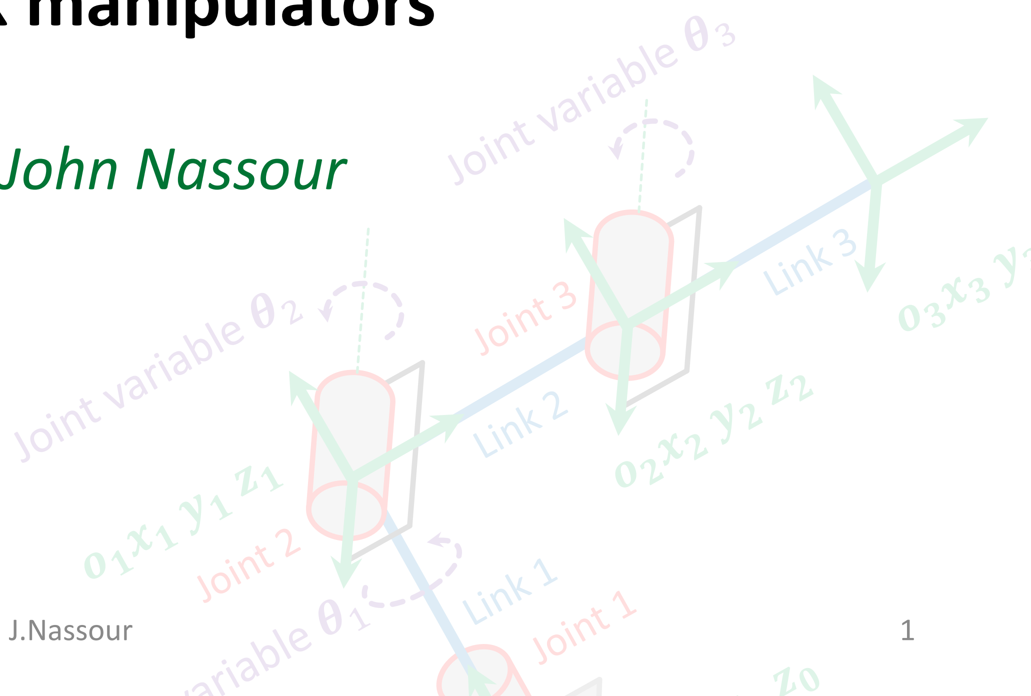
Artificial Intelligence & Neuro Cognitive Systems
Fakultät für Informatik

Forward Kinematics

Serial link manipulators

Dr.-Ing. John Nassour

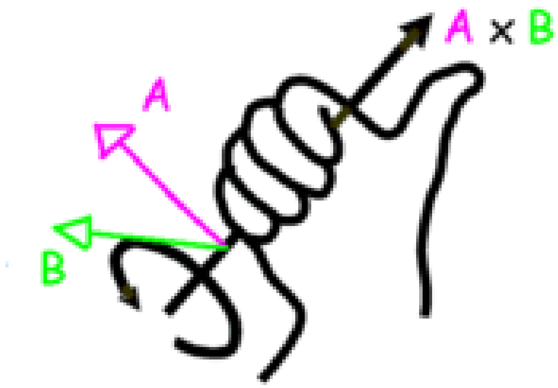


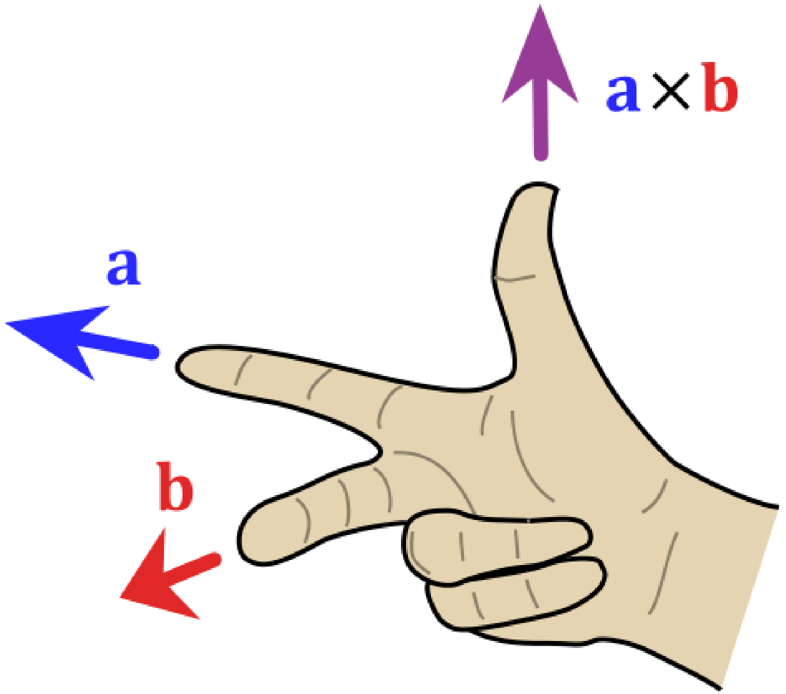
Suggested literature

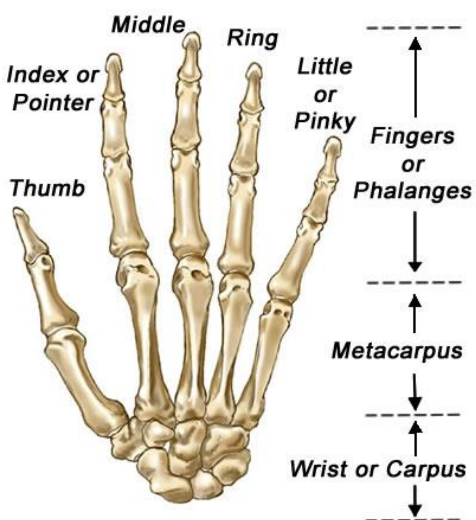
- Robot Modeling and Control
- **Robotics: Modelling, Planning and Control**

Reminder: Right Hand Rules

Cross product

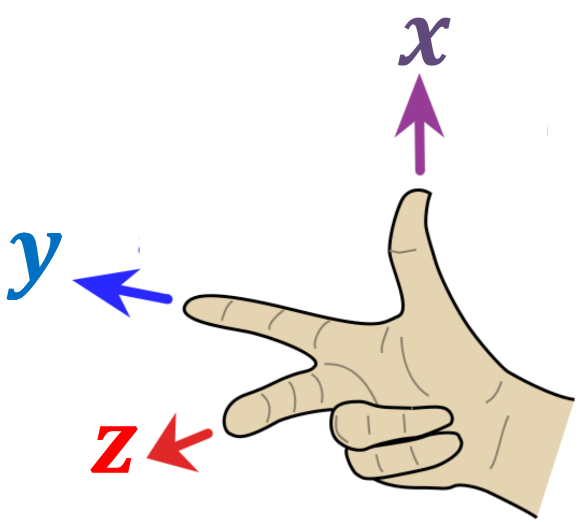


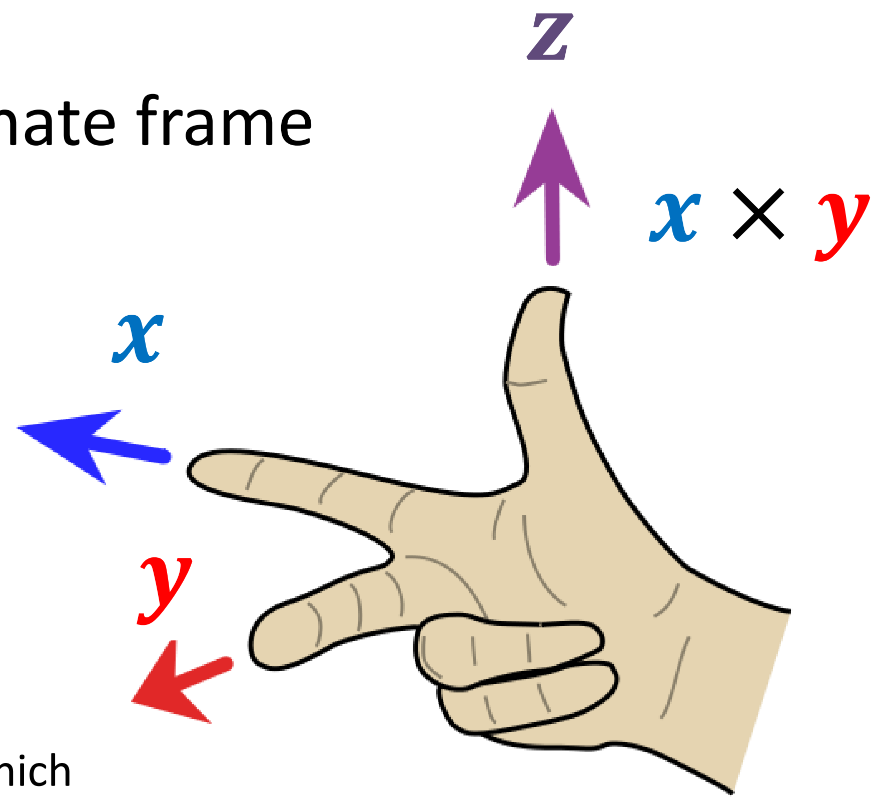




Reminder: Right Hand Rules

A right-handed coordinate frame



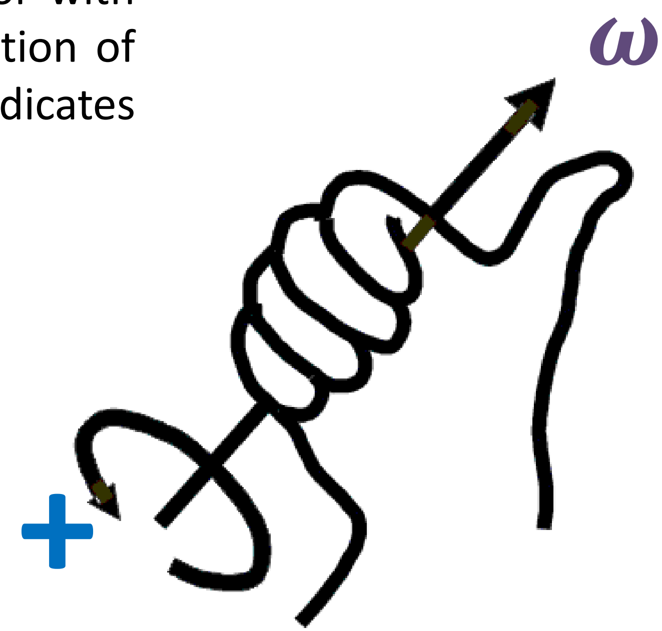


The first three fingers of your right hand which indicate the relative directions of the *x*-, *y*- and *z*-axes respectively.

Reminder: Right Hand Rules

Rotation about a vector

Wrap your right hand around the vector with your thumb (your *x*-finger) in the direction of the arrow. The curl of your fingers indicates the direction of increasing angle.



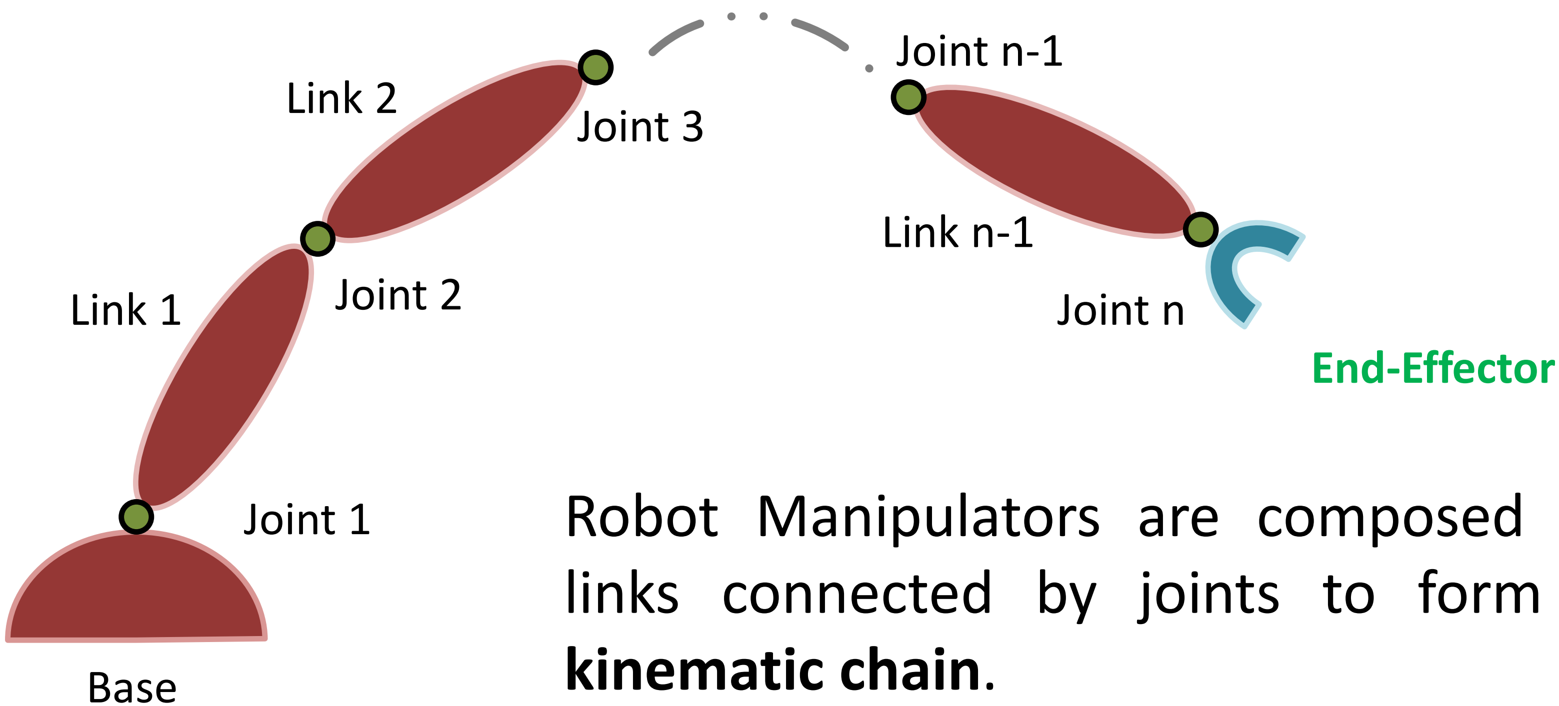
Kinematics

The problem of kinematics is to describe the motion of the manipulator without consideration of the forces and torques causing that motion.

The kinematic description is therefore a geometric one.

Forward Kinematics

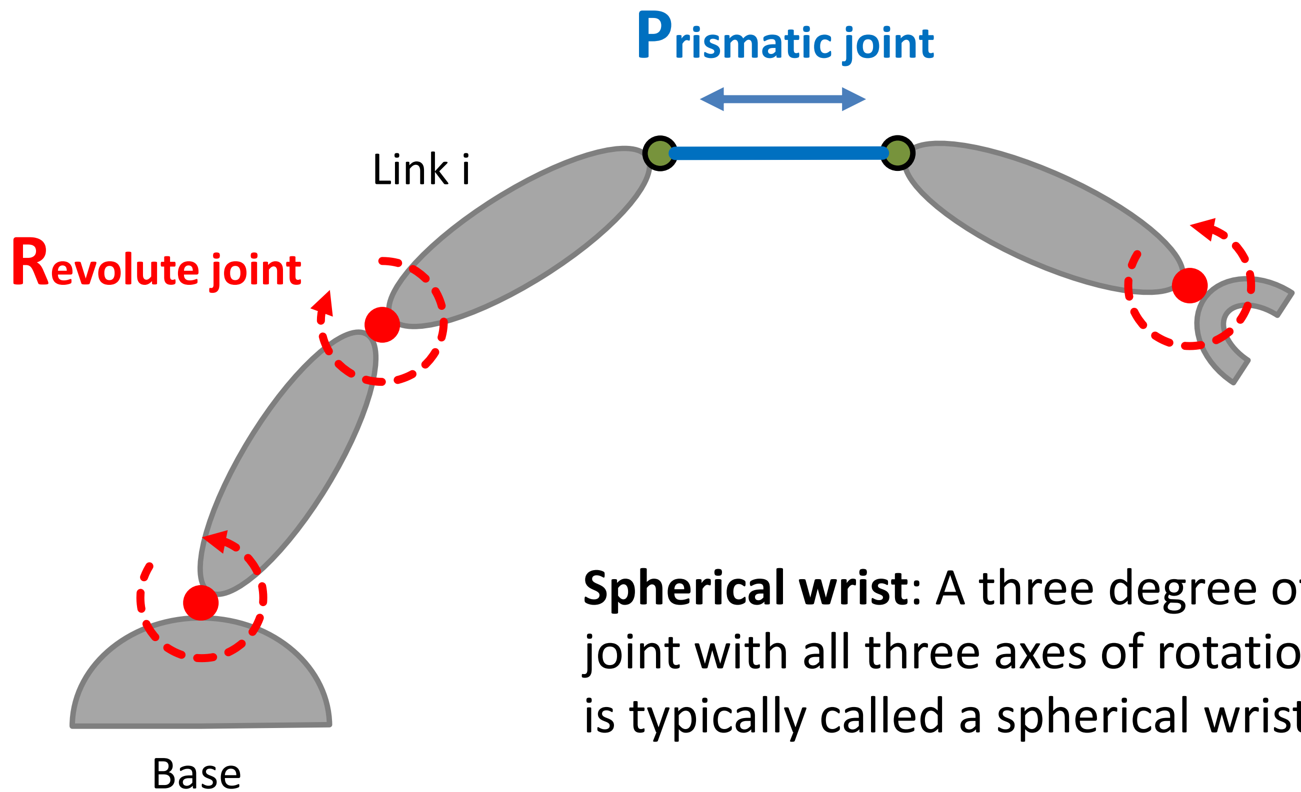
Determine the position and orientation of the end-effector given the values for the joint variables of the robot.



Robot Manipulators are composed of links connected by joints to form a **kinematic chain**.

Robot Manipulators

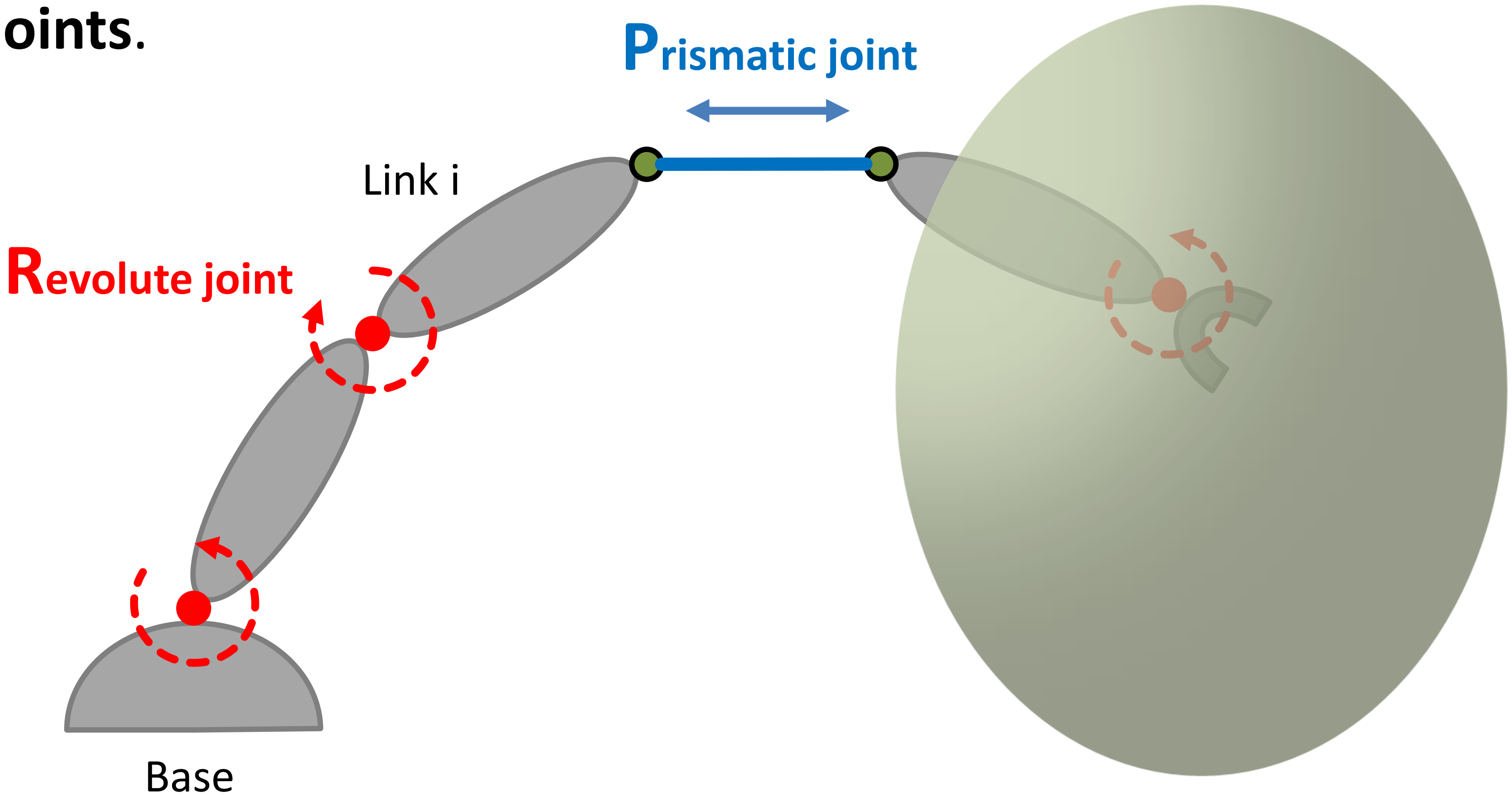
Revolute joint (R): allows a relative rotation about a single axis.
Prismatic joint (P): allows a linear motion along a single axis (extension or retraction).



Spherical wrist: A three degree of freedom rotational joint with all three axes of rotation crossing at a point is typically called a spherical wrist.

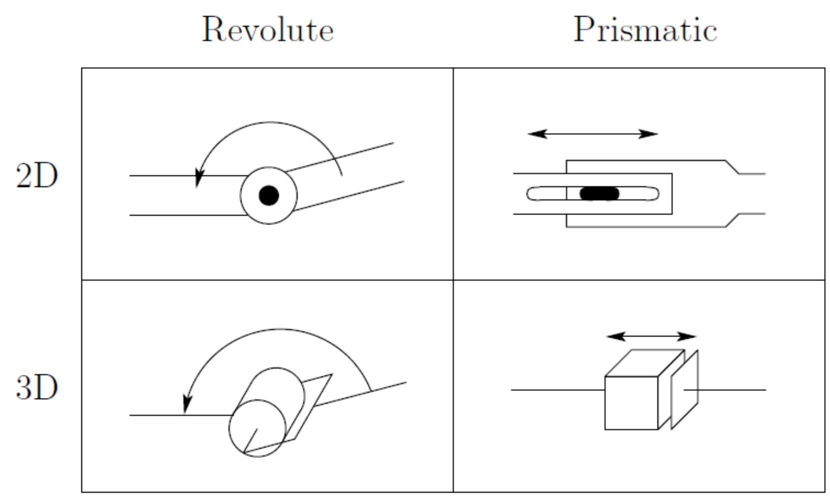
The Workspace Of A Robot

The total volume its end - effector could sweep as the robot executes all possible motions. It is constrained by **the geometry of the manipulator** as well as **mechanical limits imposed on the joints**.



Robot Manipulators

Symbolic representation of robot joints



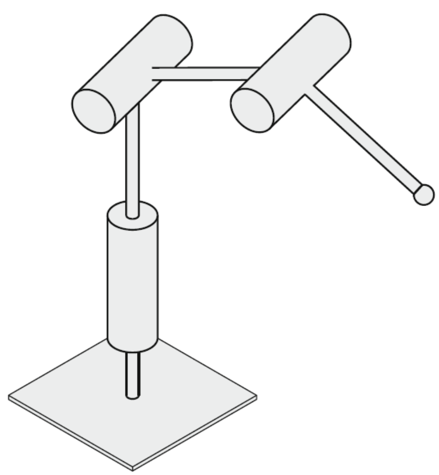
e.g. A three-link arm with three revolute joints was denoted by RRR.

Joint variables, denoted by $\boldsymbol{\theta}$ for a revolute joint and $\boldsymbol{d}$ for the prismatic joint, represent the relative displacement between adjacent links.

Articulated Manipulators (RRR)

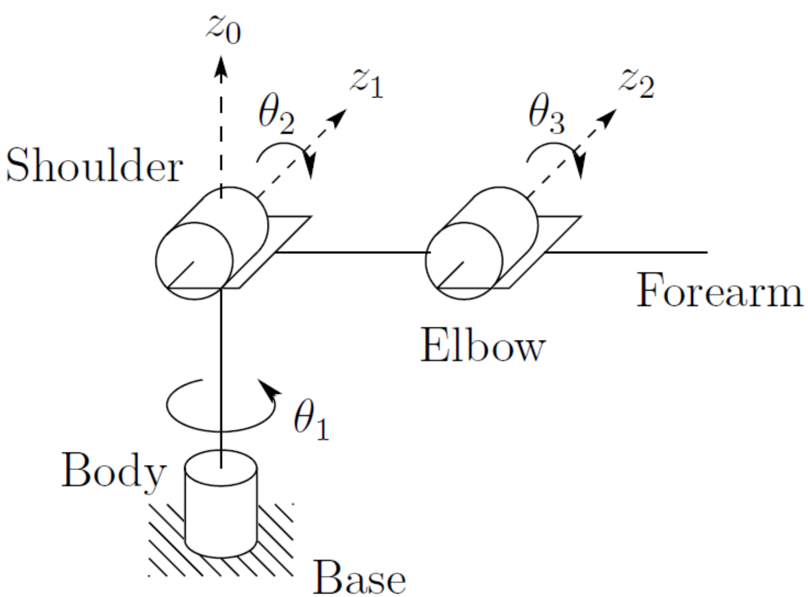
Articulated Manipulators (RRR)

Also called: Anthropomorphic Manipulators



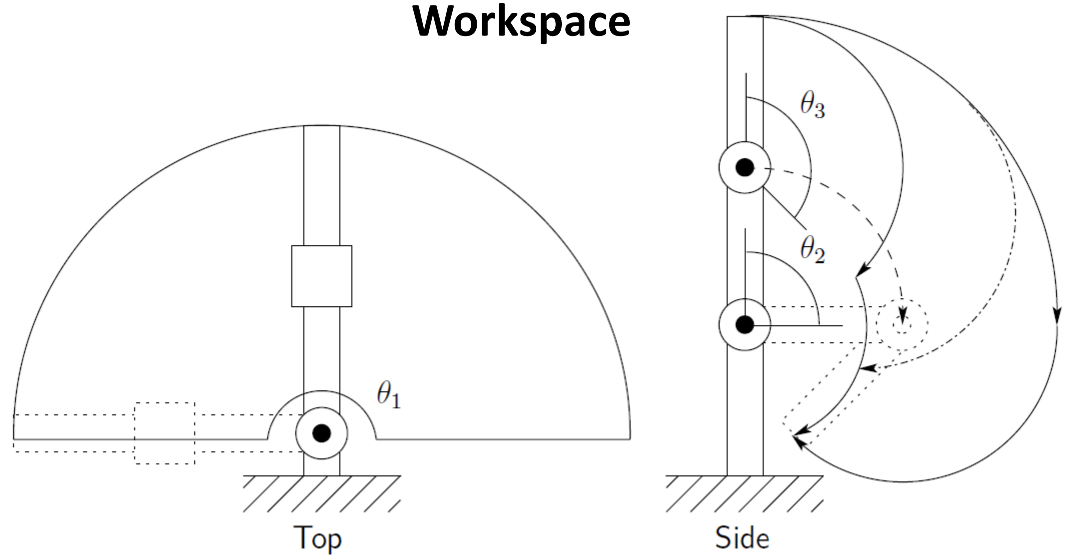
Three joints of the rotational type (**RRR**).
It resembles the human arm.
The second joint axis is perpendicular to the first one.
The third joint axis is parallel to the second one.
The workspace of the anthropomorphic robot arm, encompassing all the points that can be reached by the robot end point.

