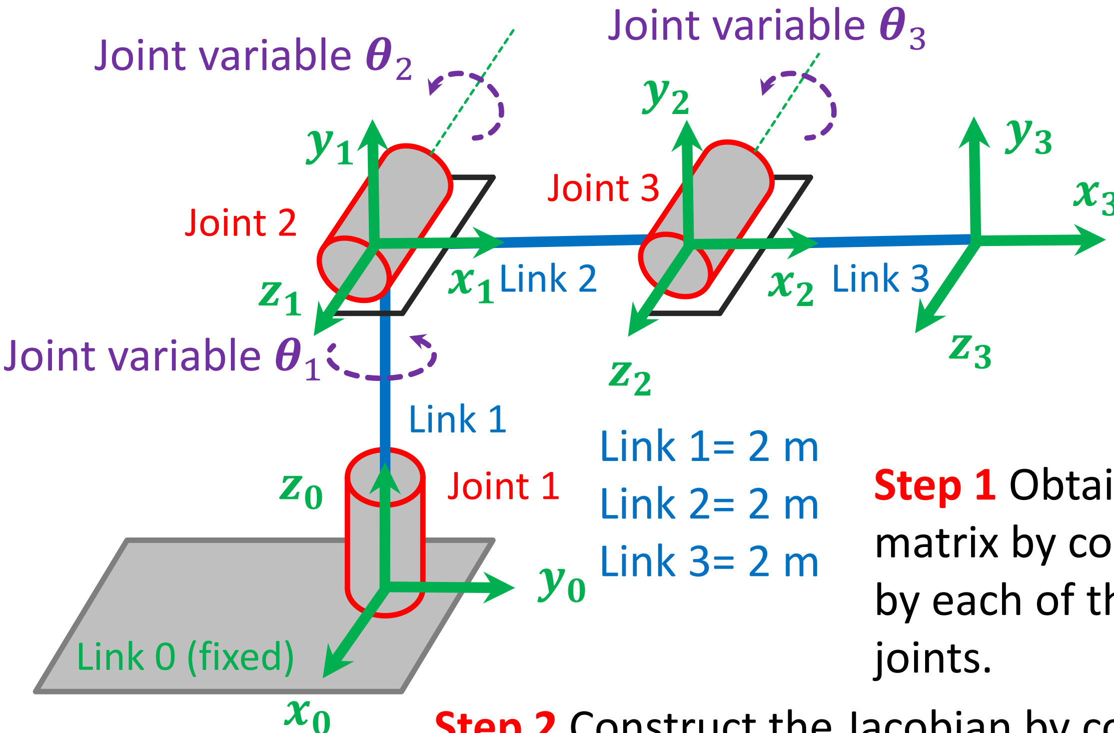
Link 1= 2 m
Link 2= 2 m
Link 3= 2 m

Work out the angular velocity Jacobian at the end-effector.

$$\mathcal{J}_\omega = [\rho_1 z_0 \quad \rho_2 z_1 \quad \cdots \quad \rho_n z_{n-1}]$$

$\rho_1 = ?$, $\rho_2 = ?$, $\rho_3 = ?$

Elbow Manipulator



The robot has three revolute joints that allow the endpoint to move in the three dimensional space. However, this robot mechanism has singular points inside the workspace. Analyze the singularity, following the procedure below.

Step 1 Obtain each column vector of the Jacobian matrix by considering the endpoint velocity created by each of the joints while immobilizing the other joints.

Step 2 Construct the Jacobian by concatenating the column vectors, and set the determinant of the Jacobian to zero for singularity: det J =0.