Elbow Manipulator (RRR)

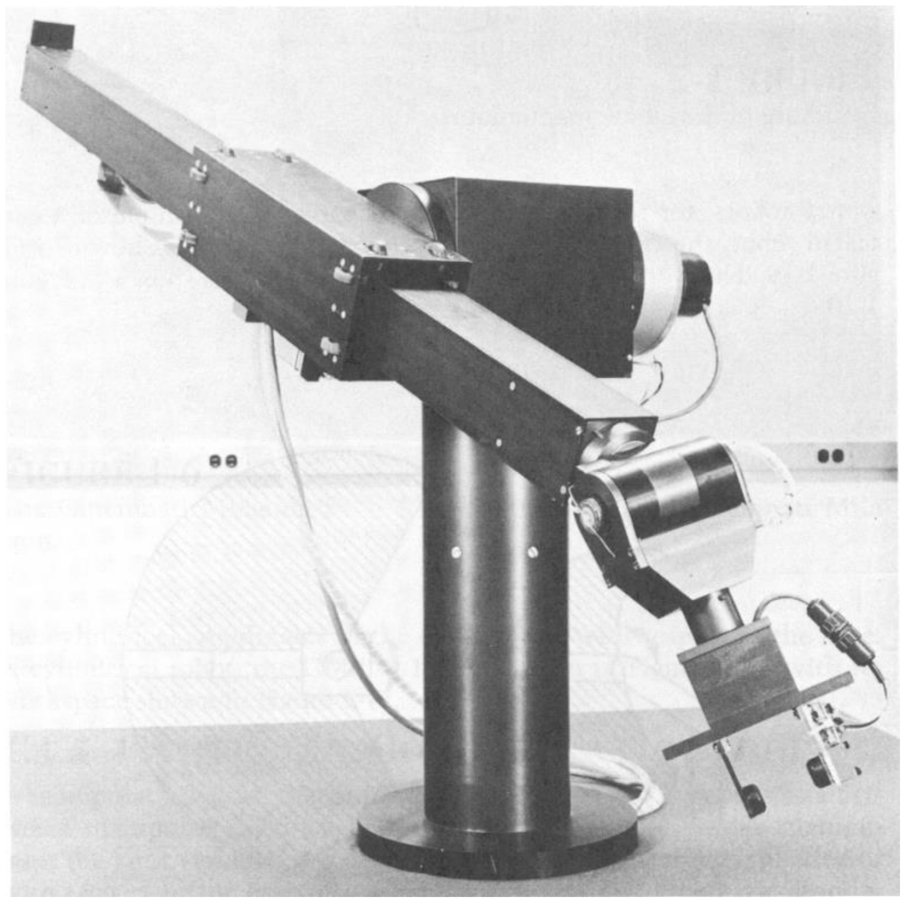


Structure

Workspace



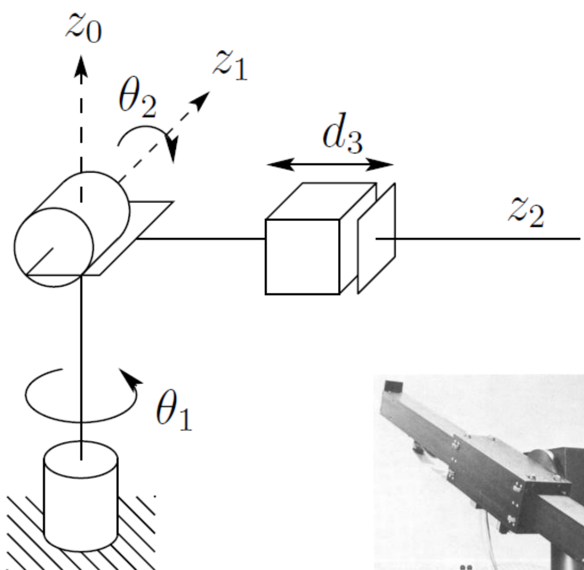
Spherical Manipulator



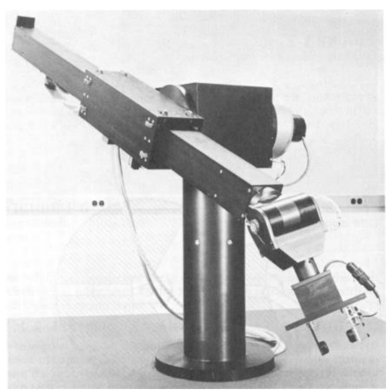
The Stanford Arm

Spherical Manipulator RRP

Two rotation and one translation (RRP).
The second joint axis is perpendicular to the first one and the third axis is perpendicular to the second one.

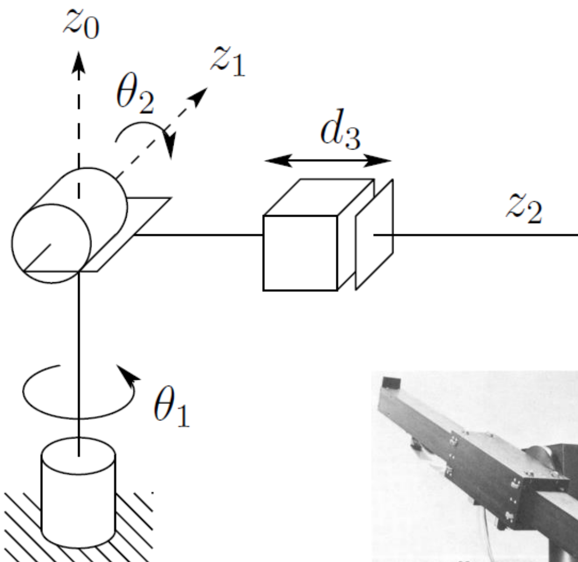


Structure

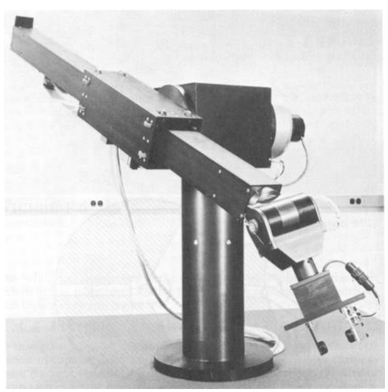


Workspace

Spherical Manipulator RRP

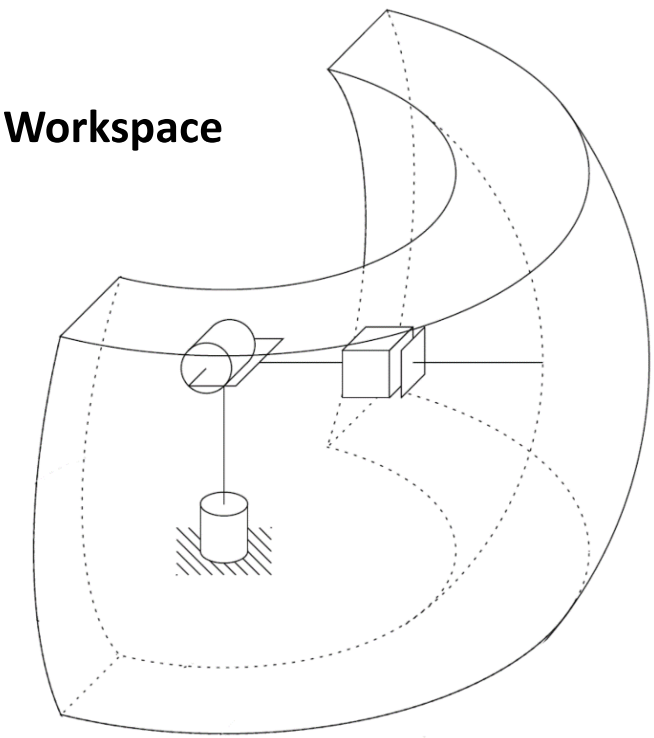


Two rotation and one translation (RRP).
The second joint axis is perpendicular to the first one and the third axis is perpendicular to the second one.
The workspace of the robot arm has a spherical shape as in the case of the anthropomorphic robot arm.



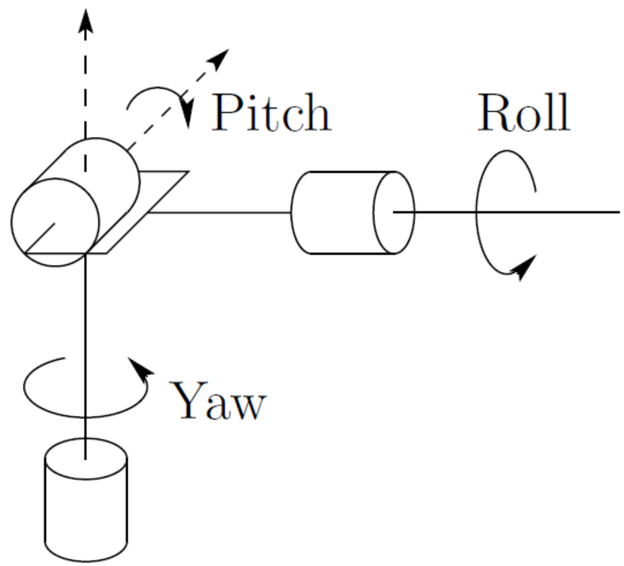
Structure

Workspace



Spherical Manipulator RRR

Workspace?

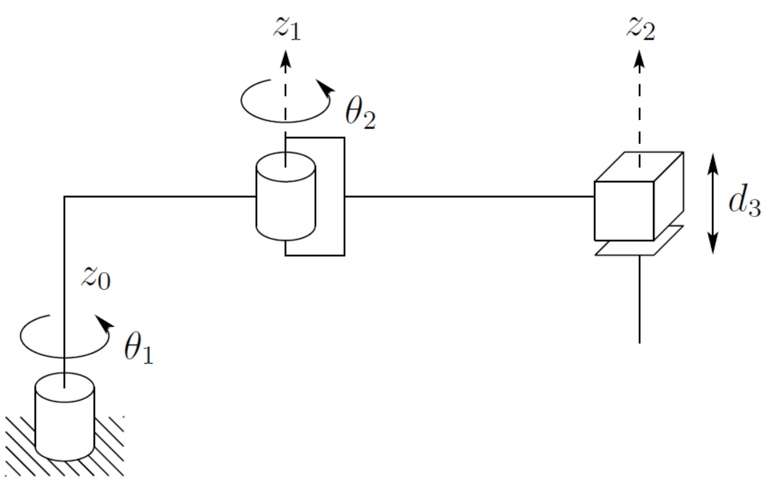


Structure

SCARA Manipulator

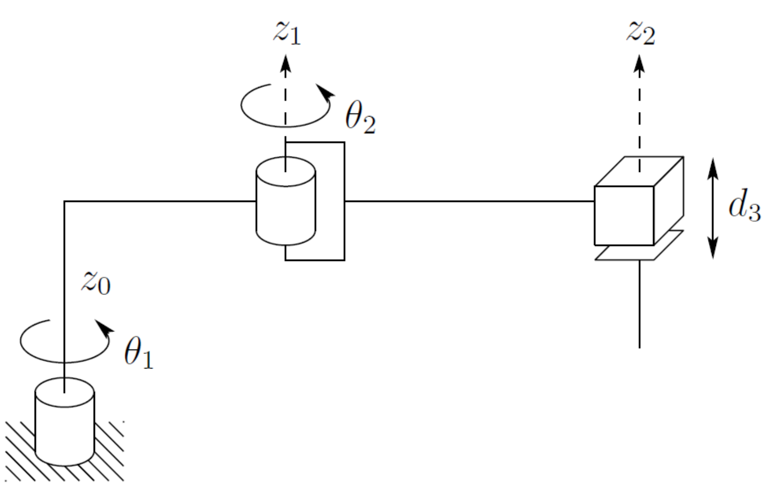
Two joints are rotational and one is translational (RRP).
The axes of all three joints are parallel.

Workspace

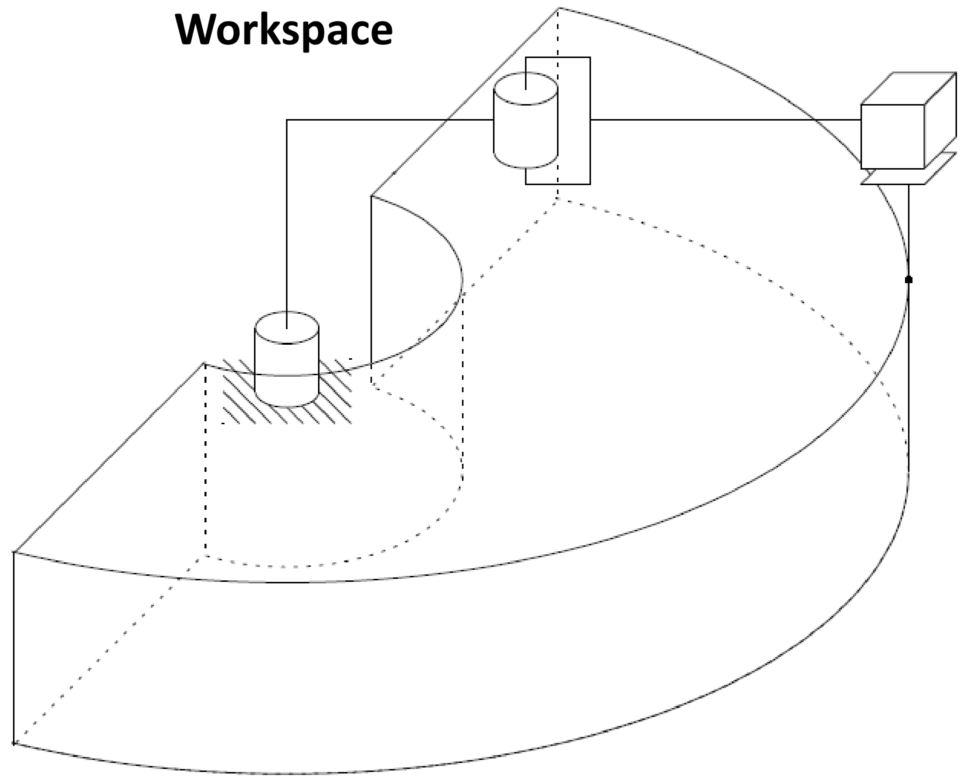


SCARA Manipulator

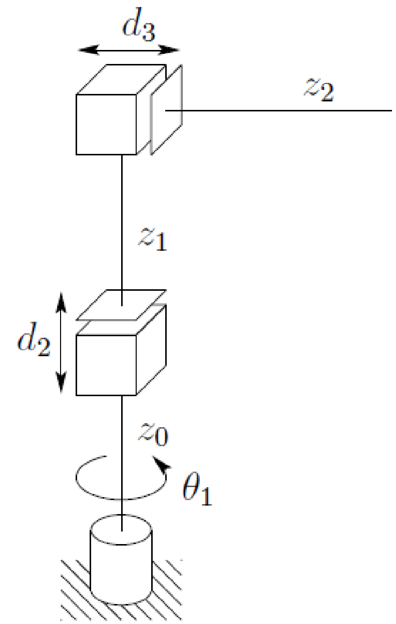
Two joints are rotational and one is translational (RRP).
The axes of all three joints are parallel.
The workspace of SCARA robot arm is of cylindrical shape.



Workspace



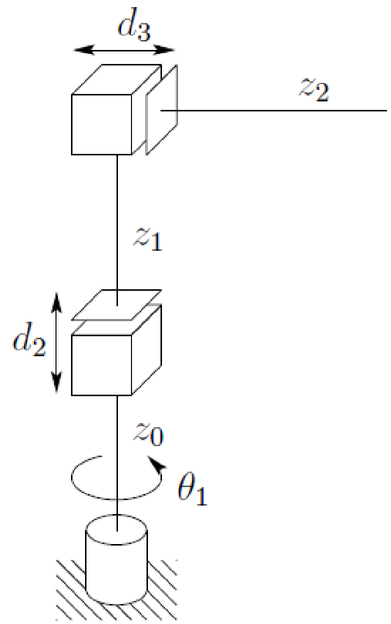
Cylindrical Manipulator

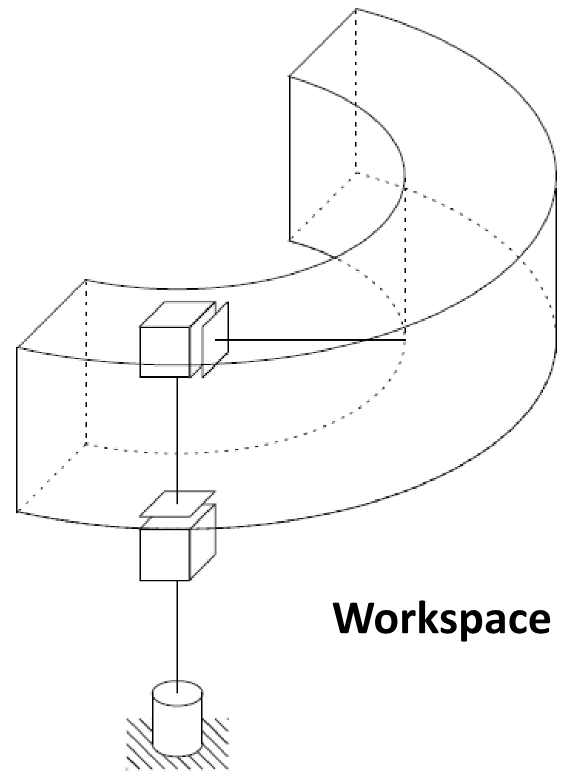


Workspace

One rotational and two translational (RPP).
The axis of the second joint is parallel to the first axis.
The third joint axis is perpendicular to the second one.

Cylindrical Manipulator





Workspace

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The axis of the second joint is parallel to the first axis.
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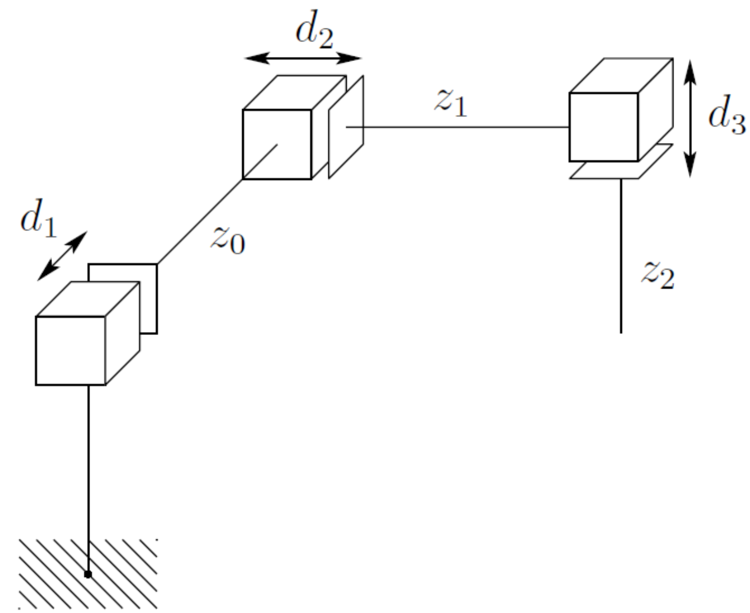
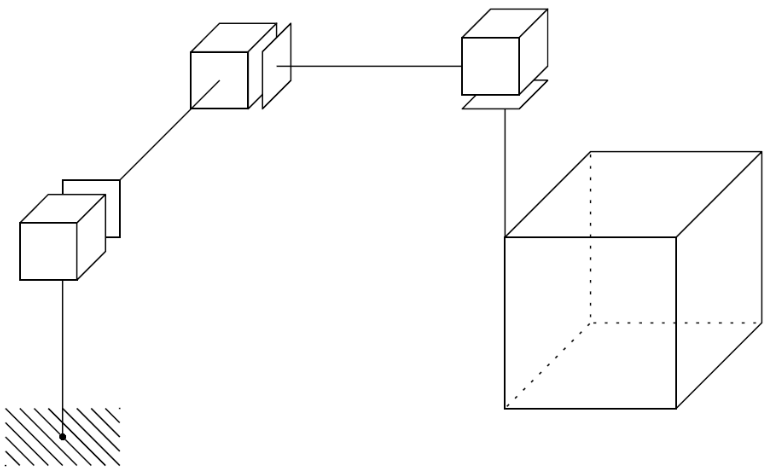
The Cartesian Manipulators

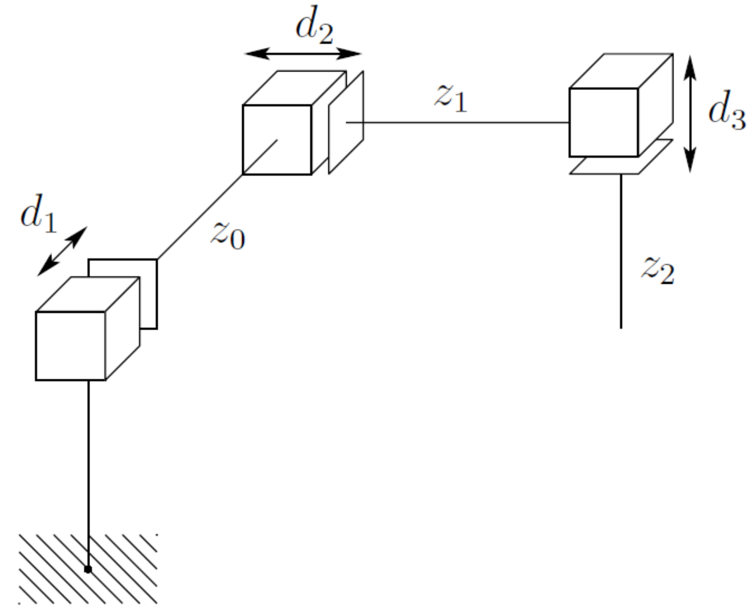


Workspace

Three joints of the translational type (PPP). The joint axes are perpendicular one to another.

The Cartesian Manipulators



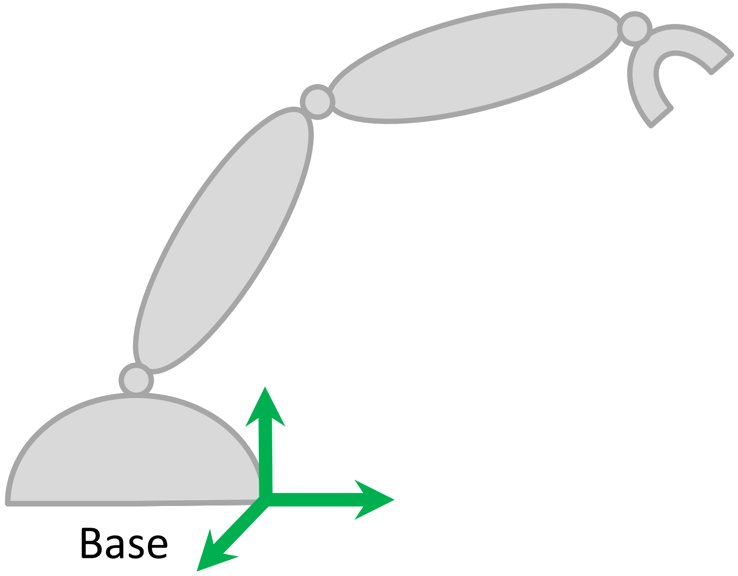


Workspace

Three joints of the translational type (PPP). The joint axes are perpendicular one to another.

Configuration Parameters

A set of **position** parameters that describes the full configuration of the system.



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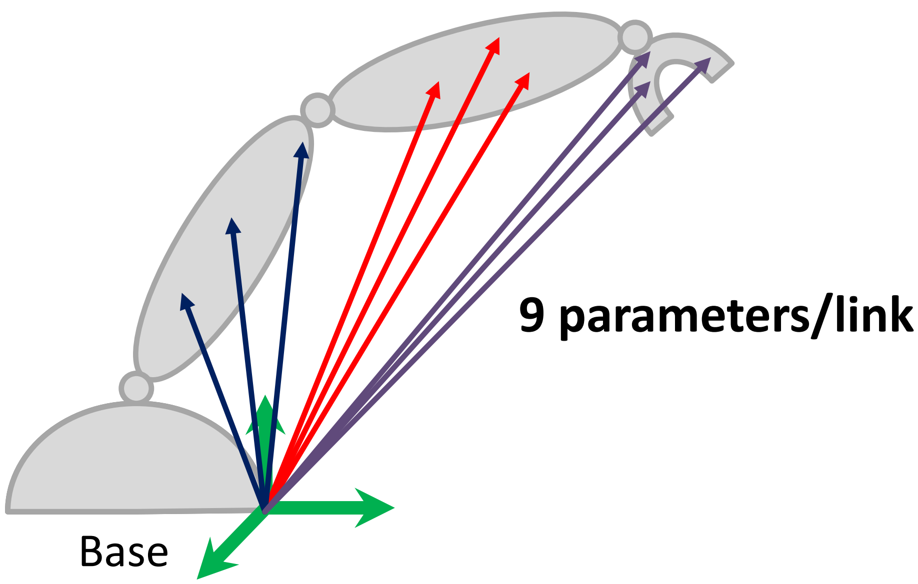
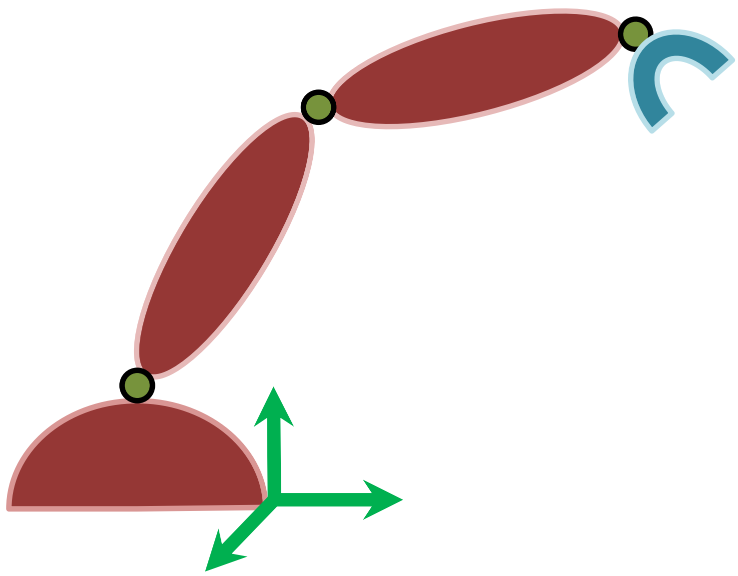


Generalized Coordinates

A set of independent configuration parameters

$$\mathbf{6}\ \boldsymbol{parameters/link} \begin{cases} \mathbf{3} \quad \boldsymbol{positions} \\ \mathbf{3}\ \boldsymbol{orientations} \end{cases}$$



Generalized Coordinates

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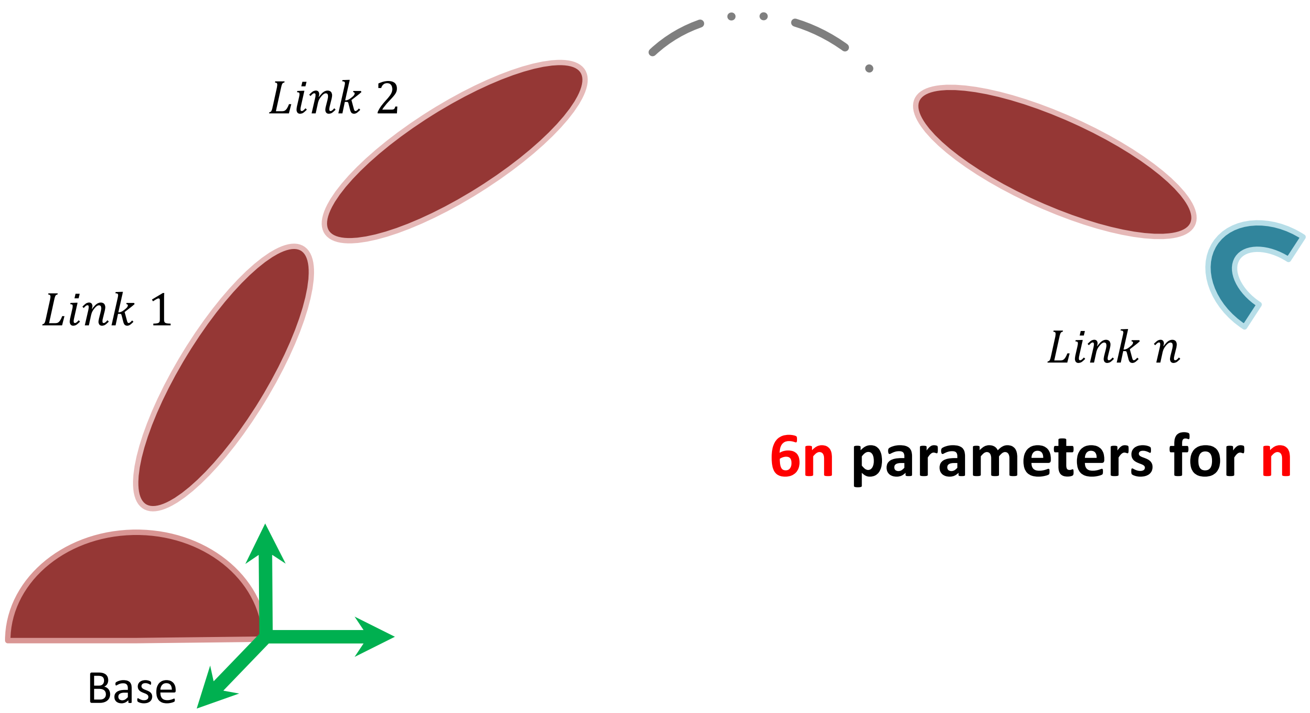
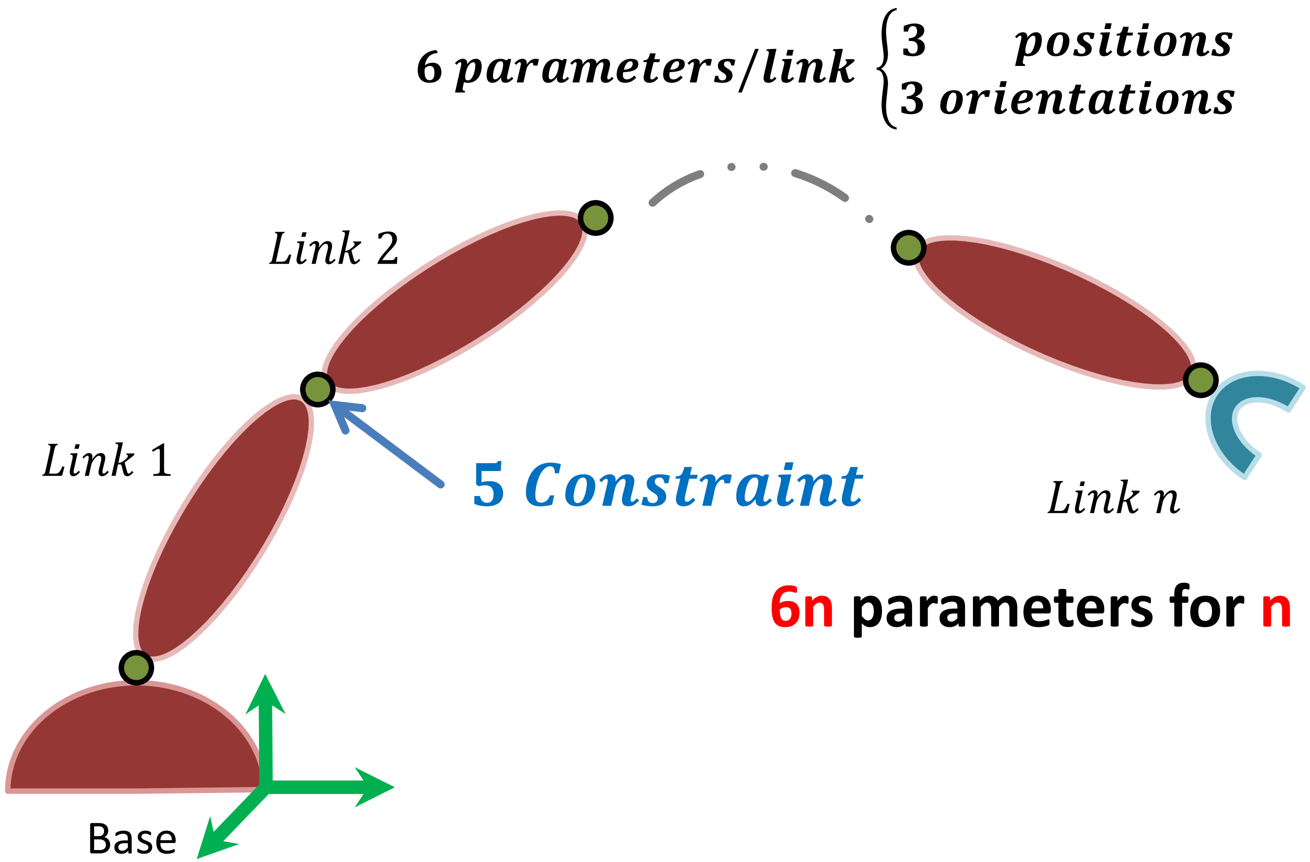
$$6\ \boldsymbol{parameters/link}\begin{cases}\boldsymbol{3\quad\ positions}\\ \boldsymbol{3\ orientations}\end{cases}$$



6n parameters for n moving links

Generalized Coordinates

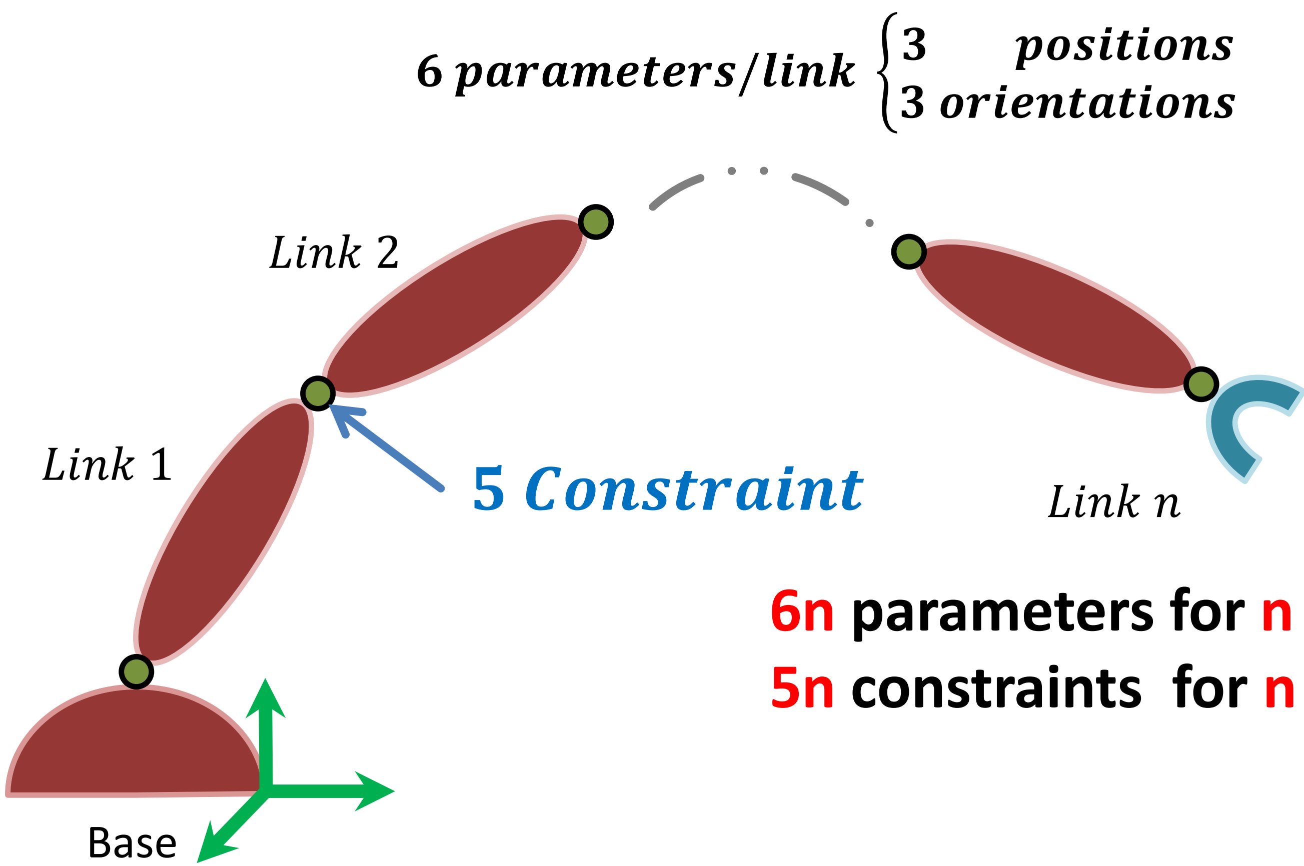
A set of independent configuration parameters



6n parameters for n moving links

Generalized Coordinates

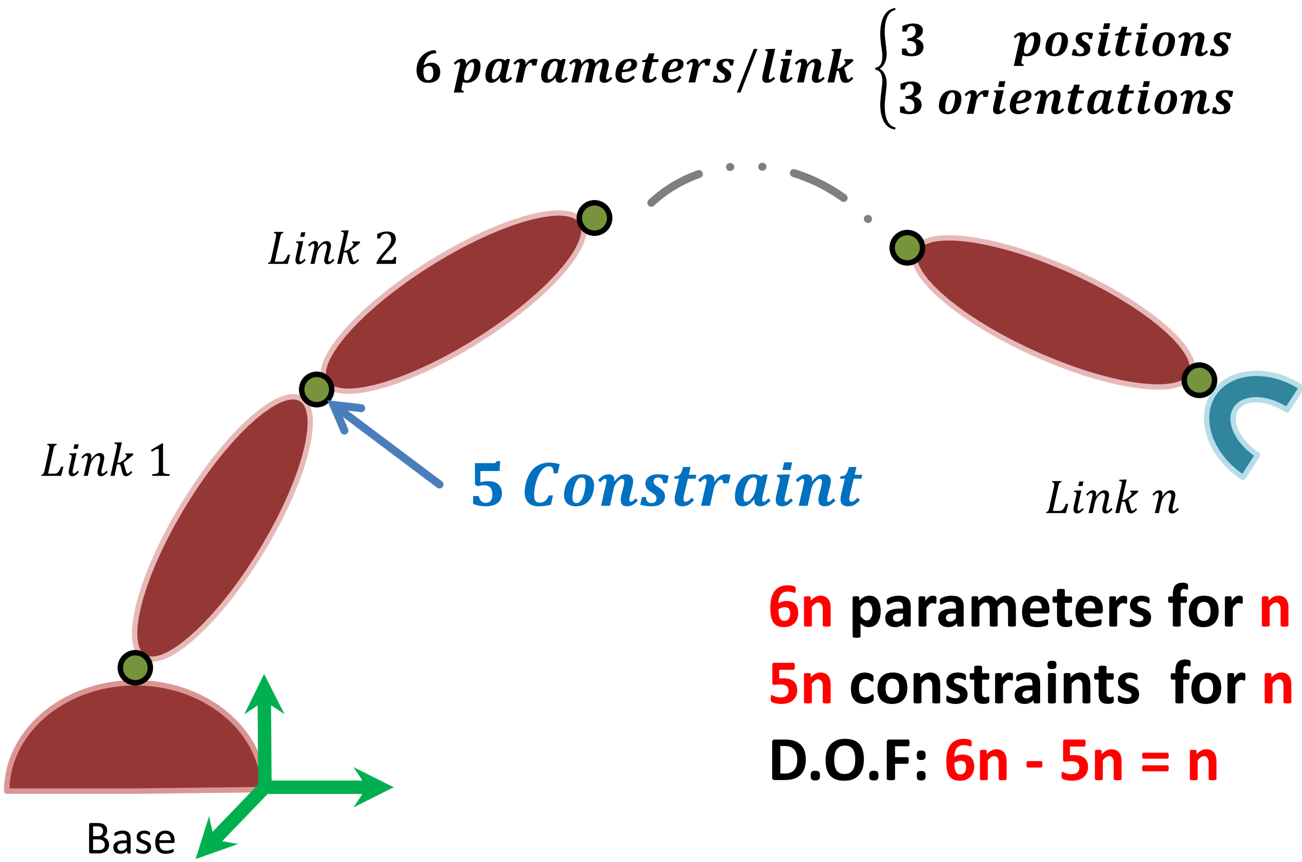
A set of independent configuration parameters



6n parameters for n moving links
5n constraints for n joints

Generalized Coordinates

A set of independent configuration parameters

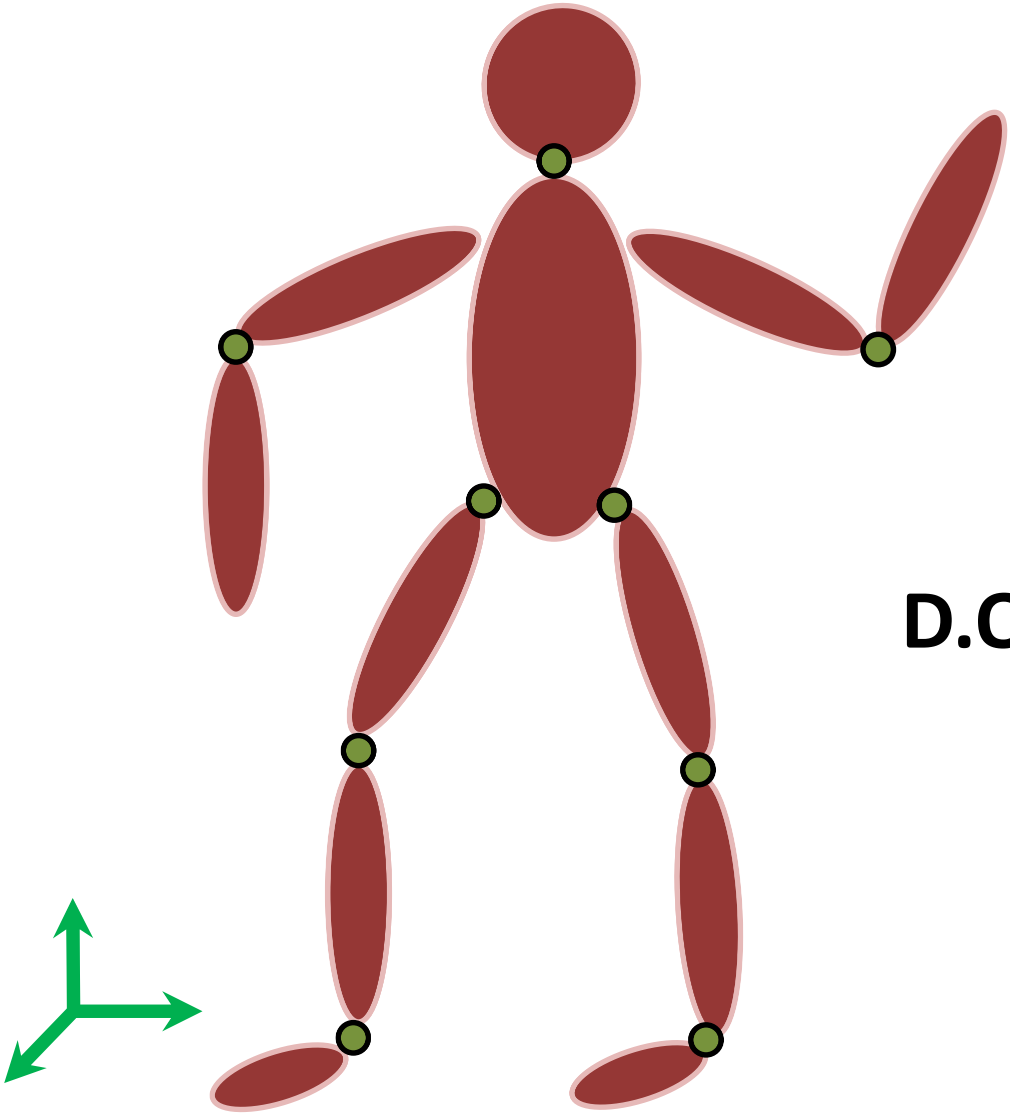


6n parameters for n moving links

5n constraints for n joints

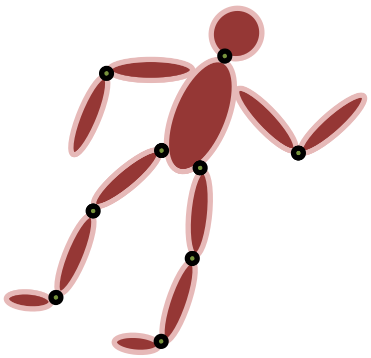
D.O.F: 6n - 5n = n

Generalized Coordinates

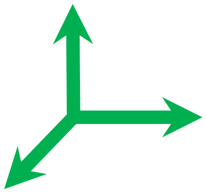


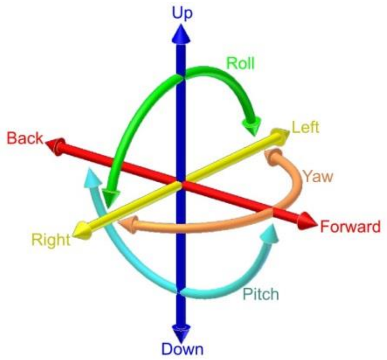
D.O.F: n joints + ?

Generalized Coordinates

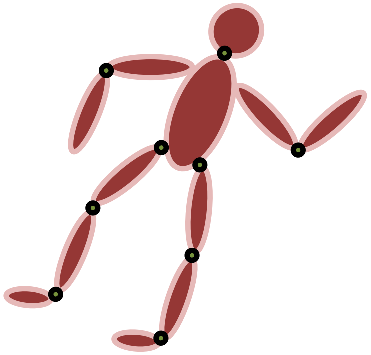


The robot is free to move forward/backward, up/down, left/right (**translation in three perpendicular axes**) combined with **rotation about three perpendicular axes**, often termed pitch, yaw, and roll.





Generalized Coordinates



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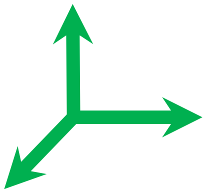
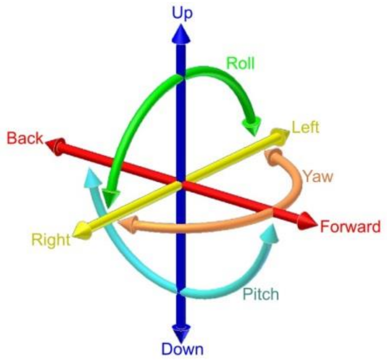
D.O.F: n joints + 6



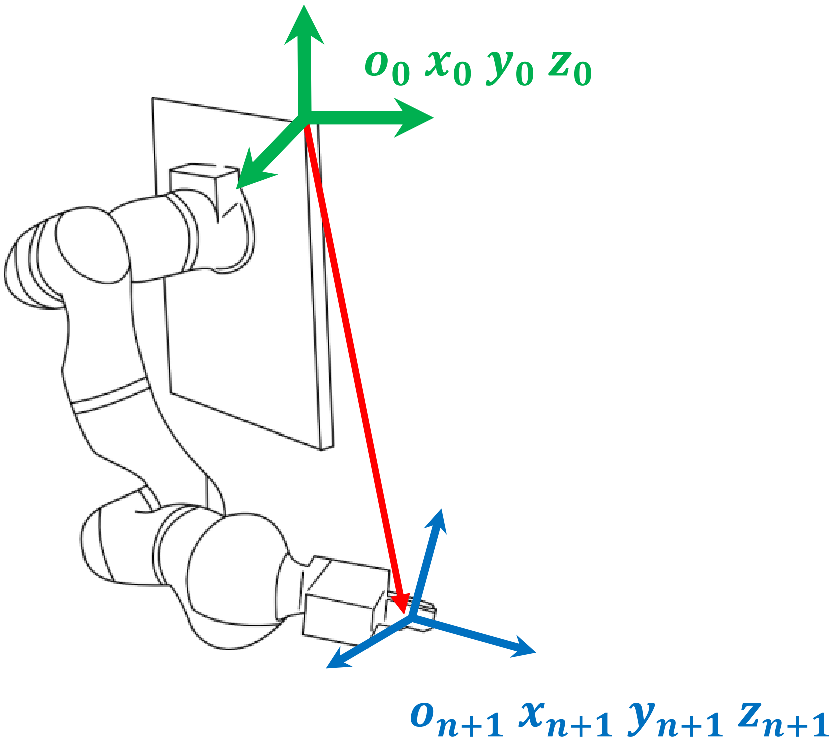


Operational Coordinates

End-effector configuration parameters are a set of m parameters $(x_1, x_2, x_3, .., x_m)$ that completely specify the end-effector position and orientation with respect to the frame $o_0\ x_0\ y_0\ z_0$.

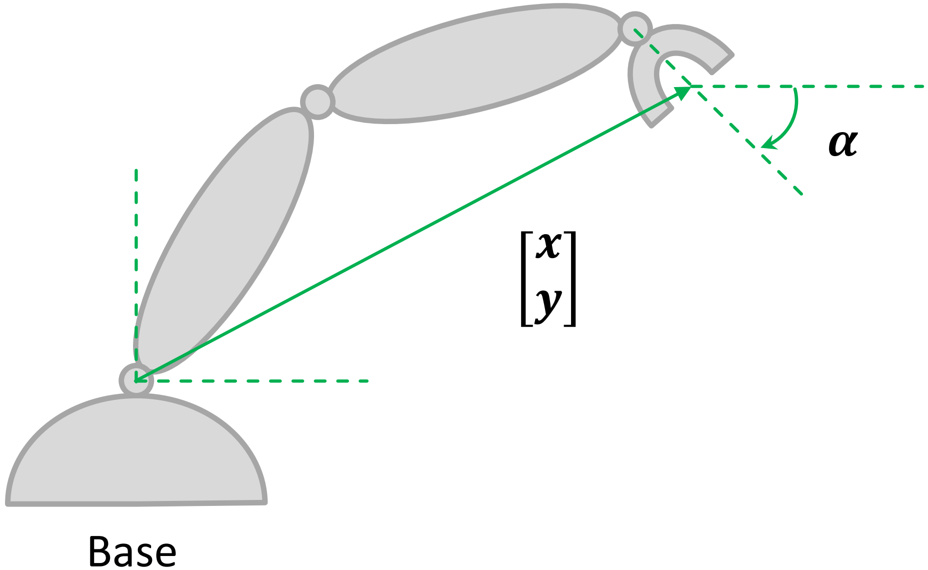
o_{n+1} is the operational point.

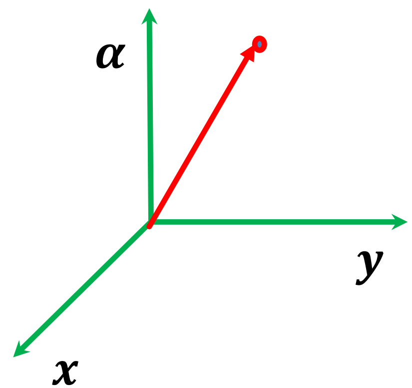
A set $(x_1, x_2, x_3, .., x_{m_0})$ of independent configuration Parameters m_0: **number of degree of freedom of the end-effector**.



Operational Coordinates

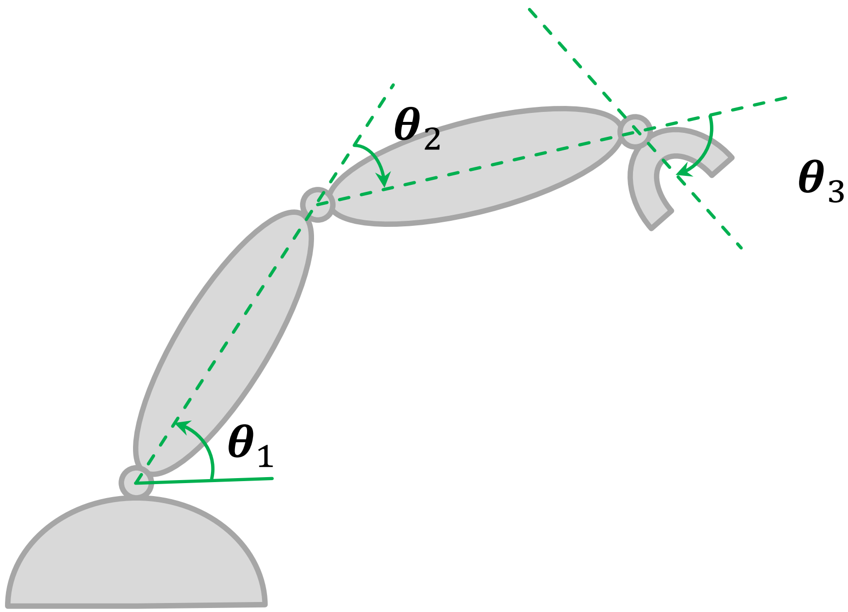
Is also called Operational Space

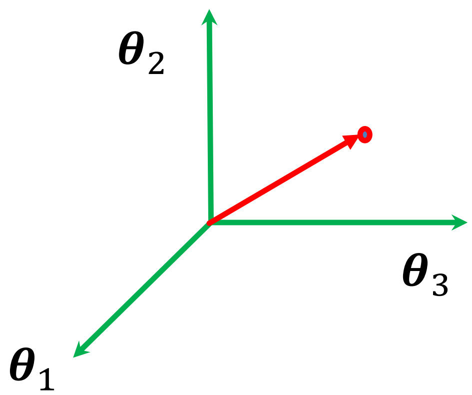




Joint Coordinates

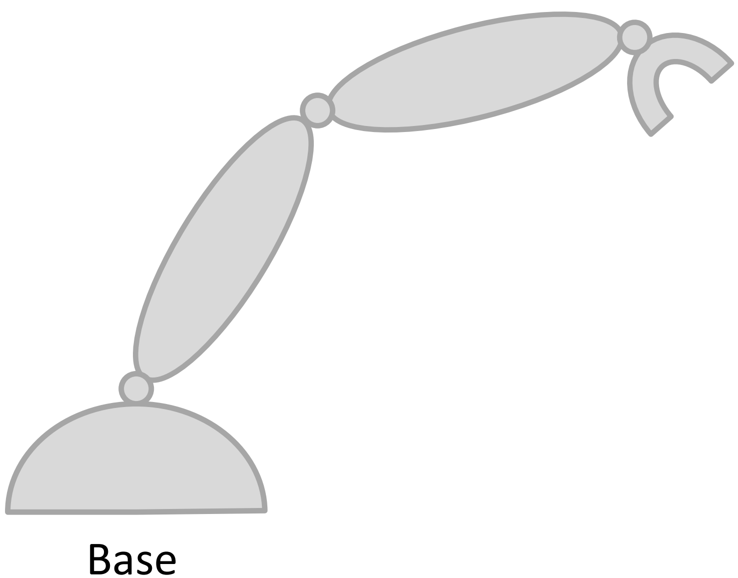
Is also called Joint Space

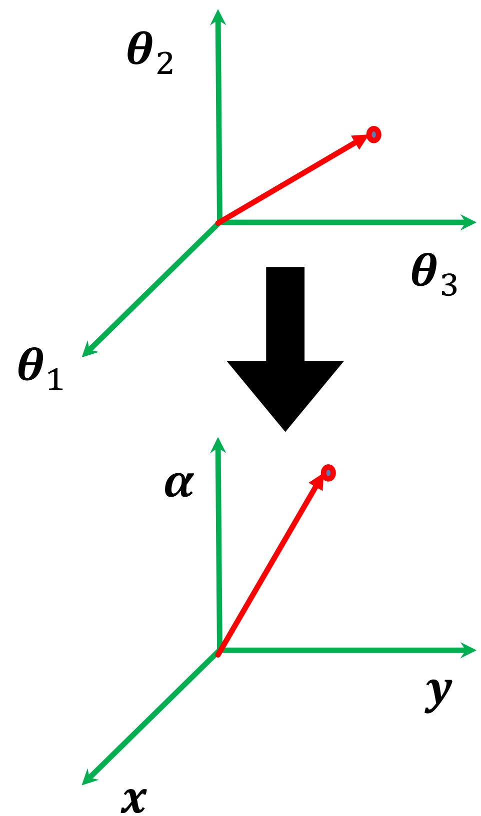




Joint Space -> Operational Space

Determine the position and orientation of the end-effector given the values for the joint variables of the robot.

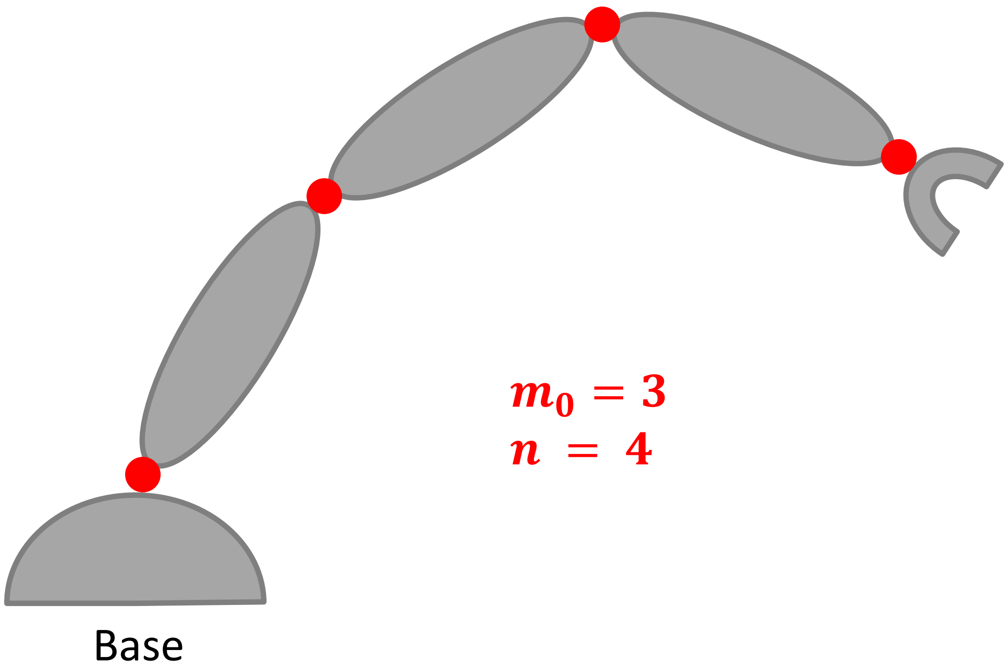




Redundancy

A robot is said to be redundant if $n > m_0$.
Degree of redundancy: $n - m_0$

how many solutions exist?