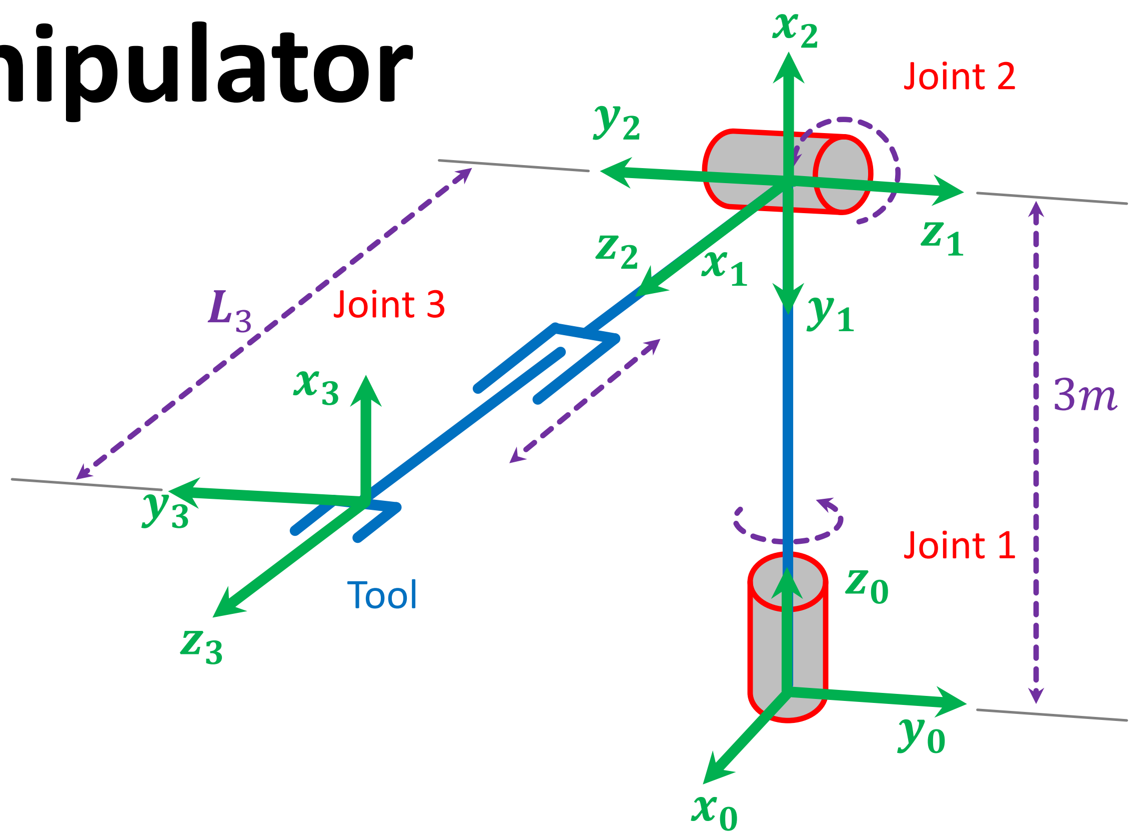
Step 3 Find the joint angles that make det J =0.

Step 4 Show the arm posture that is singular. Show where in the workspace it becomes singular. For each singular configuration, also show in which direction the endpoint cannot have a non-zero velocity.

Spherical Manipulator

Singularities …

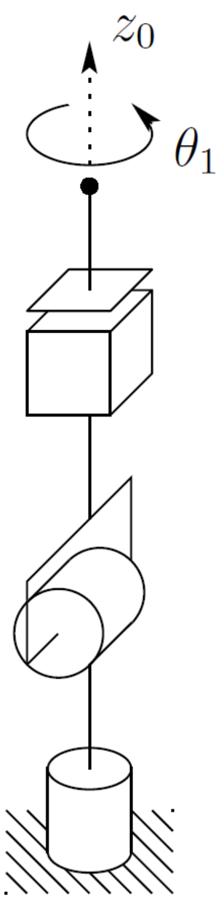
$$\mathcal{J}_v = \begin{bmatrix} L_3 s_1 s_2 & -L_3 c_1 c_2 & -c_1 s_2 \\ -L_3 c_1 s_2 & -L_3 s_1 c_2 & -s_1 s_2 \\ 0 & L_3\ s_2 & -c_2 \end{bmatrix}$$