$m_0 = 3$
$n = 4$

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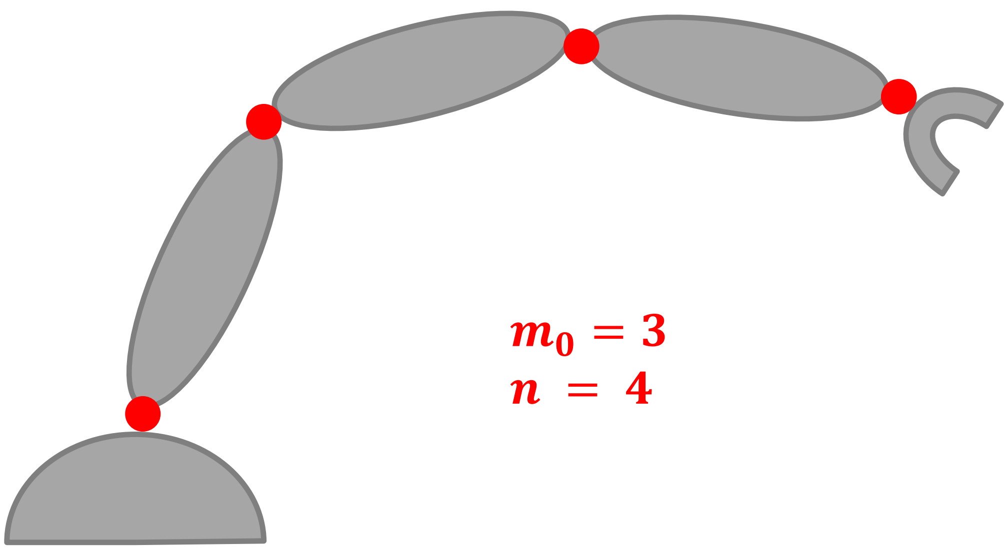

$m_0 = 3$
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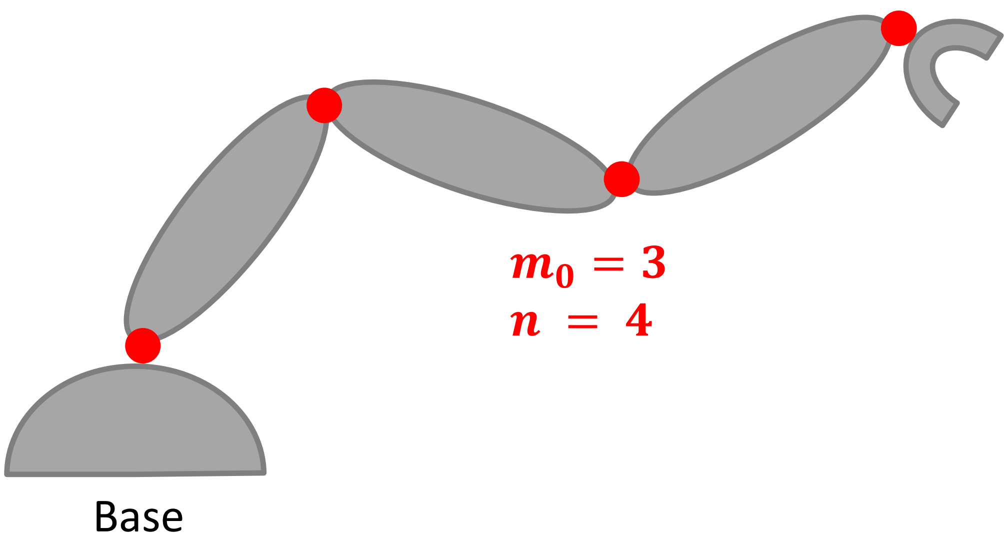

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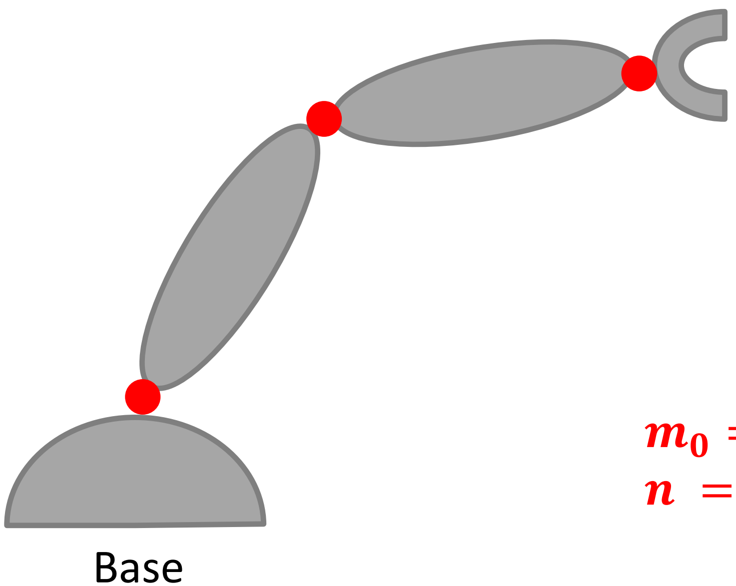

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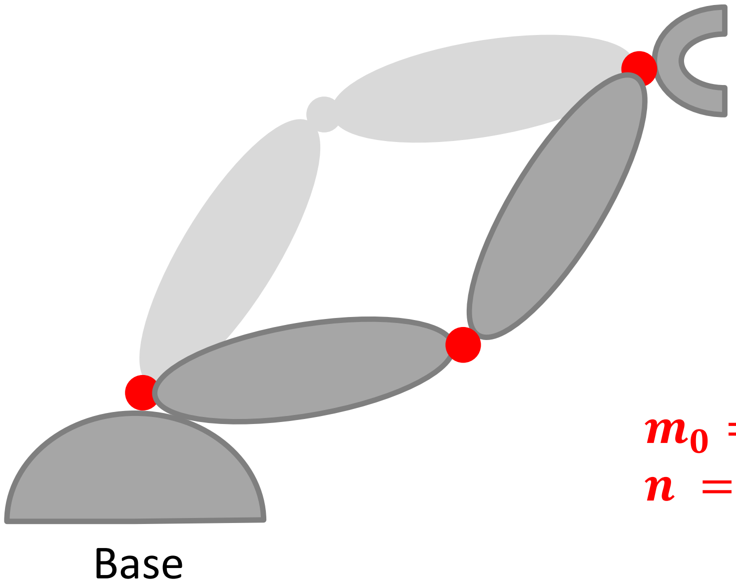

$m_0 = 3$
$n = 3$

Kinematic Arrangements

The objective of forward kinematic analysis is to determine the **cumulative effect of the entire set of joint variables**, that is, to **determine the position and orientation of the end effector** given the values of these joint variables.

We assume that each joint has **one D.O.F**

The action of each joint can be described by one real number:

the **angle of rotation** in the case of a revolute joint or

the **displacement** in the case of a prismatic joint.

When joint $\boldsymbol{i}$ is actuated, link $\boldsymbol{i}$ moves.

$\boldsymbol{q_i}$ is the **joint variable**

$$q_i = \begin{cases} \theta_i & \text{if joint } i \text{ is revolute} \\ d_i & \text{if joint } i \text{ is prismatic} \end{cases}$$

Kinematic Arrangements

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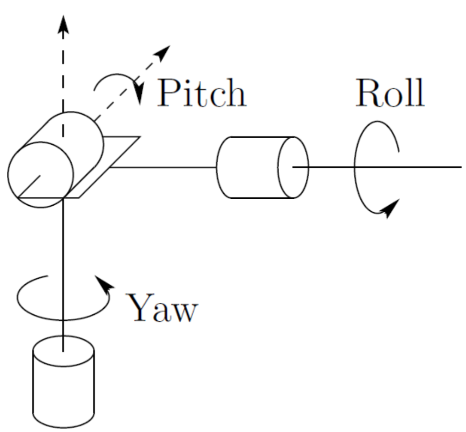
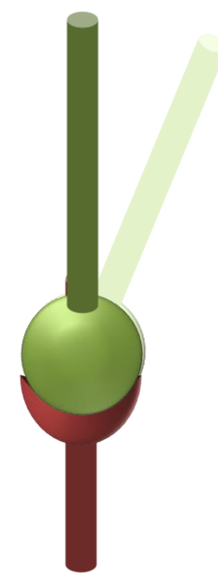


Spherical wrist 3 D.O.F

spherical wrist:

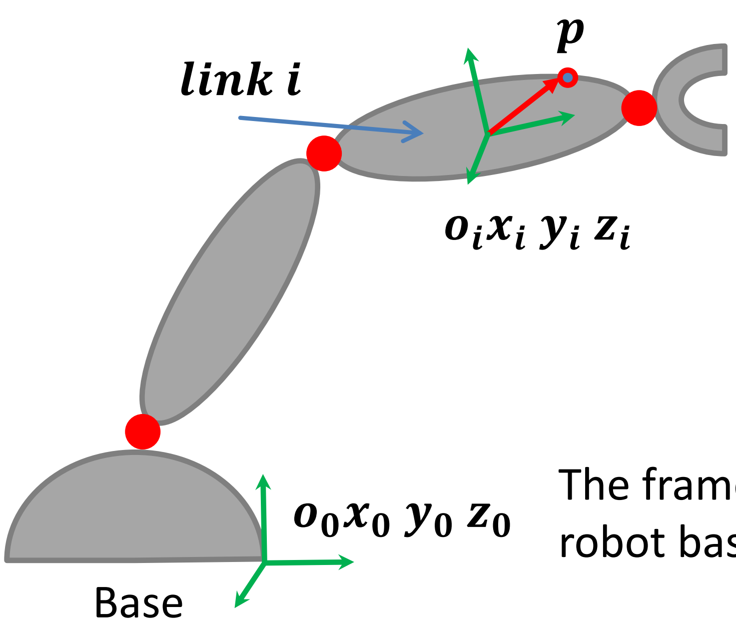
RRR

Links' lengths = 0



Kinematic Arrangements

To perform the kinematic analysis, we attach a coordinate frame rigidly to each link.
In particular, we attach $\boldsymbol{o_i x_i\ y_i\ z_i}$ to $\boldsymbol{link\ i}$.
This means that, whatever motion the robot executes, the coordinates of any point $\boldsymbol{p}$ on link $\boldsymbol{i}$ are constant when expressed in the $\boldsymbol{i^{th}}$ coordinate frame $\boldsymbol{p_i = constant}$.
When $\boldsymbol{joint\ i}$ is actuated, $\boldsymbol{link\ i}$ and its attached frame, $\boldsymbol{o_i x_i\ y_i\ z_i}$, experience a resulting motion.



The frame $\boldsymbol{o_0 x_0\ y_0\ z_0}$, which is attached to the robot base, is referred to as **the reference frame**.

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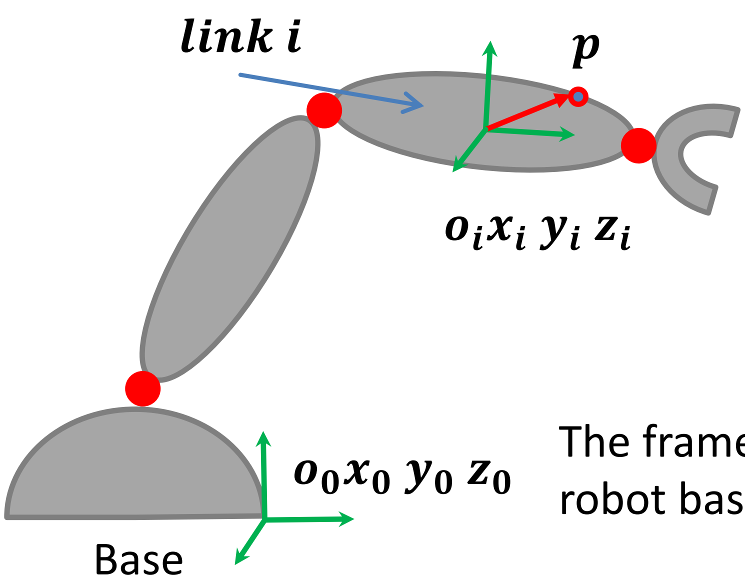

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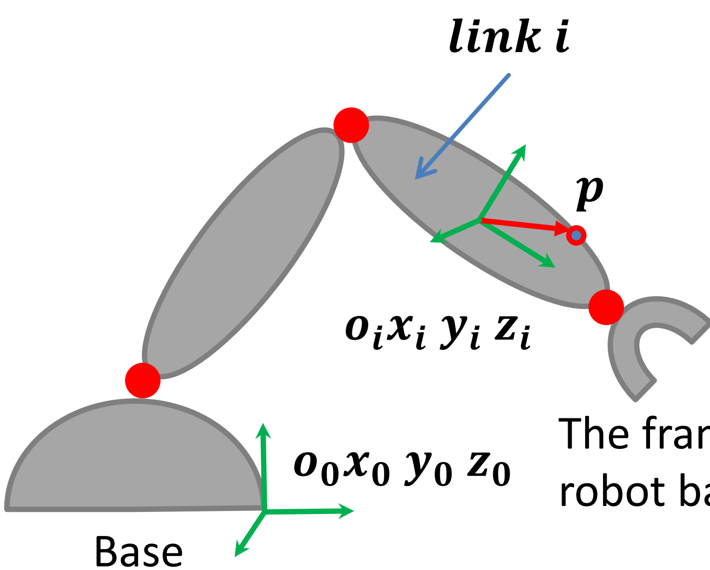

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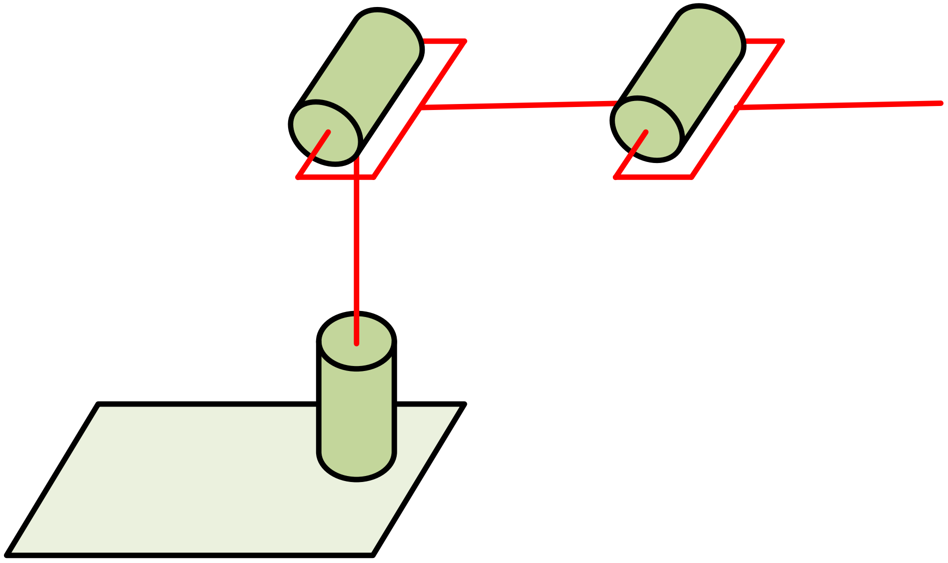
Joint And Link Labelling



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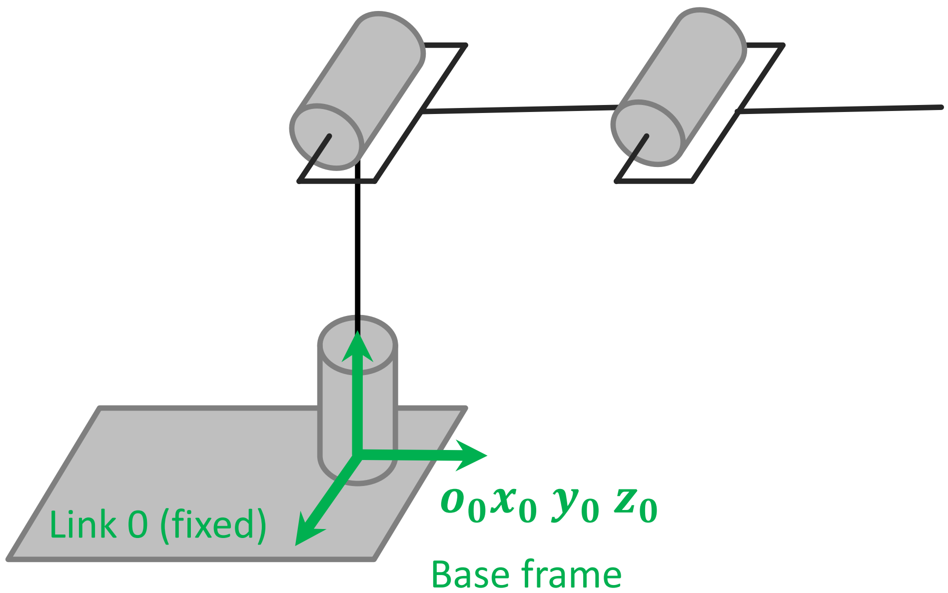

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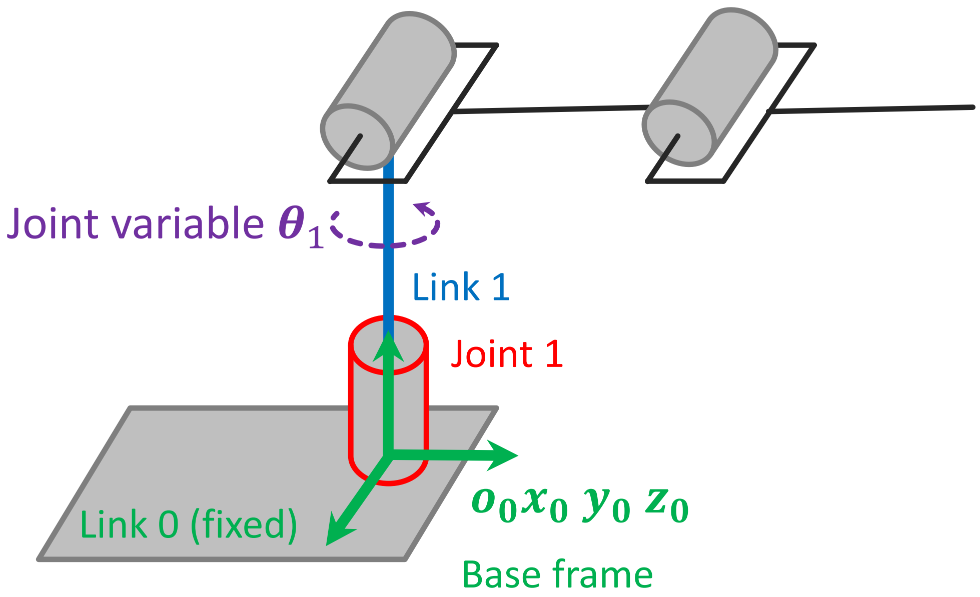

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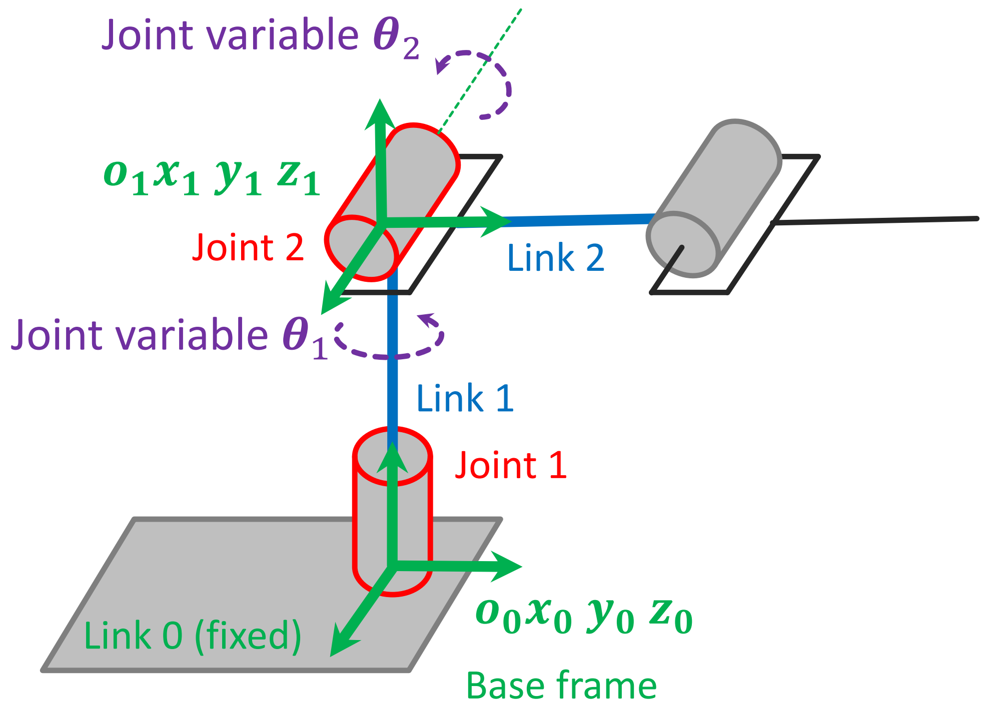

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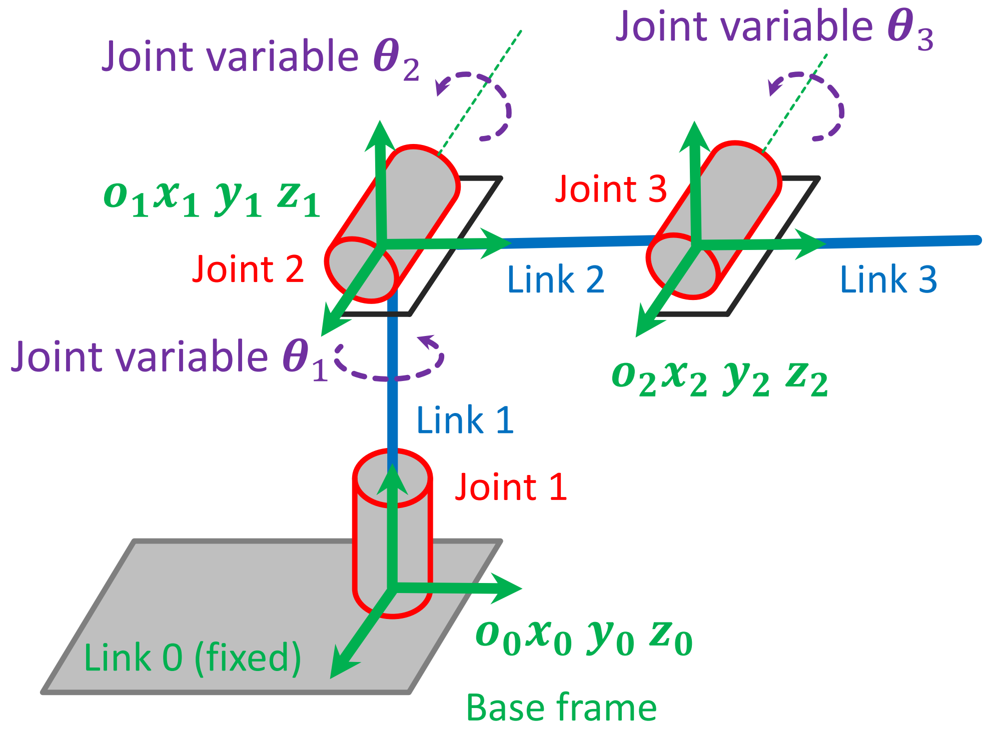

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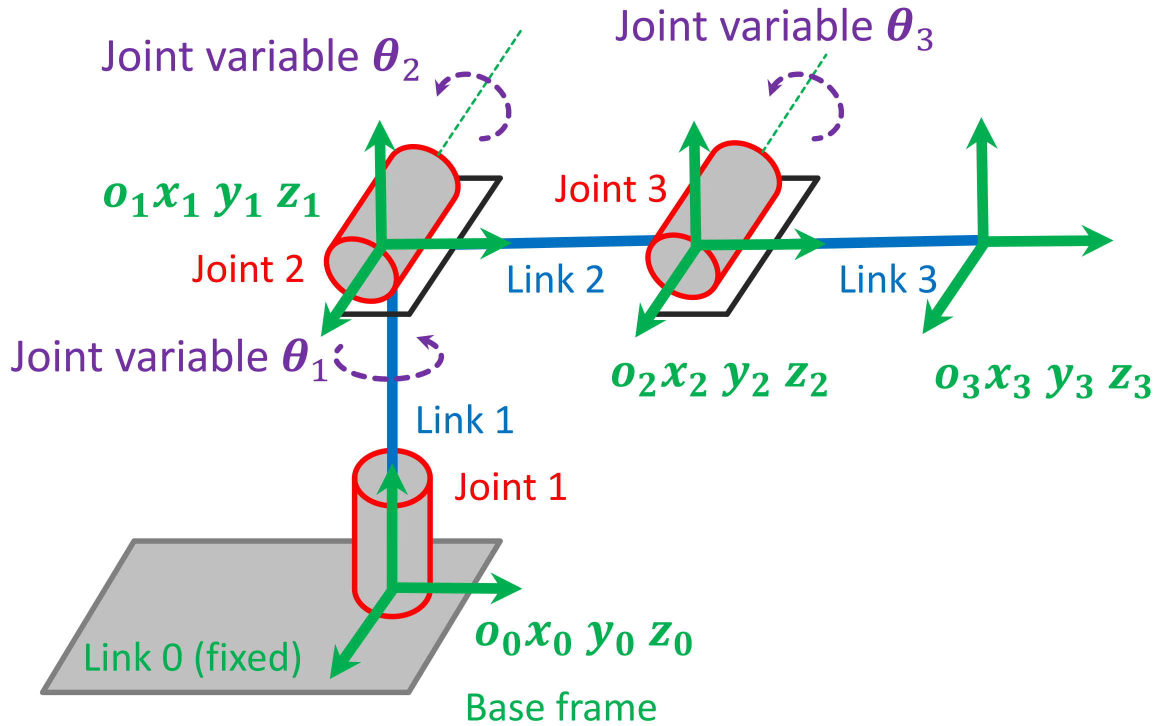


Do we need a specific way to orientate the axes?

Transformation Matrix

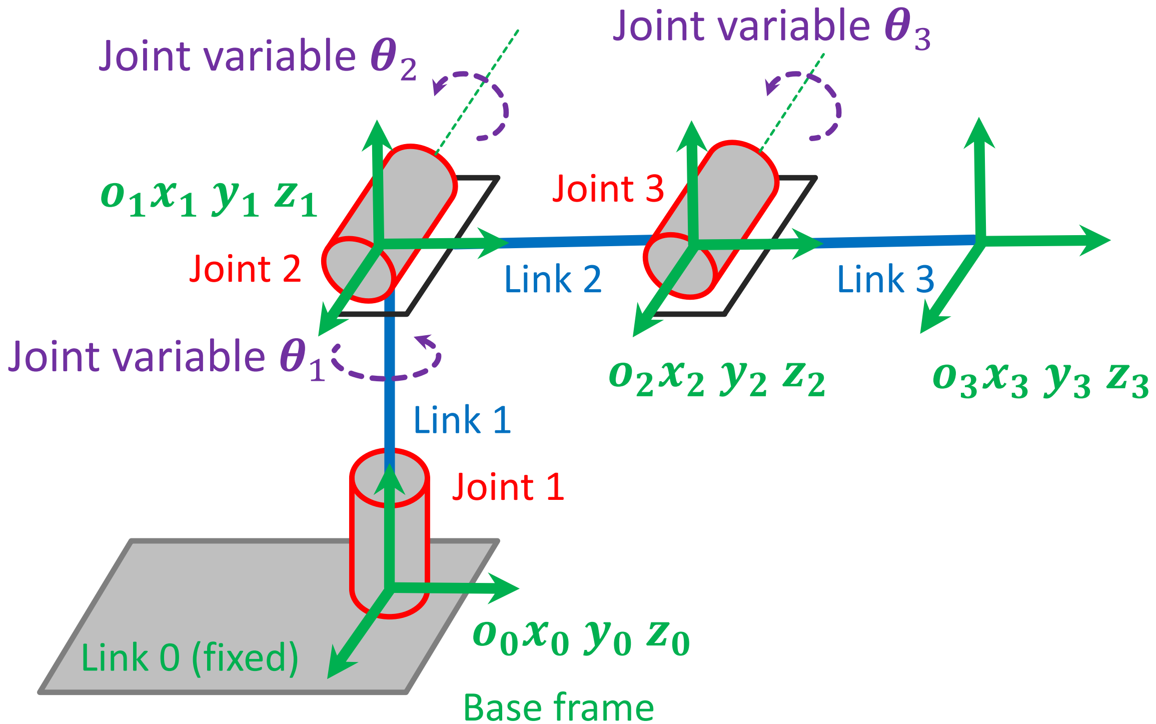
Suppose $\boldsymbol{A_i}$ is the homogeneous transformation matrix that describe the position and the orientation of $\boldsymbol{o_i x_i\ y_i\ z_i}$ with respect to $\boldsymbol{o_{i-1} x_{i-1}\ y_{i-1}\ z_{i-1}}$.

$\boldsymbol{A_i}$ is derived from joint and link i.

$\boldsymbol{A_i}$ is a function of only a single joint variable.

$$\boldsymbol{A_i = A_i(q_i)}$$

$$\boldsymbol{A_i(q_i)} = \begin{bmatrix} \boldsymbol{R^{i-1}_{\ \ i}} & \boldsymbol{o^{i-1}_{\ \ i}} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}$$



Transformation Matrix

The position and the orientation of the end effector (reference frame $\boldsymbol{o_n x_n\ y_n\ z_n}$) with respect to the base (reference frame $\boldsymbol{o_0 x_0\ y_0\ z_0}$) can be expressed by the transformation matrix:

$$\mathbf{H} = \boldsymbol{T_n^0} = \boldsymbol{A_1(q_1) \ldots A_n(q_n)} = \begin{bmatrix} \boldsymbol{R_n^0} & \boldsymbol{o_n^0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

The position and the orientation of a reference frame $\boldsymbol{o_j x_j\ y_j\ z_j}$) with respect to a reference frame $\boldsymbol{o_i x_i\ y_i\ z_i}$ can be expressed by the transformation matrix:

$$\boldsymbol{T_j^i} = \begin{cases} \boldsymbol{A_{i+1} A_{i+2} \ldots A_{j-1}\ A_j} & \boldsymbol{if\ i < j} \\ \boldsymbol{I} & \boldsymbol{if\ i = j} \\ \boldsymbol{(T_i^j)^{-1}} & \boldsymbol{if\ i > j} \end{cases}$$

Transformation Matrix

$$\boldsymbol{T_j^i} = \begin{cases} A_{i+1}A_{i+2}\dots A_{j-1}\,A_j & if\ \boldsymbol{i} < \boldsymbol{j} \\ \boldsymbol{I} & if\ \boldsymbol{i} = \boldsymbol{j} \\ (\boldsymbol{T_i^j})^{-1} & if\ \boldsymbol{i} > \boldsymbol{j} \end{cases}$$

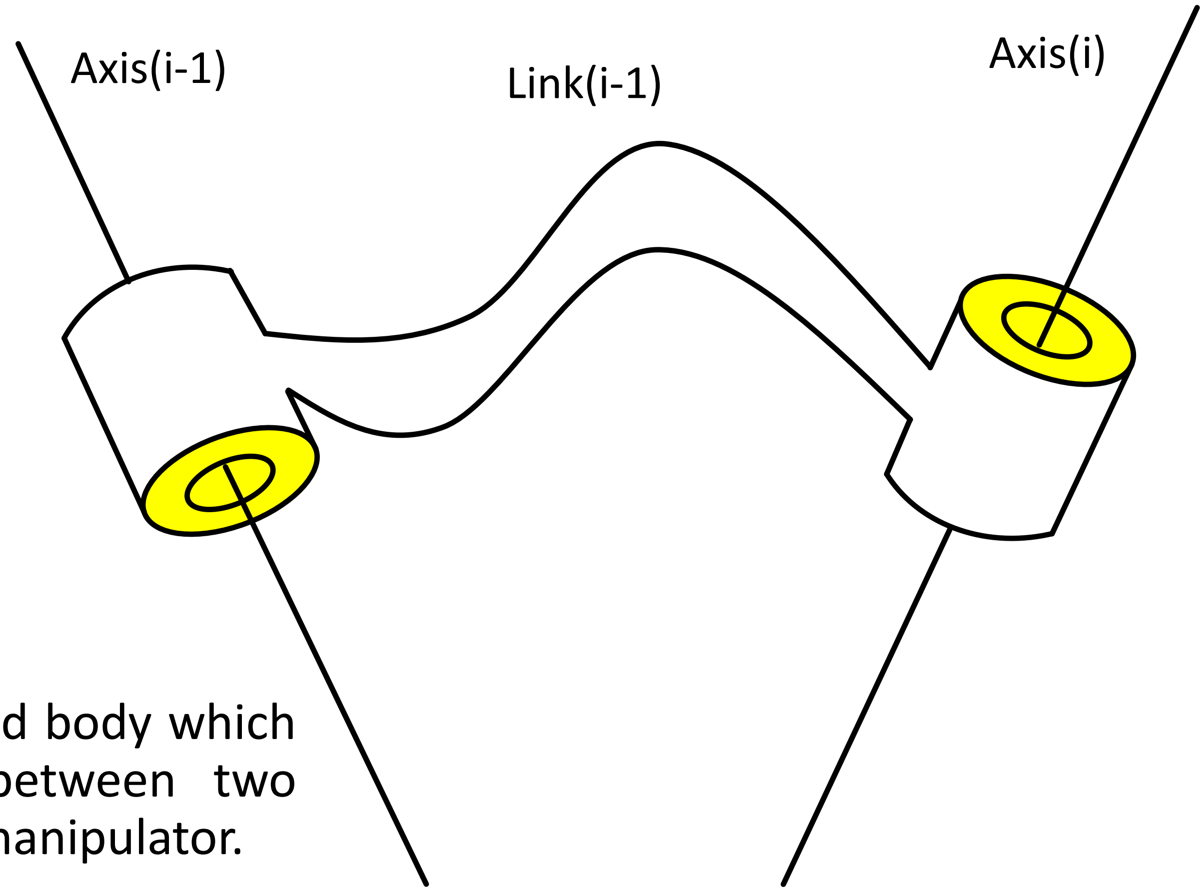
if $\boldsymbol{i} < \boldsymbol{j}$ then

$$\boldsymbol{T_j^i} = \boldsymbol{A_{i+1}A_{i+2}\dots A_{j-1}\,A_j} = \begin{bmatrix} \boldsymbol{R_j^i} & \boldsymbol{o_j^i} \\ \boldsymbol{0} & \boldsymbol{1} \end{bmatrix}$$

The orientation part: $\boldsymbol{R_j^i} = \boldsymbol{R_{i+1}}^{\boldsymbol{i}} \dots \boldsymbol{R^{j-1}}_{\boldsymbol{j}}$

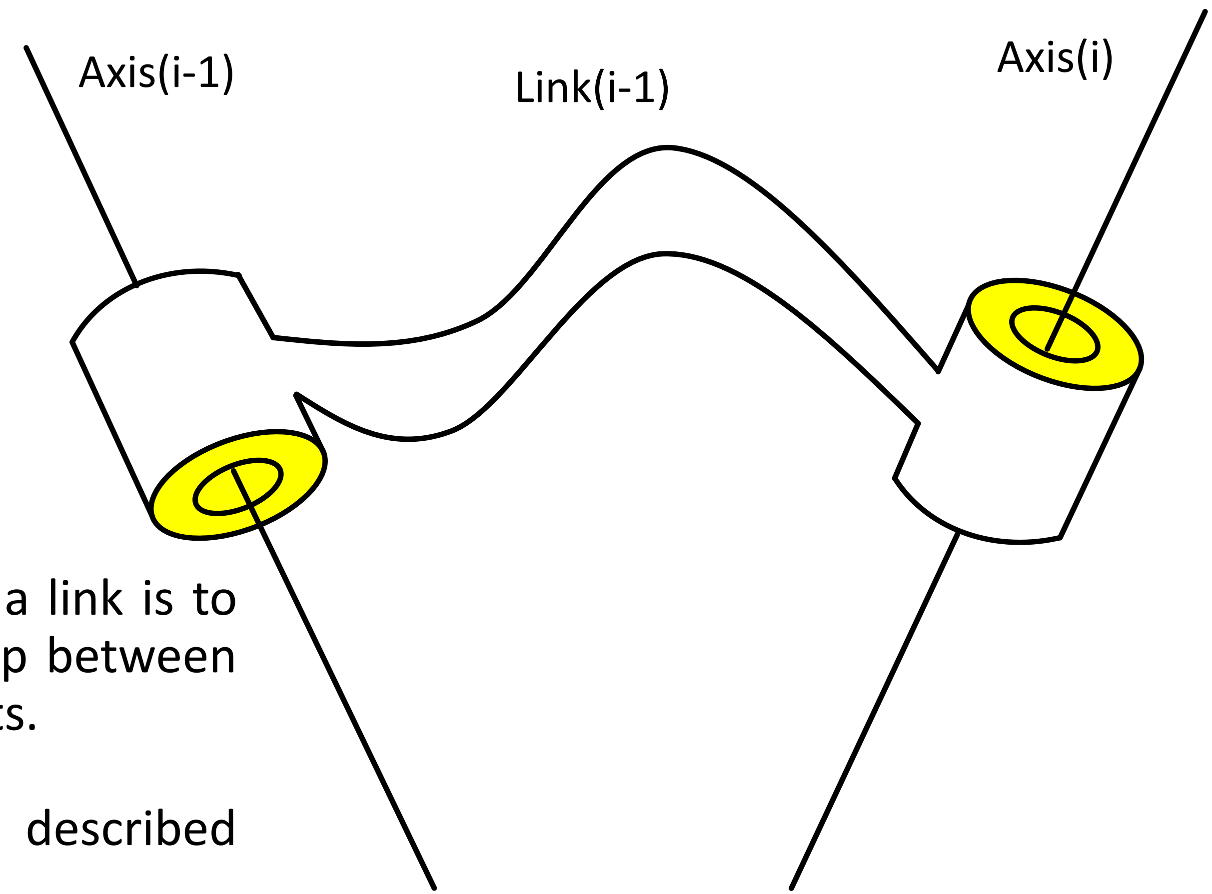
The translation part: $\boldsymbol{o_j^i} = \boldsymbol{o_{j-1}}^{\boldsymbol{i}} + \boldsymbol{R_{j-1}}^{\boldsymbol{i}}\,\boldsymbol{o^{j-1}}_{\boldsymbol{j}}$

Link Description



A link is considered as a rigid body which defines the relationship between two neighboring joint axes of a manipulator.

Link Description



The kinematics function of a link is to maintain a fixed relationship between the two joint axes it supports.

This relationship can be described with two parameters:

- **the link length a**
- **the link twist α**

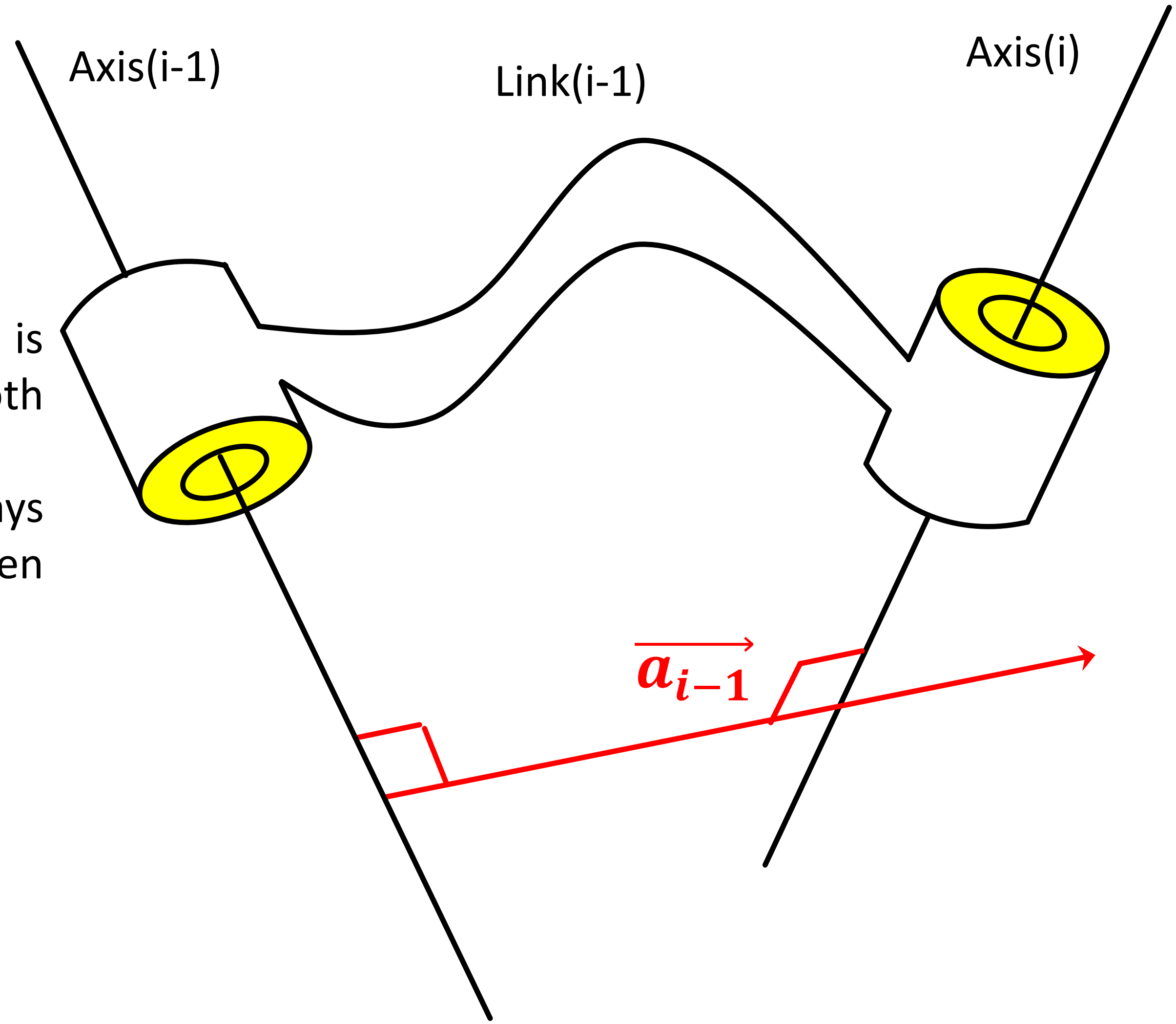
Link Description

$\overrightarrow{a_{i-1}}$ **Link Length**

mutual perpendicular

Is measured along a line which is mutually perpendicular to both axes.

The mutually perpendicular always exists and is unique except when both axes are parallel.

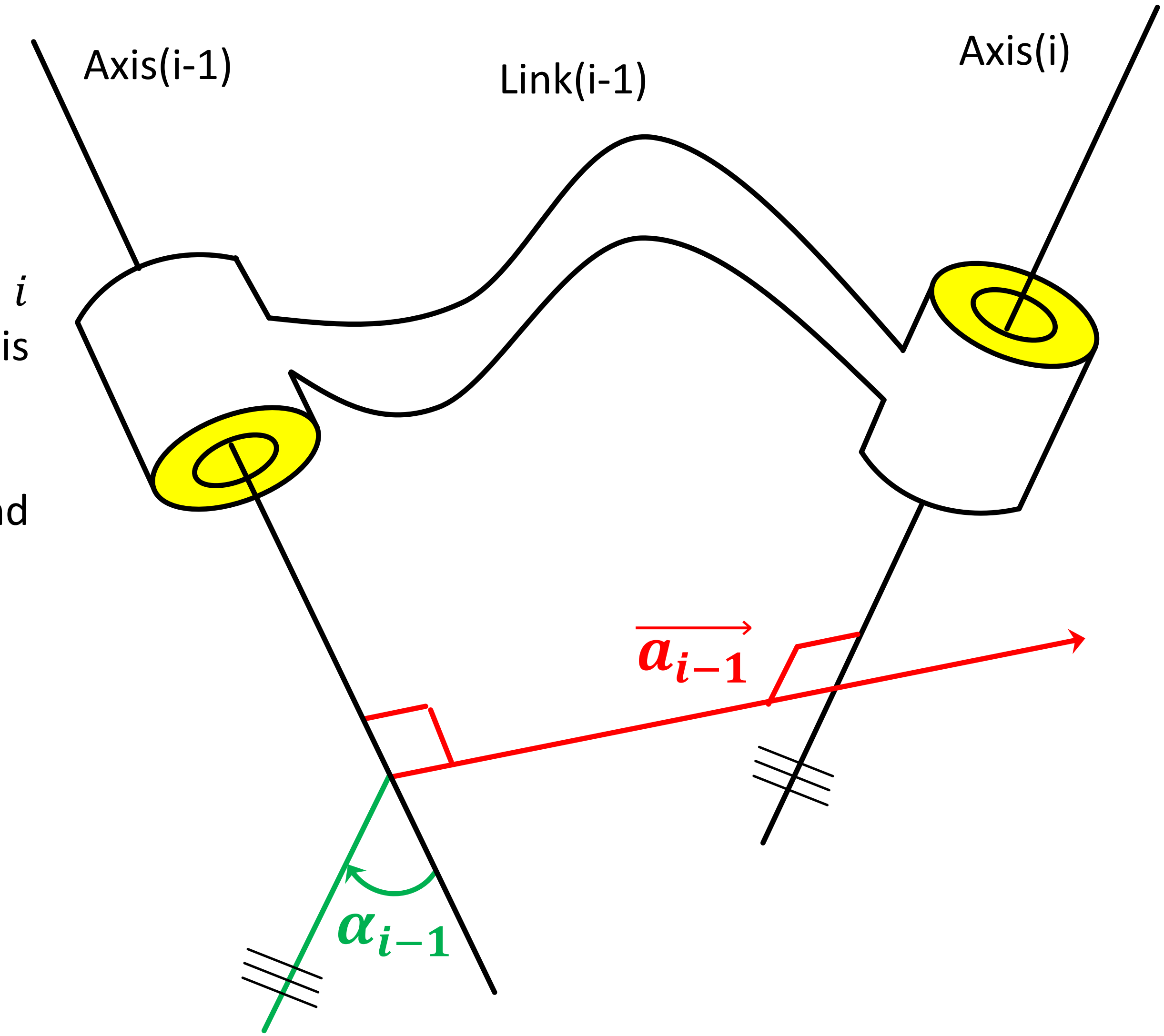


Link Description

α_{i-1} Link Twist

Project both axes $i-1$ and i onto the plane whose normal is the mutually perpendicular line.

Measured in the right-hand sense about $\overrightarrow{a_{i-1}}$.



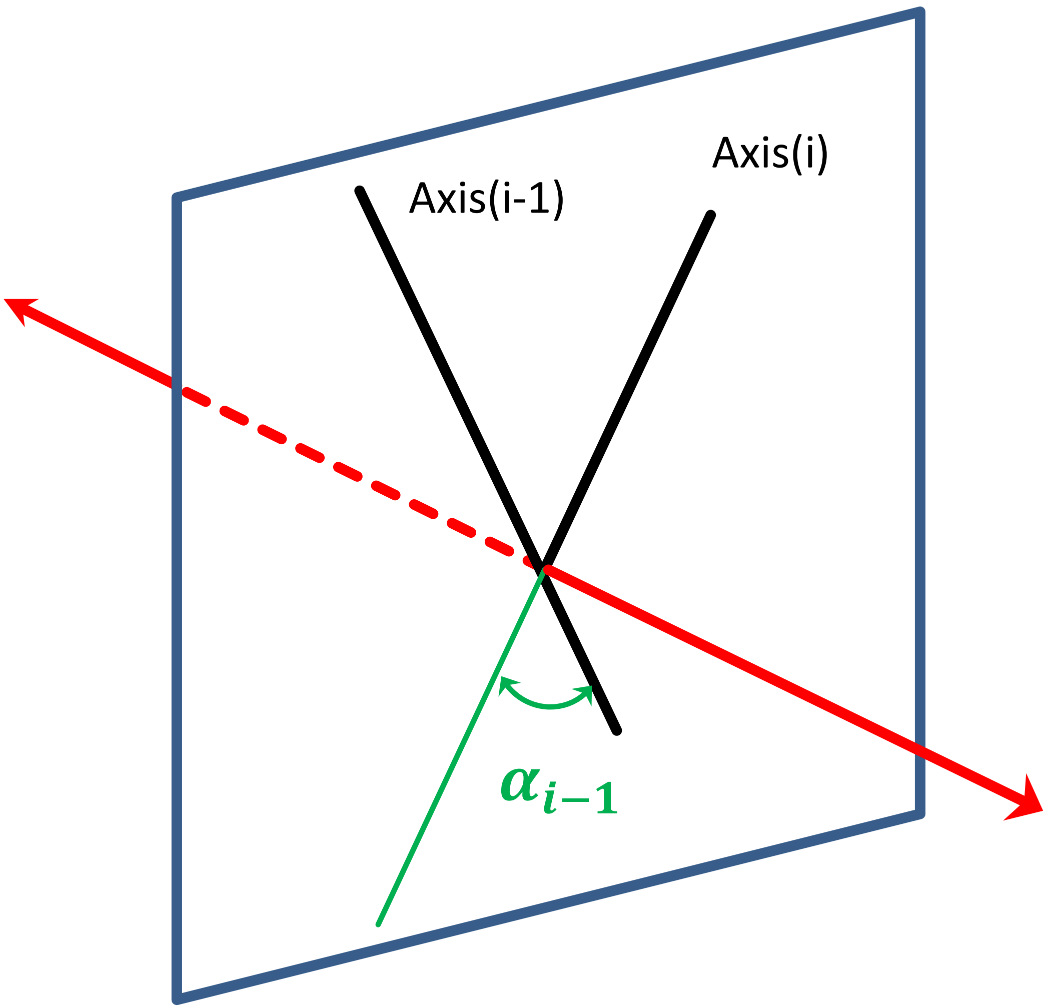
Link Description

Intersecting joint axis !

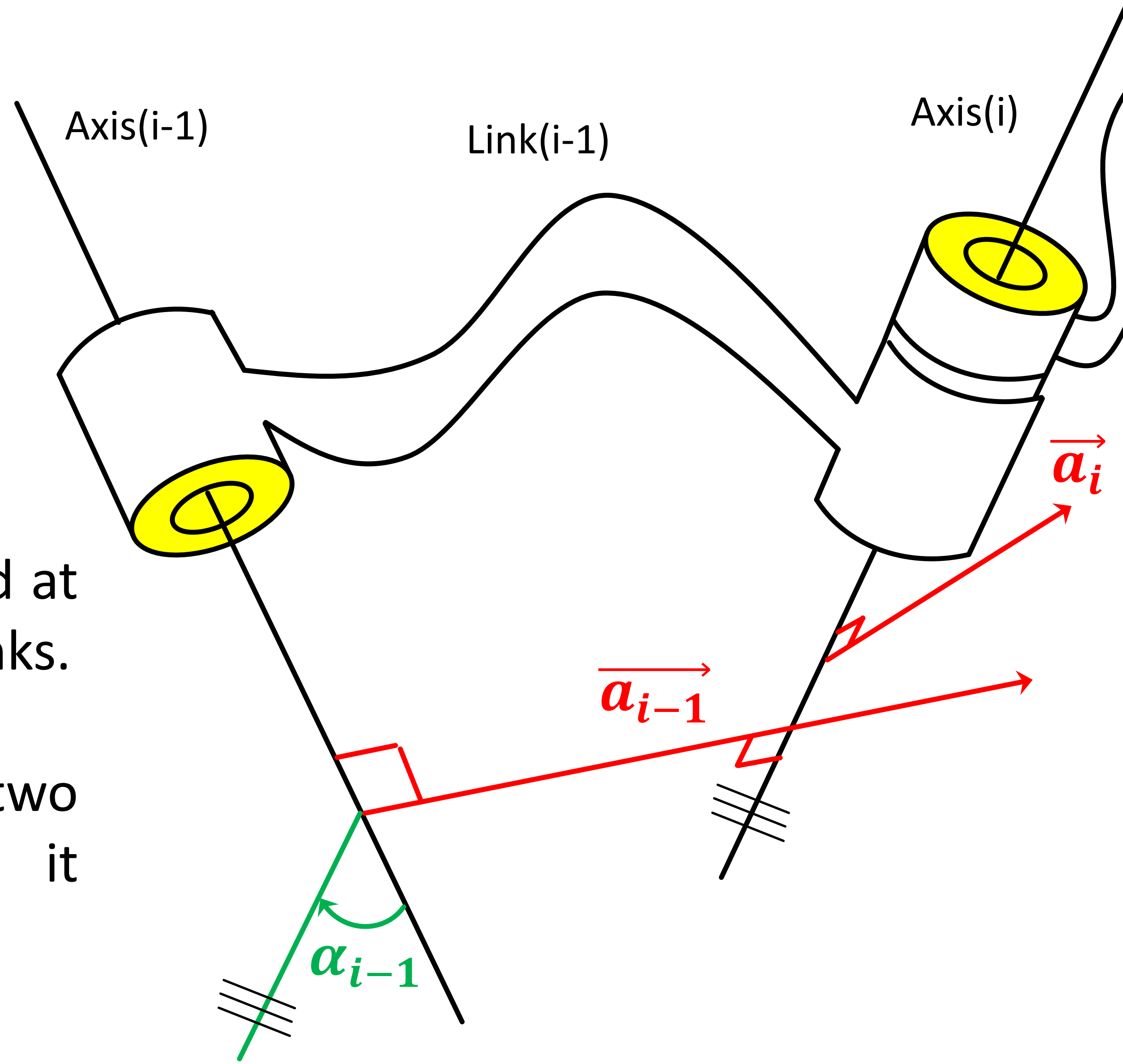
$\overrightarrow{a_{i-1}}$ **Link length ?**

α_{i-1} **Link Twist ?**

The sense of α_{i-1} is free.



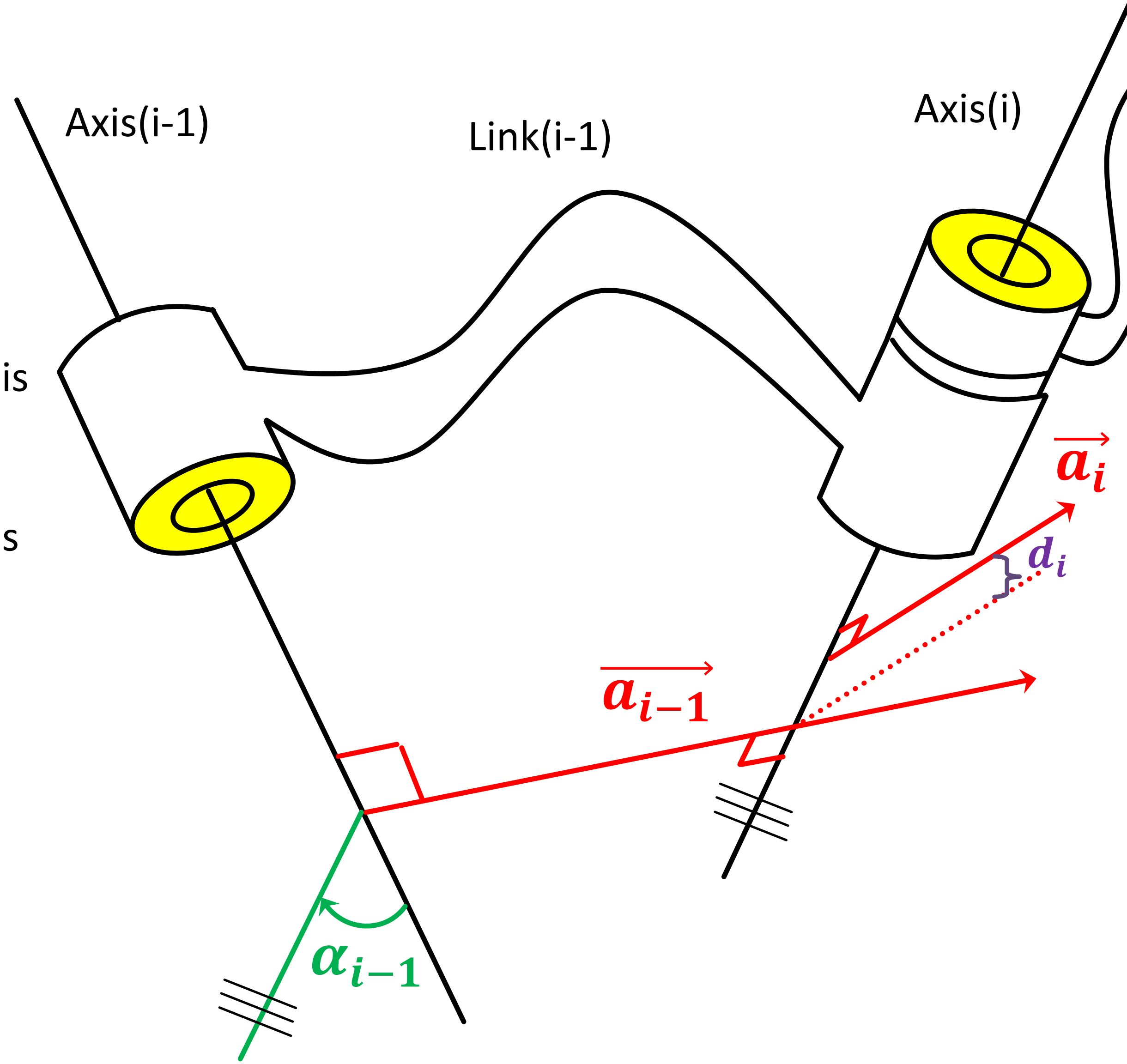
Joint Parameters



A joint axis is established at the connection of two links.