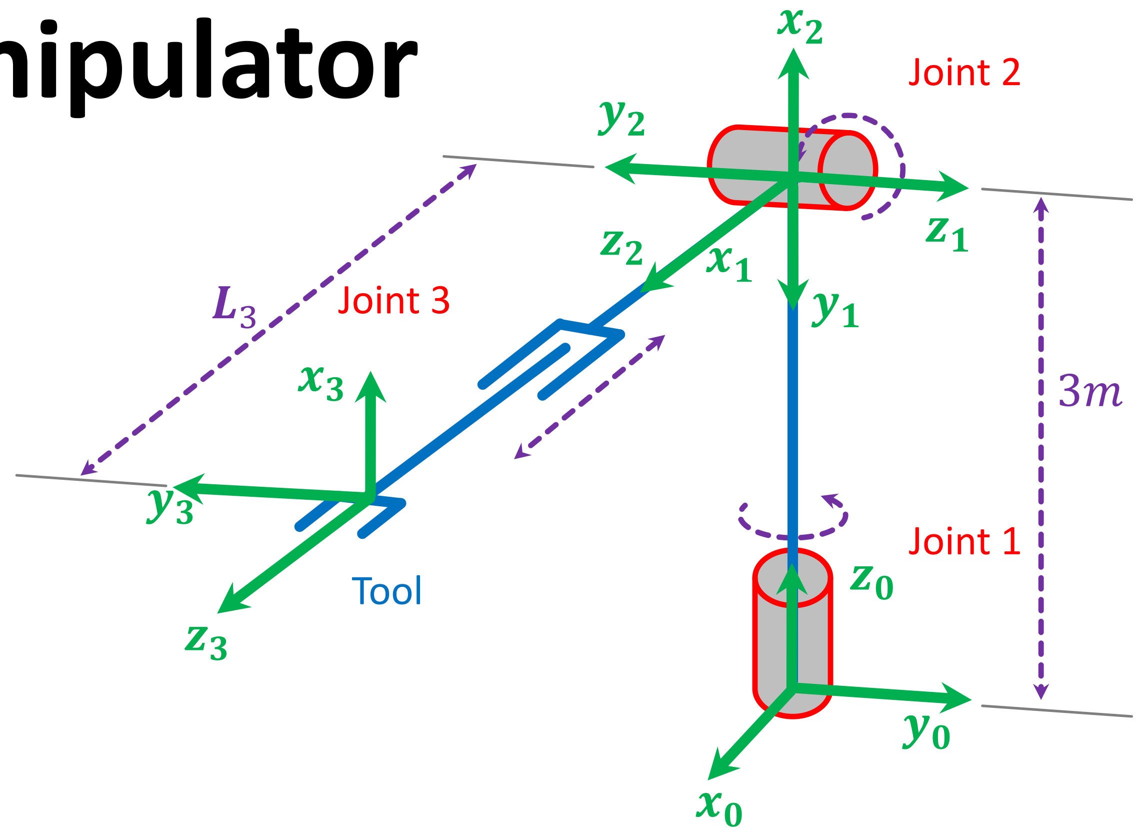


$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$
$$= aei + bfg + cdh - ceg - bdi - afh.$$

Spherical Manipulator

Singularities …





The manipulator is in a singular configuration when the wrist center intersects z_0, any rotation about the base leaves this point fixed.

Reminder

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh.$$

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{\begin{bmatrix} ei - fh & hc - ib & bf - ce \\ gf - di & ai - gc & dc - af \\ dh - ge & gb - ah & ae - db \end{bmatrix}}{|\mathbf{A}|}$$

$$= \frac{\begin{bmatrix} ei - fh & hc - ib & bf - ce \\ gf - di & ai - gc & dc - af \\ dh - ge & gb - ah & ae - db \end{bmatrix}}{aei + bfg + cdh - gec - hfa - idb}$$