This joint will have two normals connected to it one for each of the links.

Joint Parameters

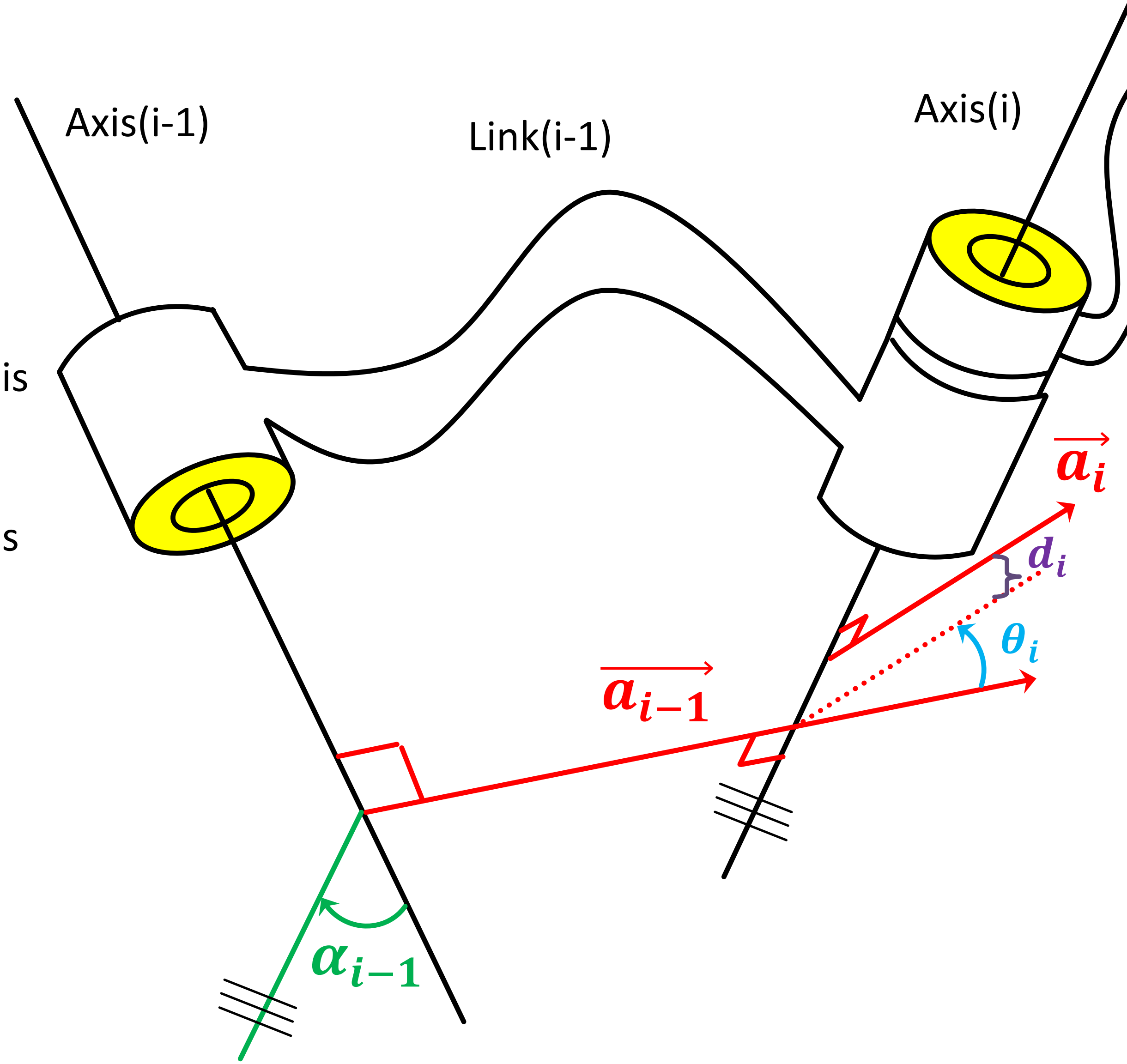


d_i Link Offset

Variable if joint is prismatic.

The relative position of two links is called link offset whish is the distance between the links (the displacement, along the joint axes between the links).

Joint Parameters



d_i Link Offset

Variable if joint is prismatic.

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θ_i Joint Angle

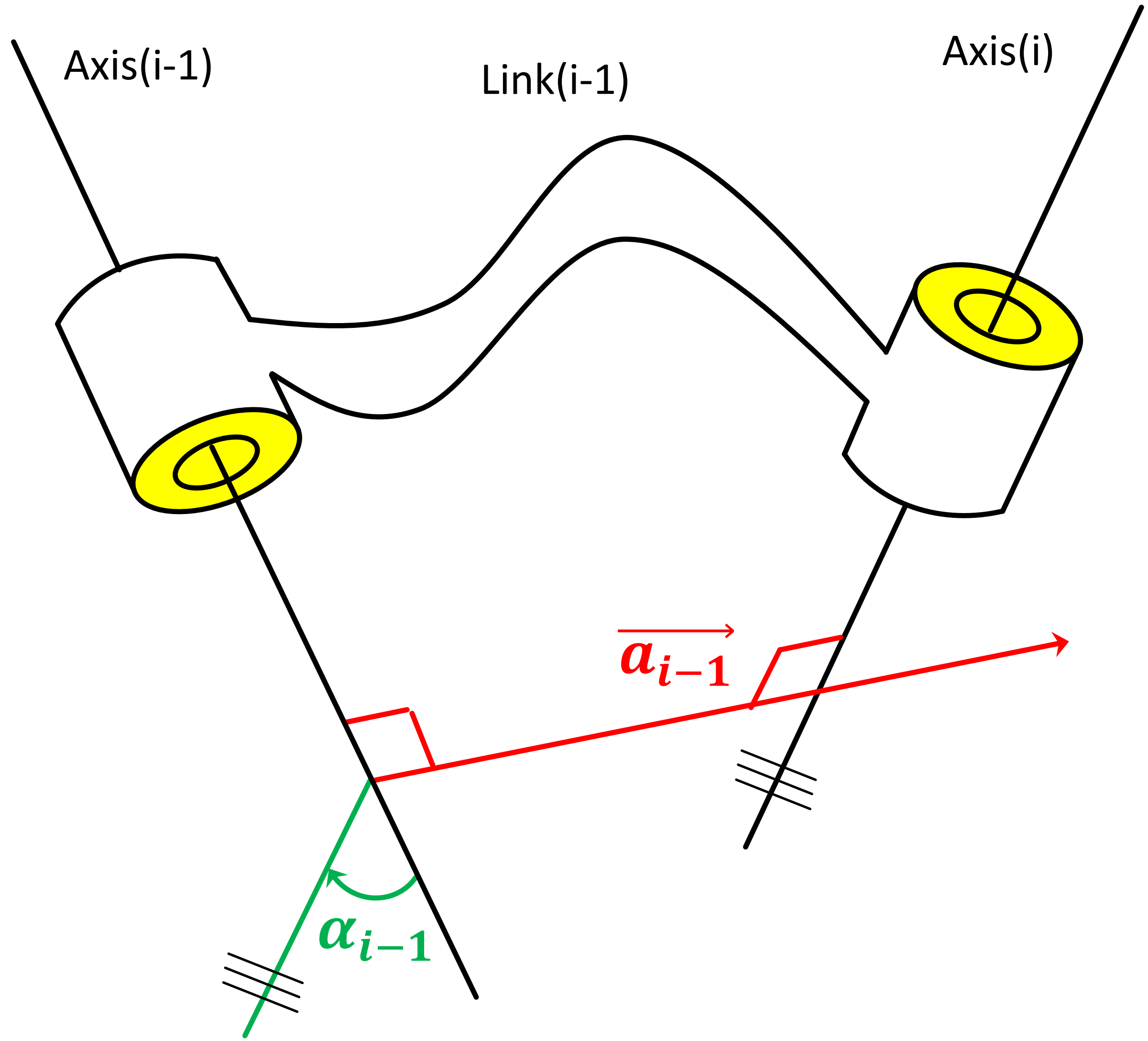
Variable if joint is revolute.

The joint angle between the normals is measured in a plane normal to the joint axis.

Link Description

$\overrightarrow{a_{i-1}}$ **Link Length**
and
α_{i-1} **Link Twist**

depend on joint axes
$i - 1$ and i.



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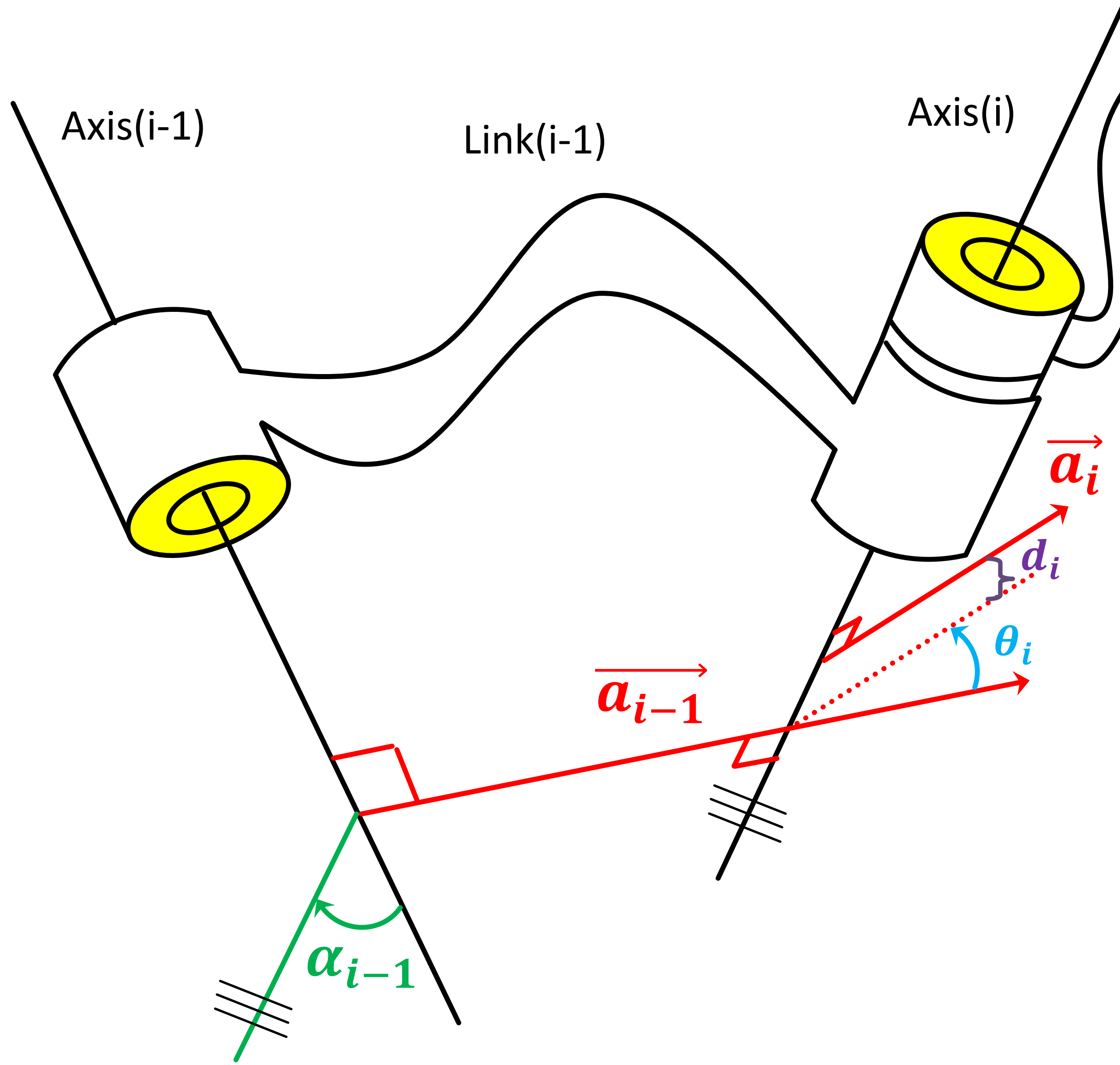

Denavit-Hartenberg Convention

Each A matrix has 6 variables- 3 in the rotation matrix and 3 in the position vector.

DH parameters collapse 6 variables to 4 link and joint parameters if we follow a certain procedure for setting coordinate frames.

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$\boldsymbol{\theta_i}$ **is joint angle** of link i (revolute variable)

Denavit-Hartenberg Matrix

Each homogeneous transformation A_i is represented as a product of four basic transformations:

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i} \, \mathrm{Trans}_{z,d_i} \, \mathrm{Trans}_{x,a_i} \, Rot_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Reminder:

a_i **is link length**
α_i **is link twist**
d_i **is link offset**
θ_i **is joint angle**

where the four quantities are parameters associated with *link* i and *joint* i.

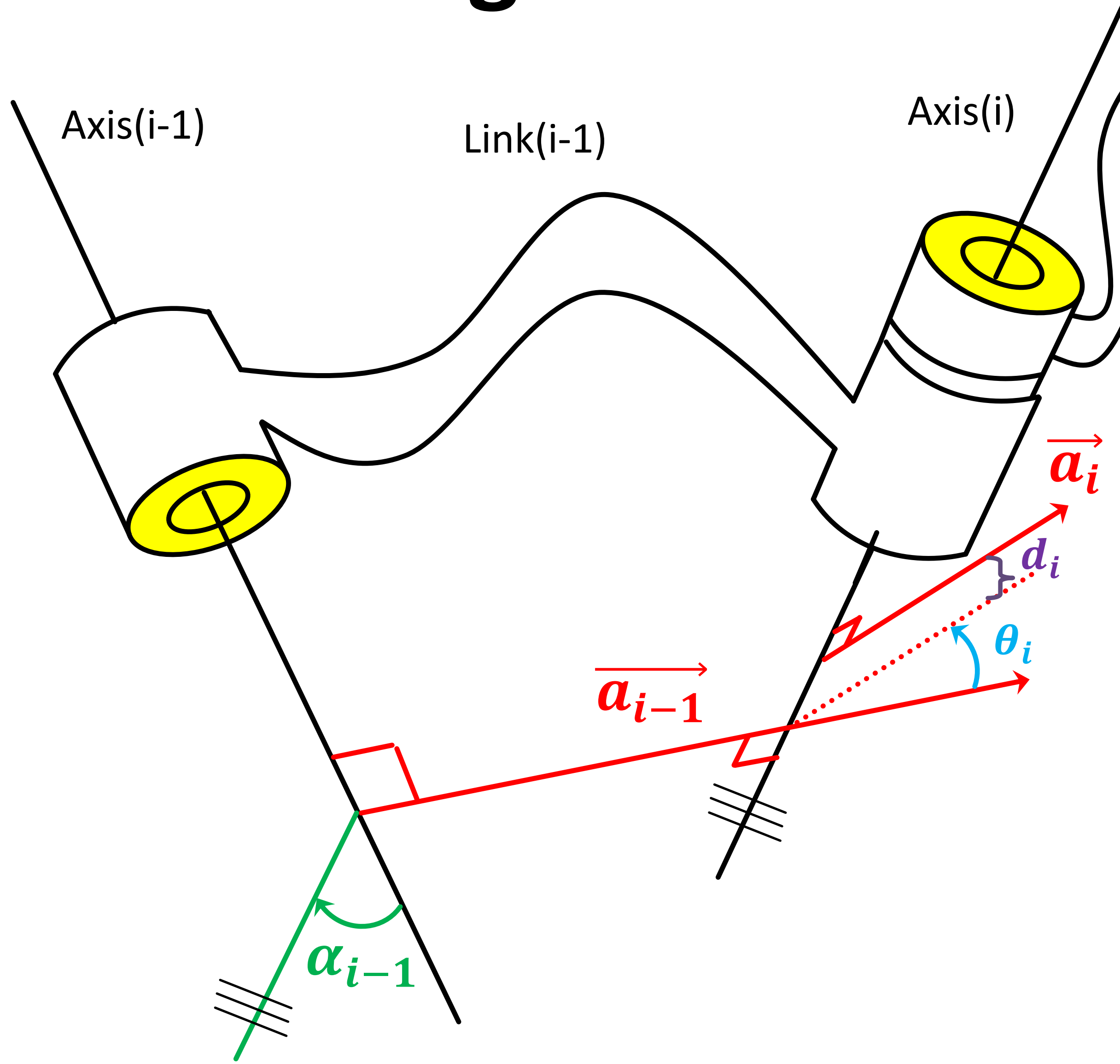
Denavit-Hartenberg Matrix

a_i is link length

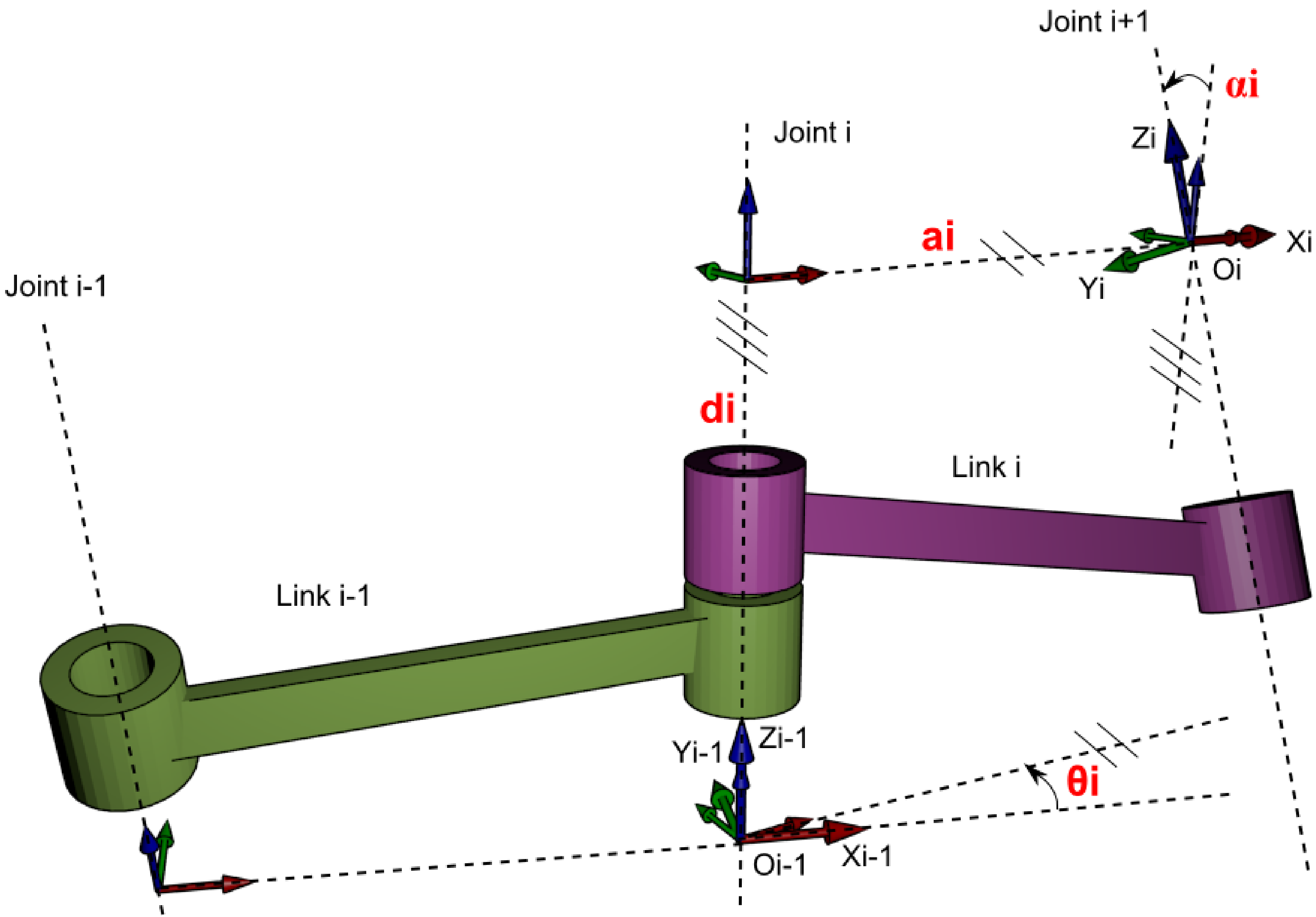
α_i is link twist

d_i is link offset

θ_i is joint angle



Denavit-Hartenberg Matrix



Denavit-Hartenberg Convention

it is not necessary that the origin of $frame\ i$ be placed at the physical end of $link\ i$.

it is not necessary that frame i be placed within the physical link; $frame\ i$ could lie in free space — so long as $frame\ i$ is **rigidly attached** to $link\ i$.

By a clever choice of the origin and the coordinate axes, it is possible to cut down the number of parameters needed from six to four (or even fewer in some cases).

Denavit-Hartenberg Convention

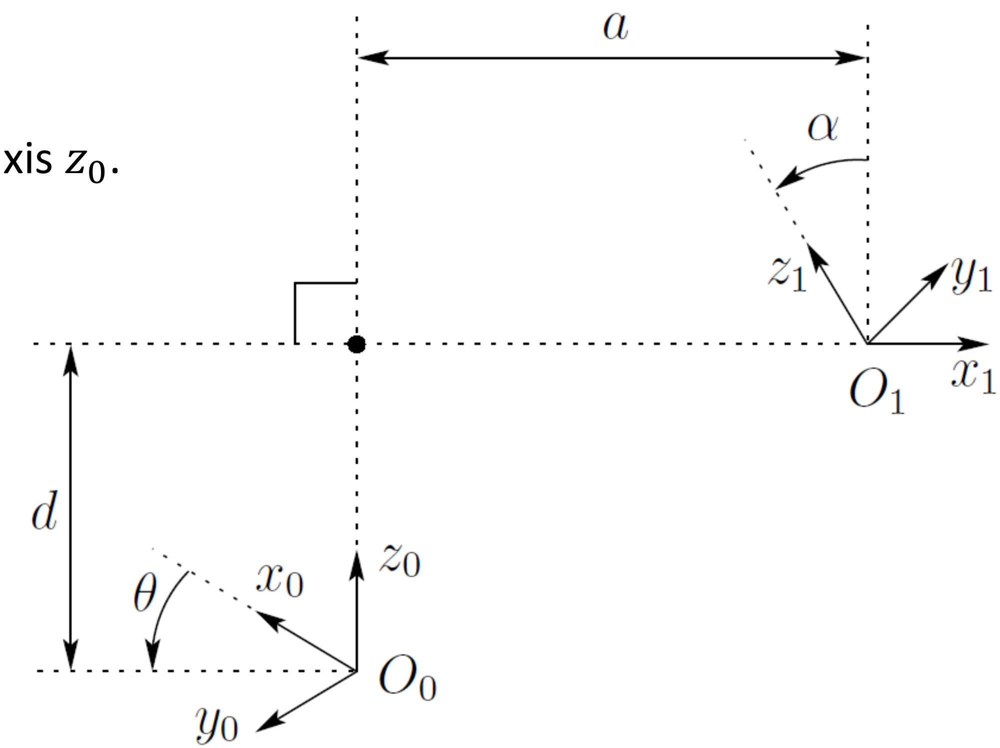
DH Coordinate Frame Assumptions

(DH1) The axis x_1 is perpendicular to the axis z_0.
(DH2) The axis x_1 intersects the axis z_0.

Under these conditions, there exist unique numbers **a, d, $\boldsymbol{\theta}$, $\boldsymbol{\alpha}$** such that:

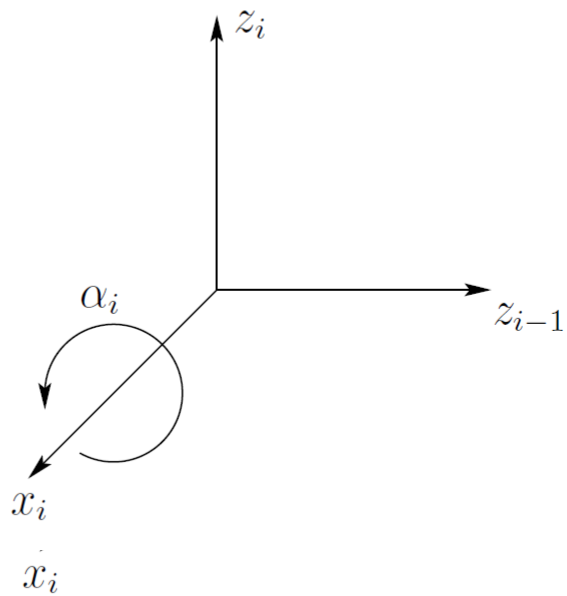
$$A = Rot_{z,\theta}\mathrm{Trans}_{z,d}\mathrm{Trans}_{x,a}Rot_{x,\alpha}$$

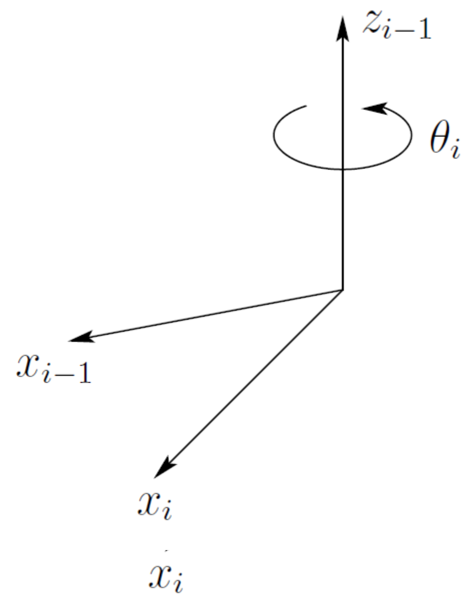
$$A = \begin{bmatrix} R_1^0 & o_1^0 \\ 0 & 1 \end{bmatrix}$$



Denavit-Hartenberg Convention

Positive sense for θ and α





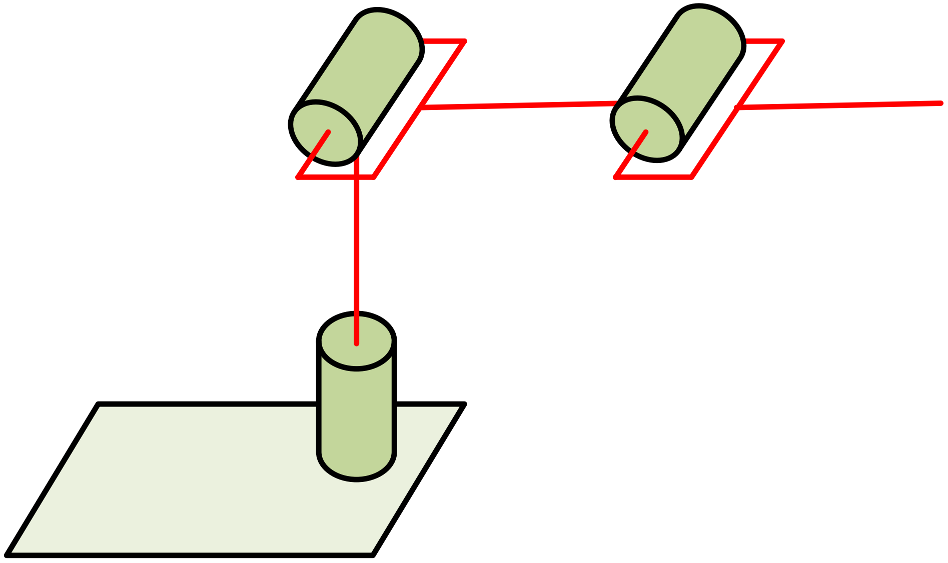
Rules For Assigning Frames

Rule 1: z_{i-1} is axis of actuation of joint i.
Axis of revolution of revolute joint
Axis of translation of prismatic joint

Rule 2: Axis x_i is set so it is perpendicular to and intersects z_{i-1}.

Rule 3: Derive y_i from x_i and z_i.

Rules For Assigning Frames



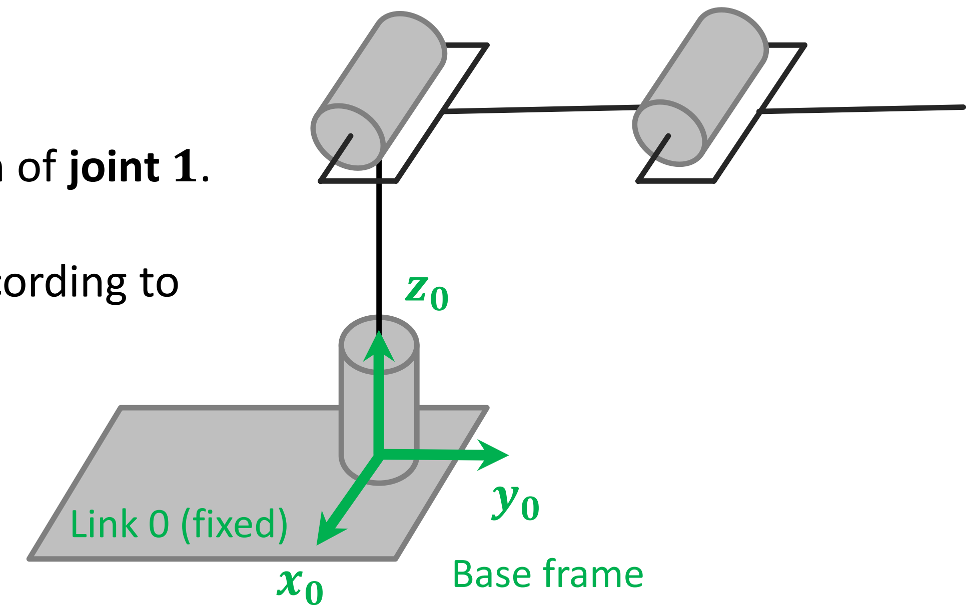
Rules For Assigning Frames

Rule 1: z_{i-1} is axis of actuation of joint i

Base frame

z_0 is axis of actuation of **joint 1**.

x_0 and y_0 are set according to the right hand rule.

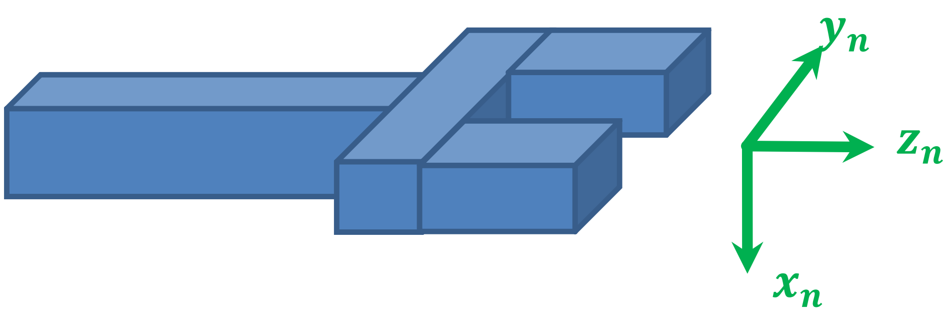


Rules For Assigning Frames

Rule 1: z_{i-1} is axis of actuation of joint i

Tool frame

z_n is the **approach** direction of the tool.
y_n is the **slide** direction of the gripper.
x_n is the **normal** direction to other axes.

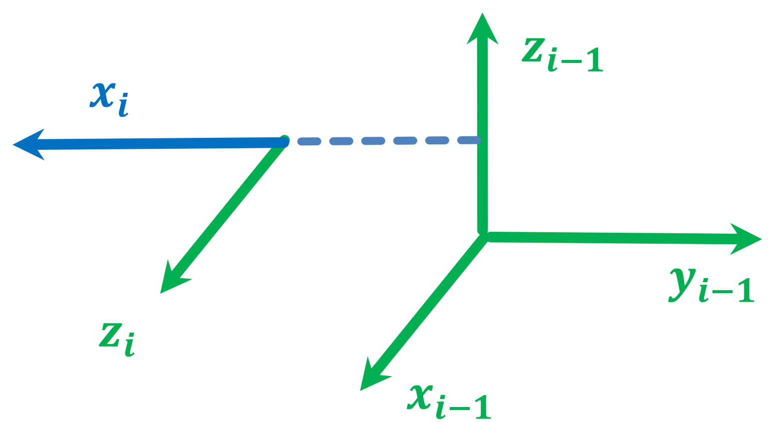


Rules For Assigning Frames

Rule 2: Axis x_i is set so it is perpendicular to and intersects z_{i-1}

Case 1: z_{i-1} and z_i are not coplanar.

- There is only one line possible for x_i, which is the shortest line from z_{i-1} to z_i.
- o_i is at intersection of x_i and z_i.



Rules For Assigning Frames

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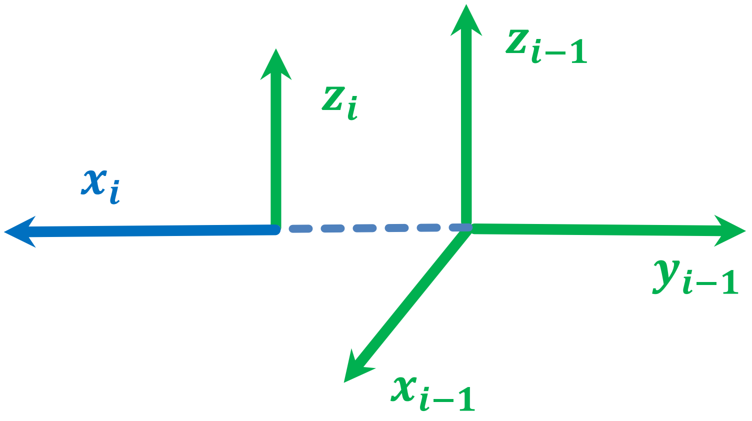
Case 2: z_{i-1} and z_i are parallel.

- There are an infinite number of possibilities for x_i from z_{i-1} to z_i.
- Usually easiest to choose an x_i that passes through o_{i-1} (so that $d_i = 0$).
- o_i is at intersection of x_i and z_i.
- $\alpha_i = 0$ always for this case.



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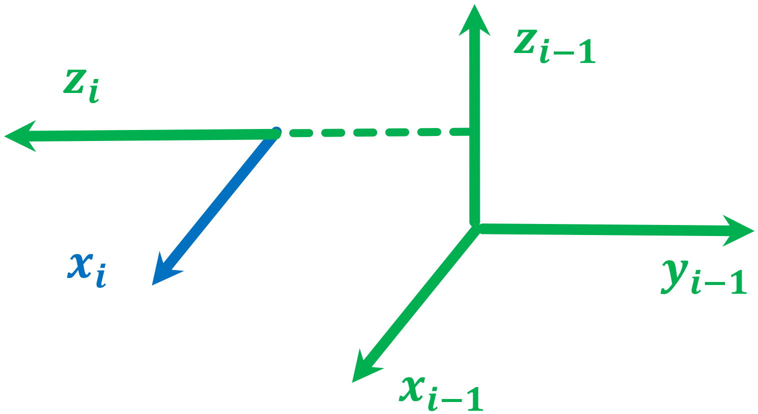
Case 3: z_{i-1} intersects z_i.

- x_i is normal to the plane of z_{i-1} and z_i.
- Positive direction of x_i is arbitrary.
- o_i naturally sits at intersection of z_{i-1} and z_i but can be anywhere on z_i.
- $a_i = 0$ always for this case.



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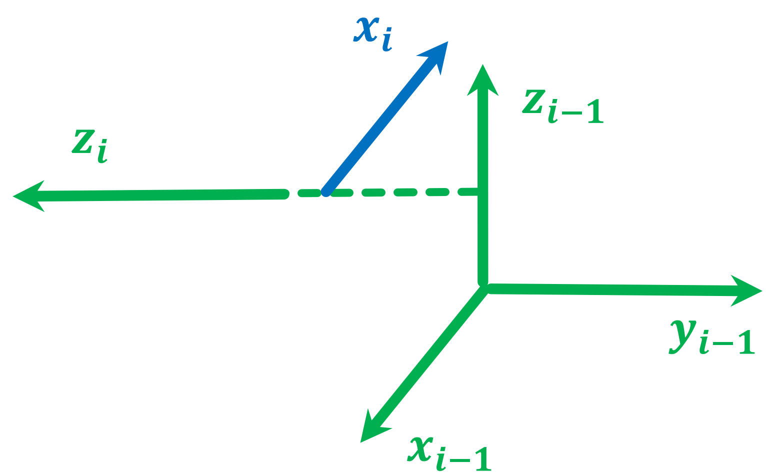

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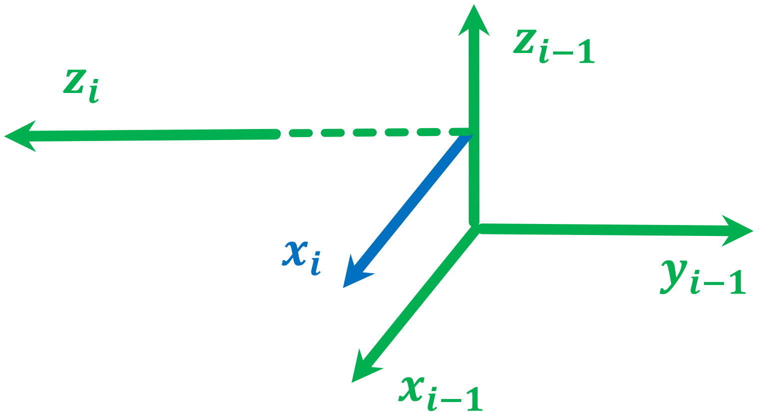
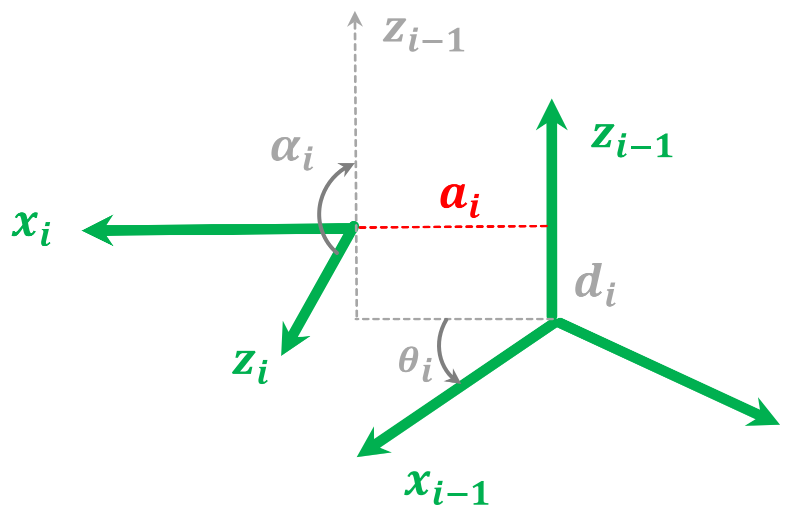


D-H Parameters

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.

$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.



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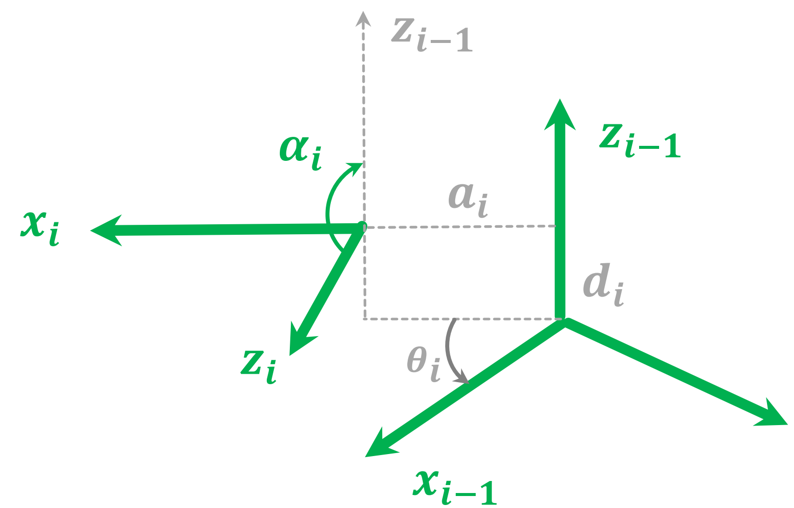
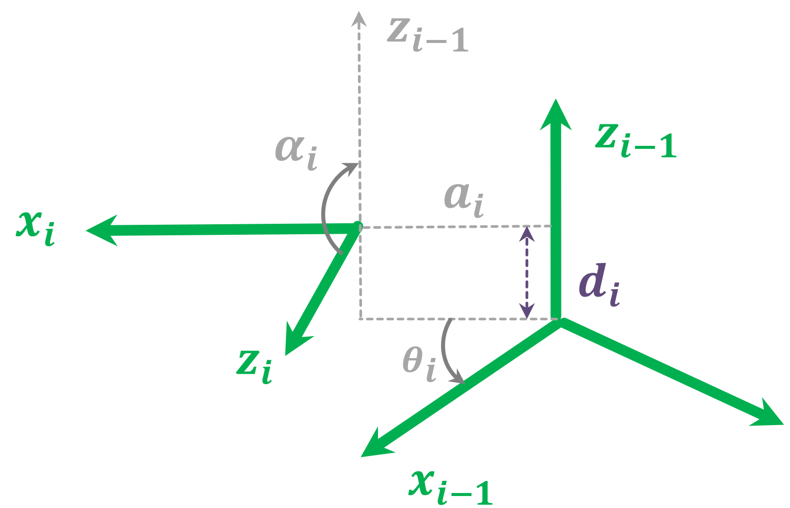
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.

$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.



D-H Parameters

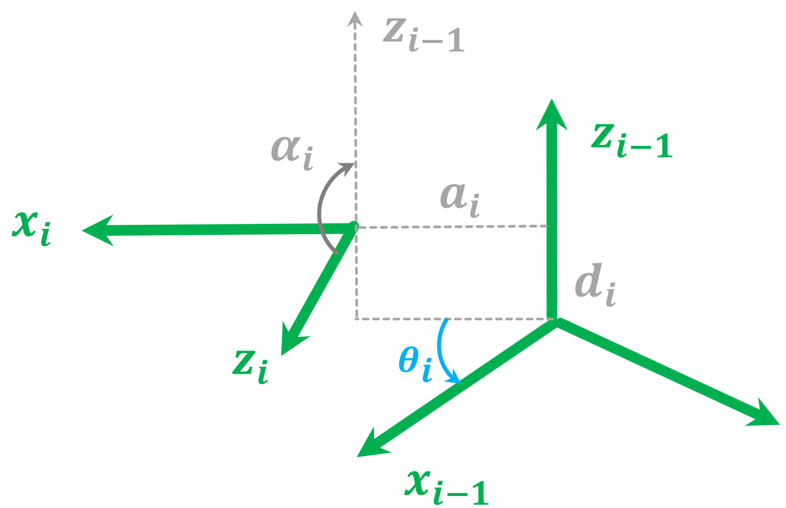
$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.



D-H Parameters

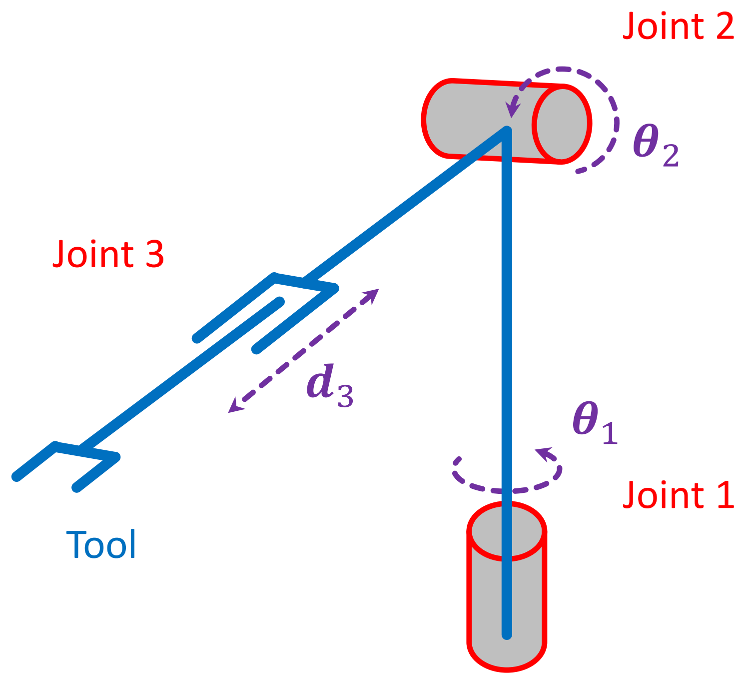
$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.

$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.



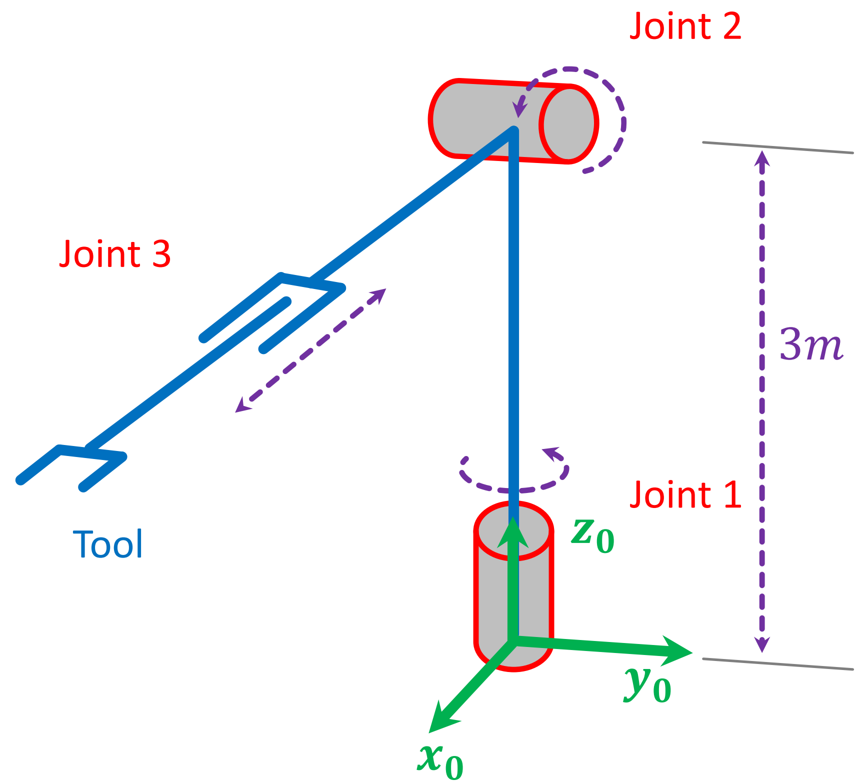
Example: RRP Robot

Assign coordinate frames so that we can find DH parameters for this robot.



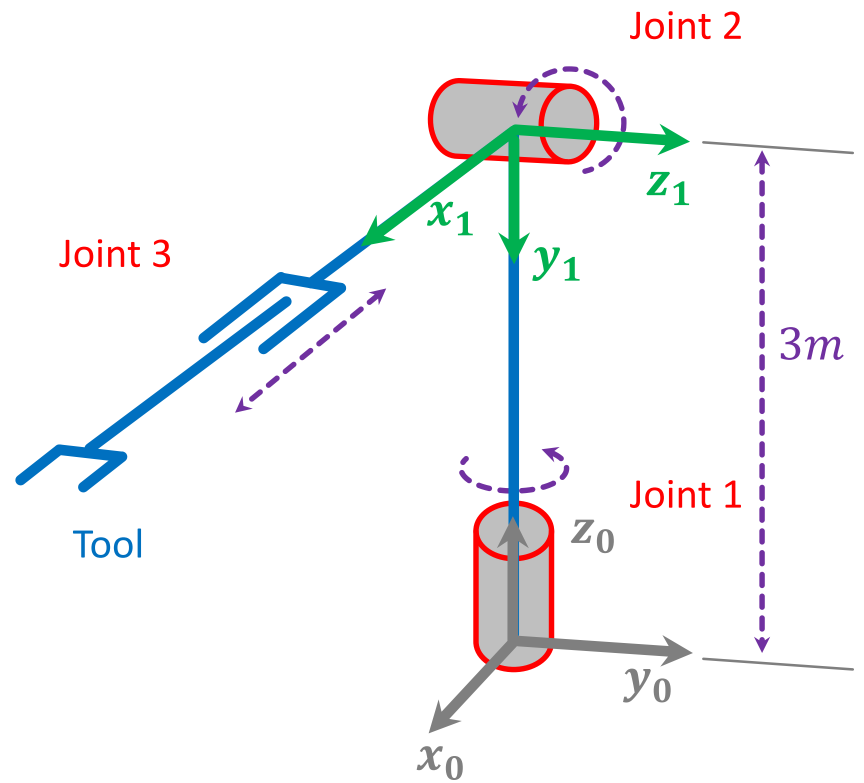
Example: RRP Robot

Assign coordinate frames so that we can find DH parameters for this robot.



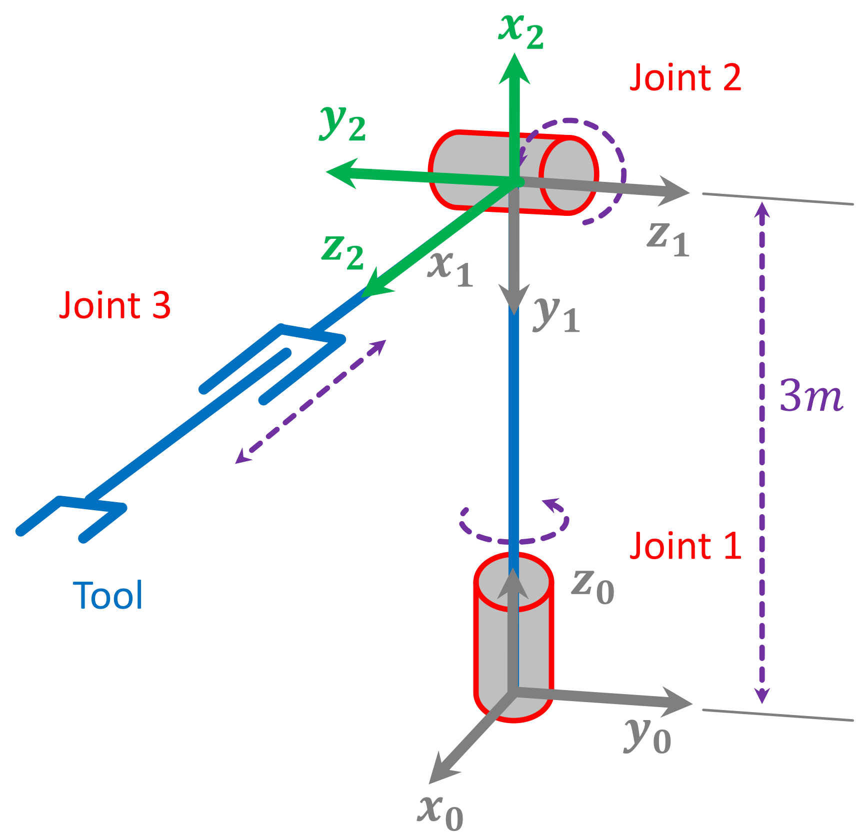
Example: RRP Robot

Assign coordinate frames so that we can find DH parameters for this robot.



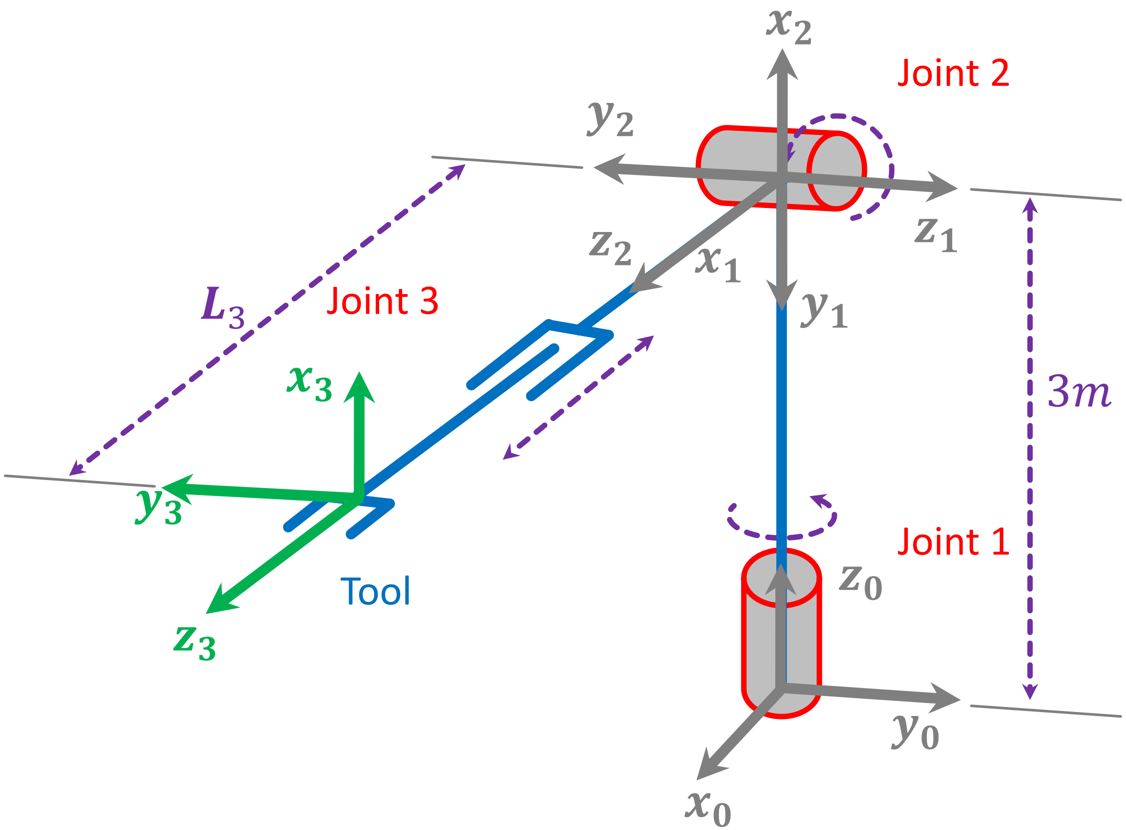
Example: RRP Robot

Assign coordinate frames so that we can find DH parameters for this robot.



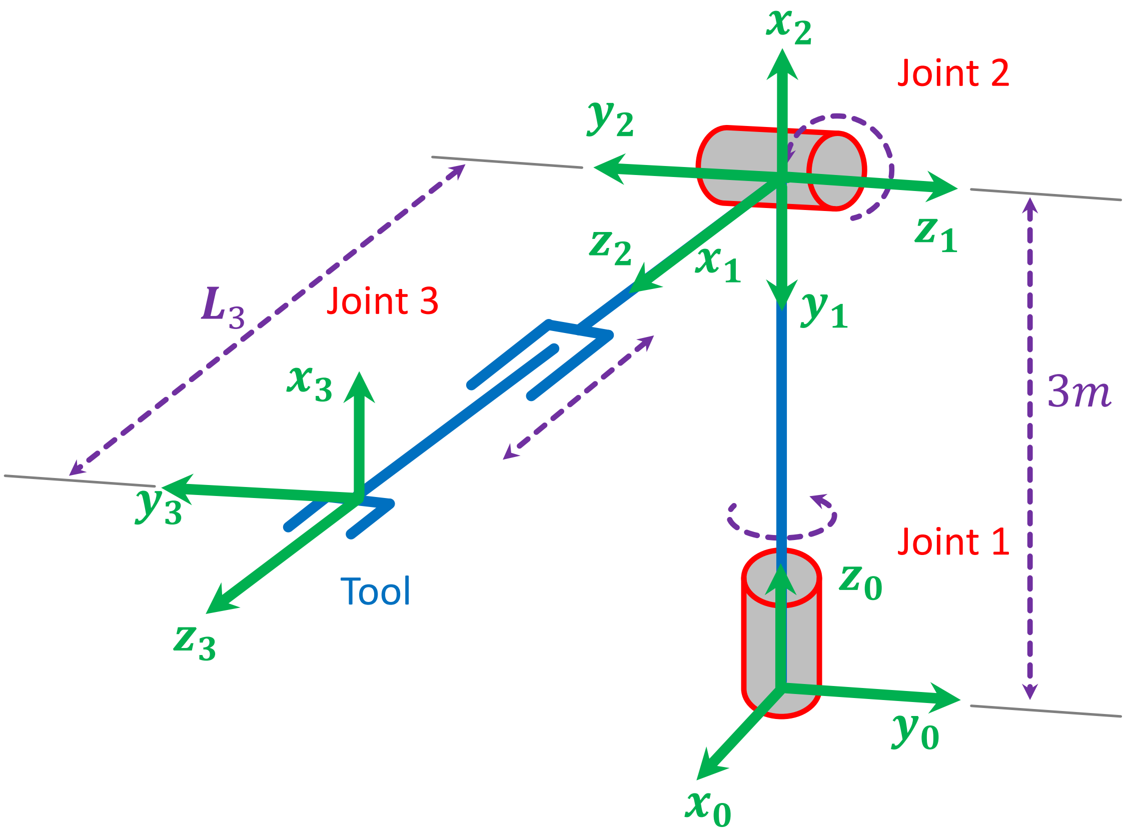
Example: RRP Robot

Assign coordinate frames so that we can find DH parameters for this robot.



Example: RRP Robot

Find DH parameters for this robot. Identify the joint variables.

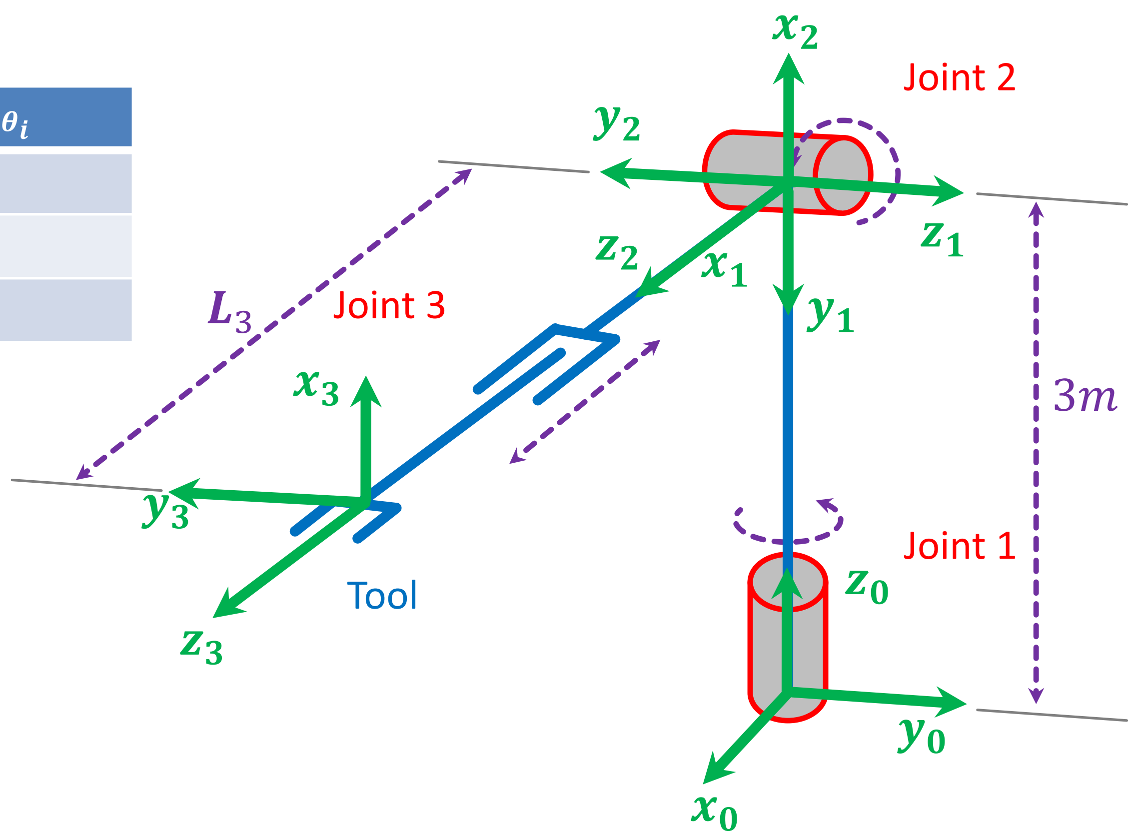


Example: RRP Robot

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

Find DH parameters for this robot. Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1				
2				
3				

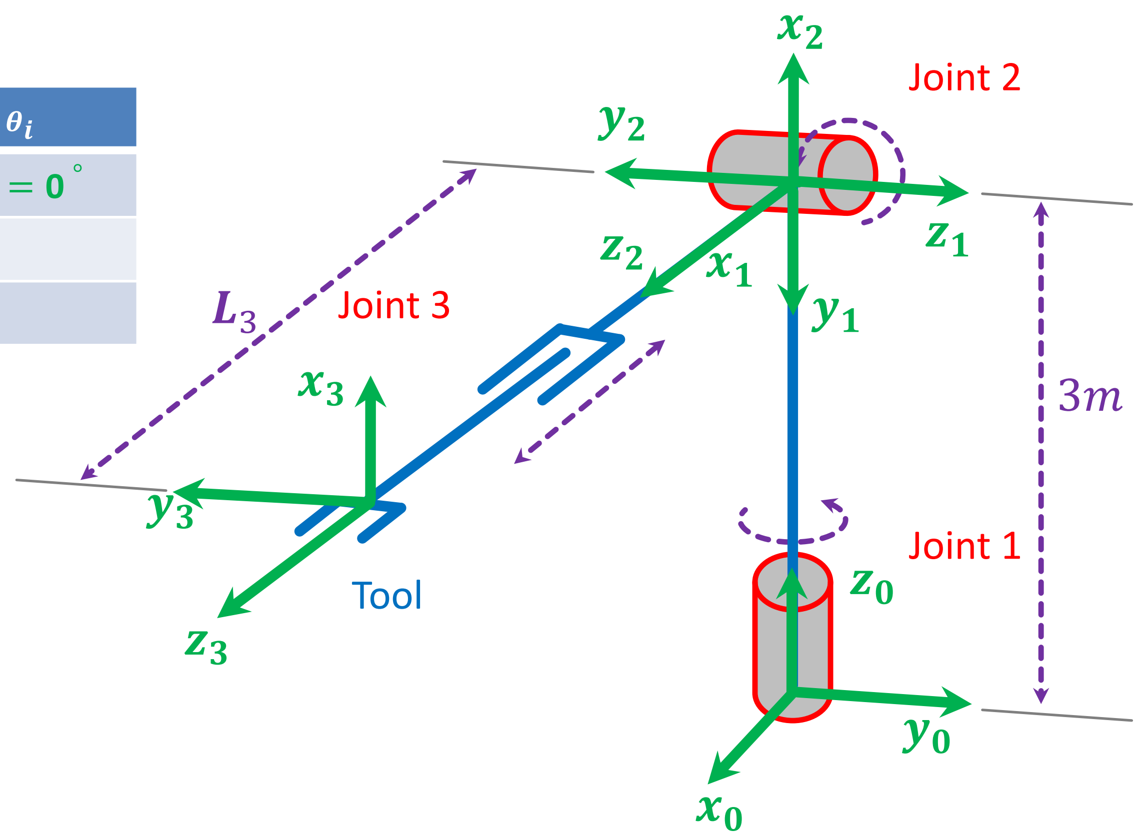


Example: RRP Robot

Find DH parameters for this robot. Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\theta_1 = 0^\circ$
2				
3				

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

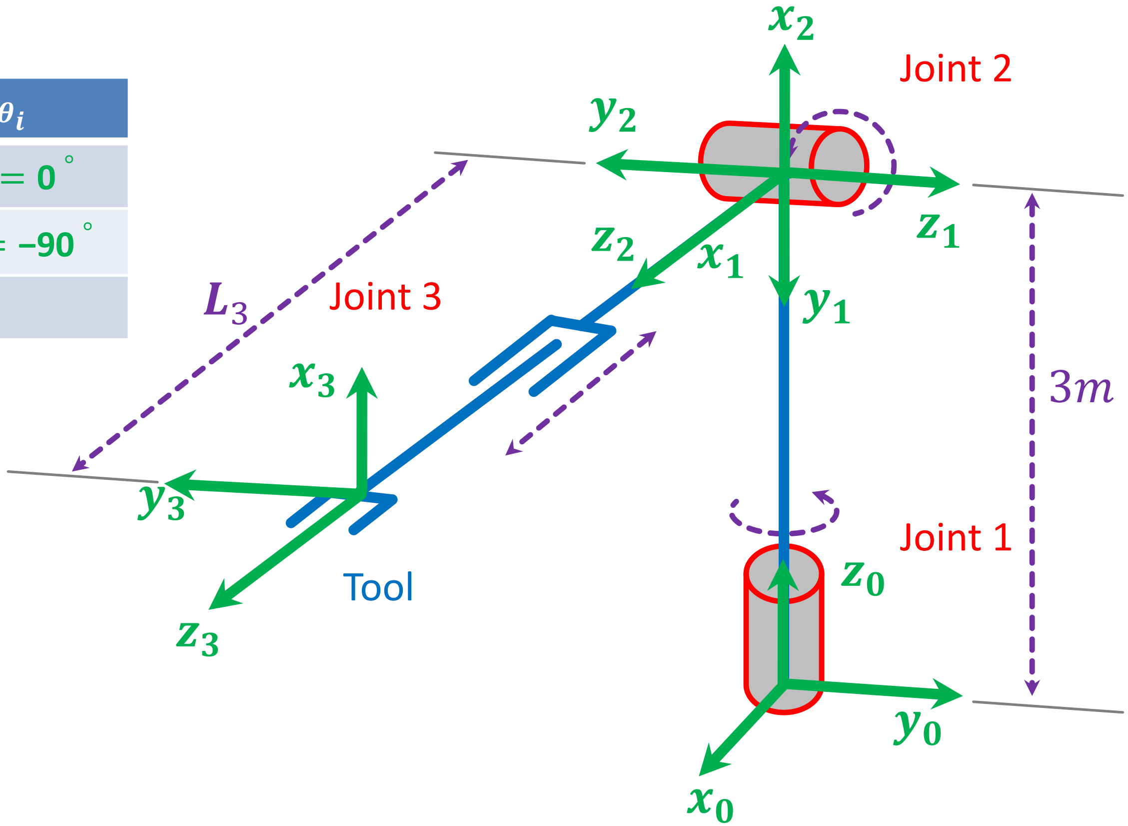


Example: RRP Robot

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

Find DH parameters for this robot. Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1	0	-90°	3m	$\theta_1 = 0^\circ$
2	0	-90°	0	$\theta_2 = -90^\circ$
3				

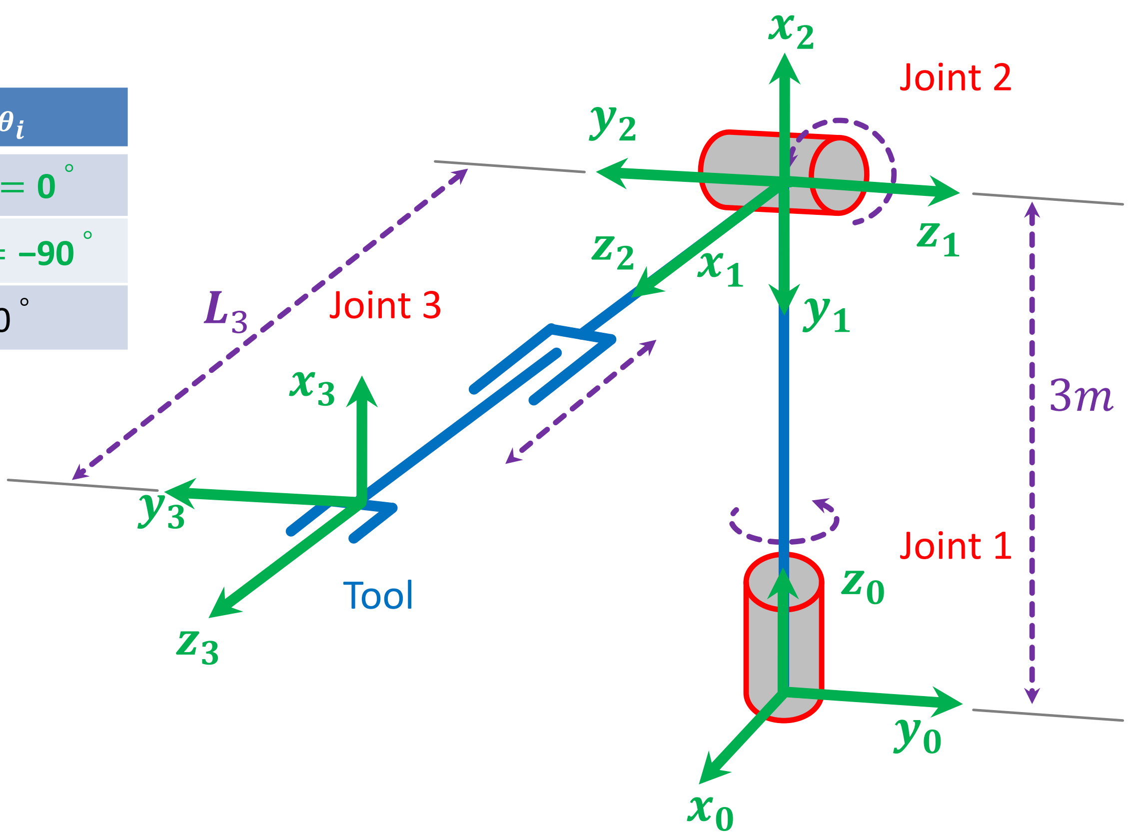


Example: RRP Robot

Find DH parameters for this robot. Identify the joint variables.

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

i	a_i	α_i	d_i	θ_i
1	0	-90°	3m	$\theta_1 = 0^\circ$
2	0	-90°	0	$\theta_2 = -90^\circ$
3	0	0°	$d_3 = L_3$	0°



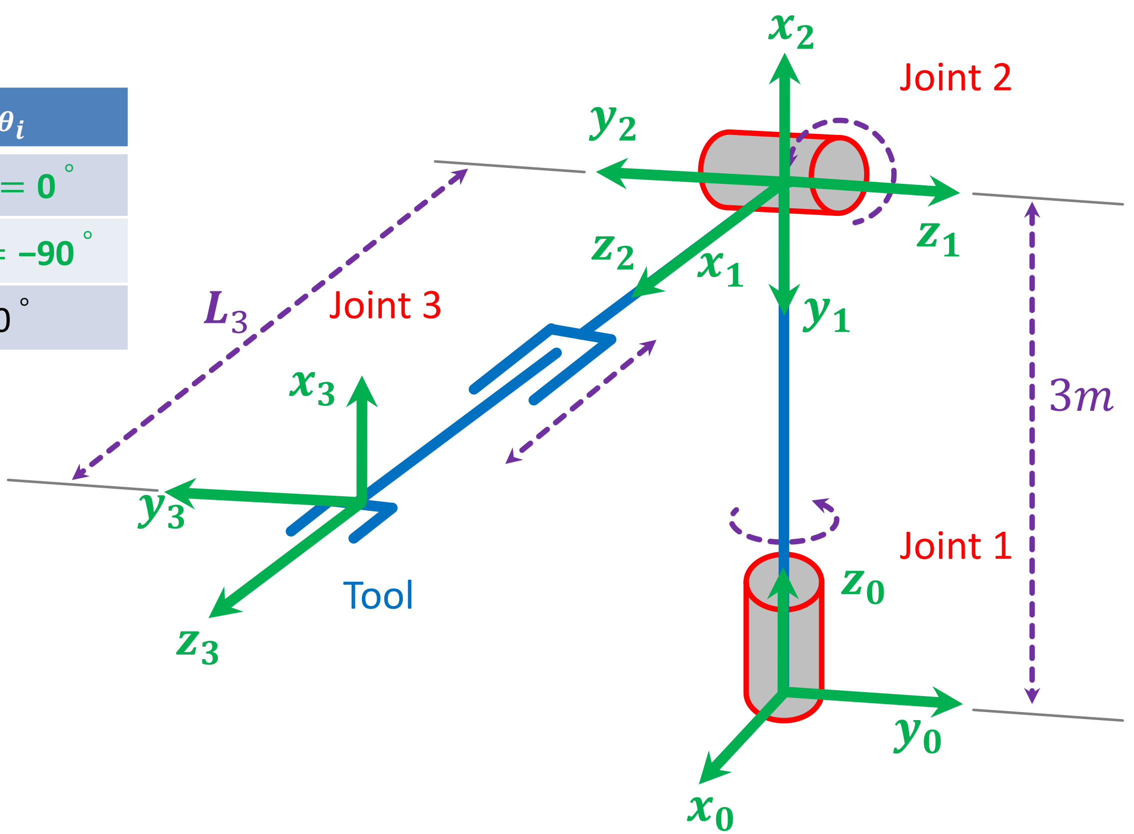
Example: RRP Robot

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

Find DH parameters for this robot. Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1	0	-90°	3m	$\theta_1 = 0^\circ$
2	0	-90°	0	$\theta_2 = -90^\circ$
3	0	0°	$d_3 = L_3$	0°

Find the A matrices



Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	-90°	3m	$\theta_1 = 0^{\circ}$
2	0	-90°	0	$\theta_2 = -90^{\circ}$
3	0	0°	$d_3 = L_3$	0°

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_1 =$

$A_2 =$

$A_3 =$

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\theta_1 = 0$ °
2	0	–90 °	0	$\theta_2 = -90$ °
3	0	0 °	$d_3 = L_3$	0 °

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \boldsymbol{L_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	-90 °	3m	$\theta_1 = 0°$
2	0	-90 °	0	$\theta_2 = -90°$
3	0	0 °	$d_3 = L_3$	0 °

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

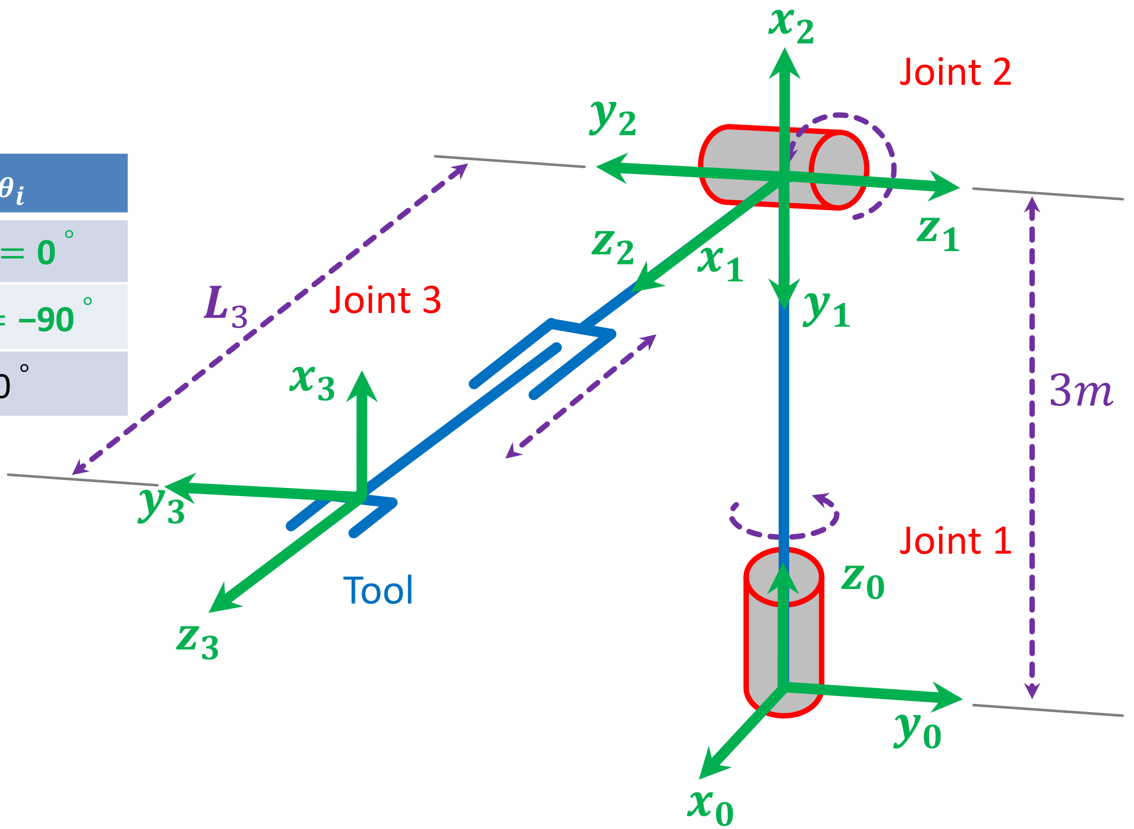
$T^0_1 =$

$T^0_2 =$ $\quad _2 =$

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\theta_1 = 0°$
2	0	–90 °	0	$\theta_2 = -90°$
3	0	0 °	$d_3 = L_3$	0 °



$$T_1^0 = A_1$$

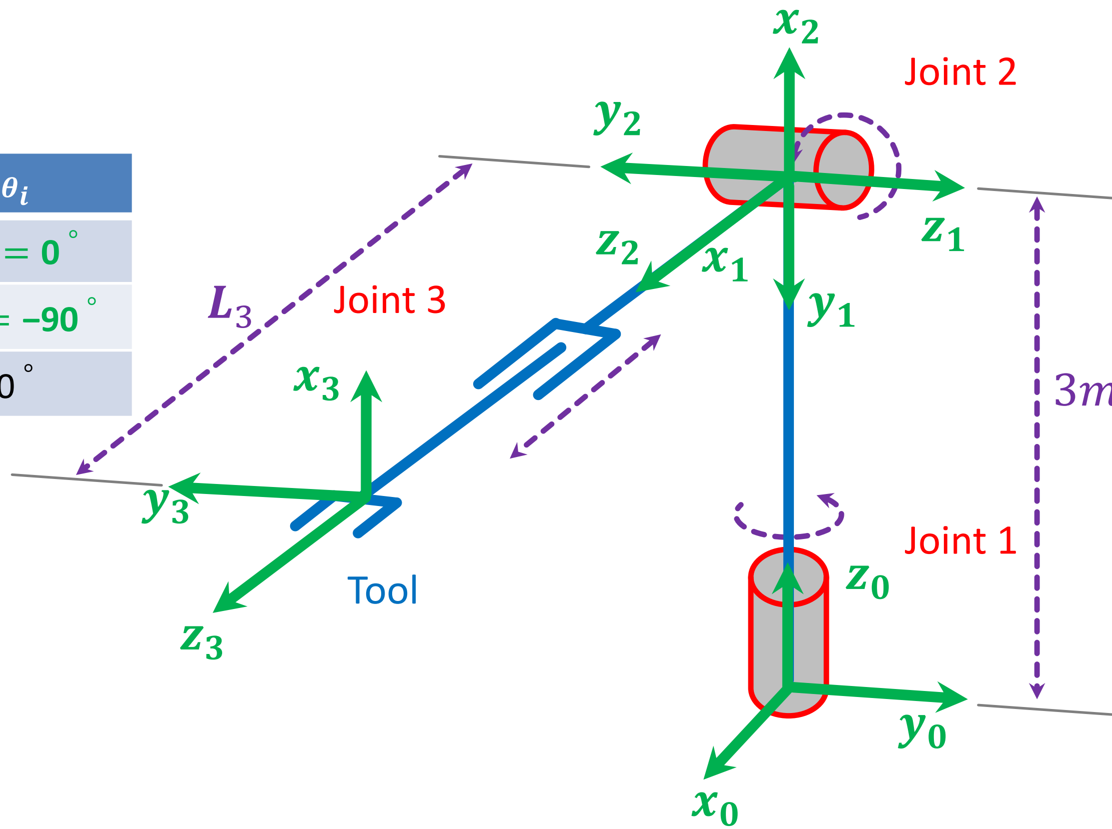
$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_1c_2 & s_1 & -c_1s_2 & 0 \\ s_1c_2 & -c_1 & -s_1s_2 & 0 \\ -s_2 & 0 & -c_2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? & 0 \\ ? & ? & ? & 0 \\ ? & 0 & ? & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the current configuration

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\theta_1 = 0$ °
2	0	–90 °	0	$\theta_2 = -90$ °
3	0	0 °	$d_3 = L_3$	0 °



$$T_1^0 = A_1$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_1c_2 & s_1 & -c_1s_2 & 0 \\ s_1c_2 & -c_1 & -s_1s_2 & 0 \\ -s_2 & 0 & -c_2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \text{[illegible]} & \text{[illegible]} & 1 & 0 \\ \text{[illegible]} & 1 & 0 & 0 \\ \text{[illegible]} & \text{[illegible]} & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the current configuration

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	-90 °	3m	$\theta_1 = 0$ °
2	0	-90 °	0	$\theta_2 = -90$ °
3	0	0 °	$d_3 = L_3$	0 °

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

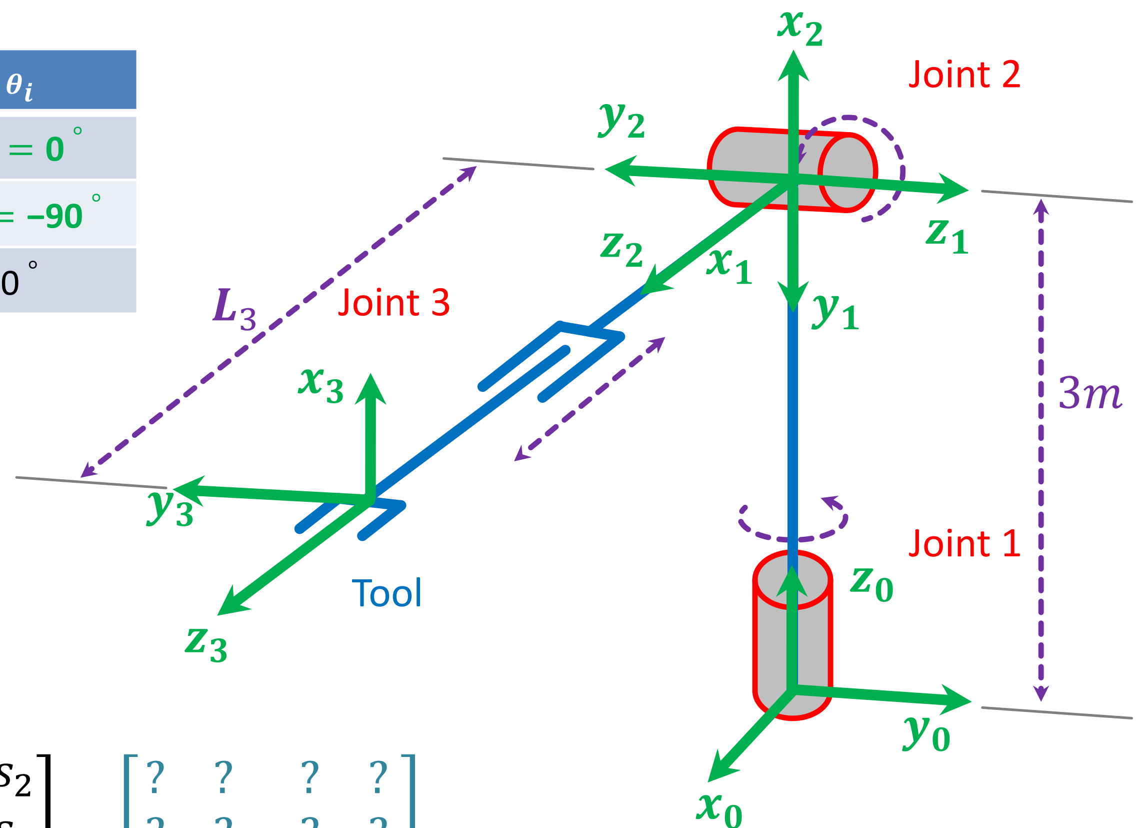
$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 =$$

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\boldsymbol{\theta_1 = 0}$ °
2	0	–90 °	0	$\boldsymbol{\theta_2 = -90}$ °
3	0	0 °	$\boldsymbol{d_3 = L_3}$	0 °



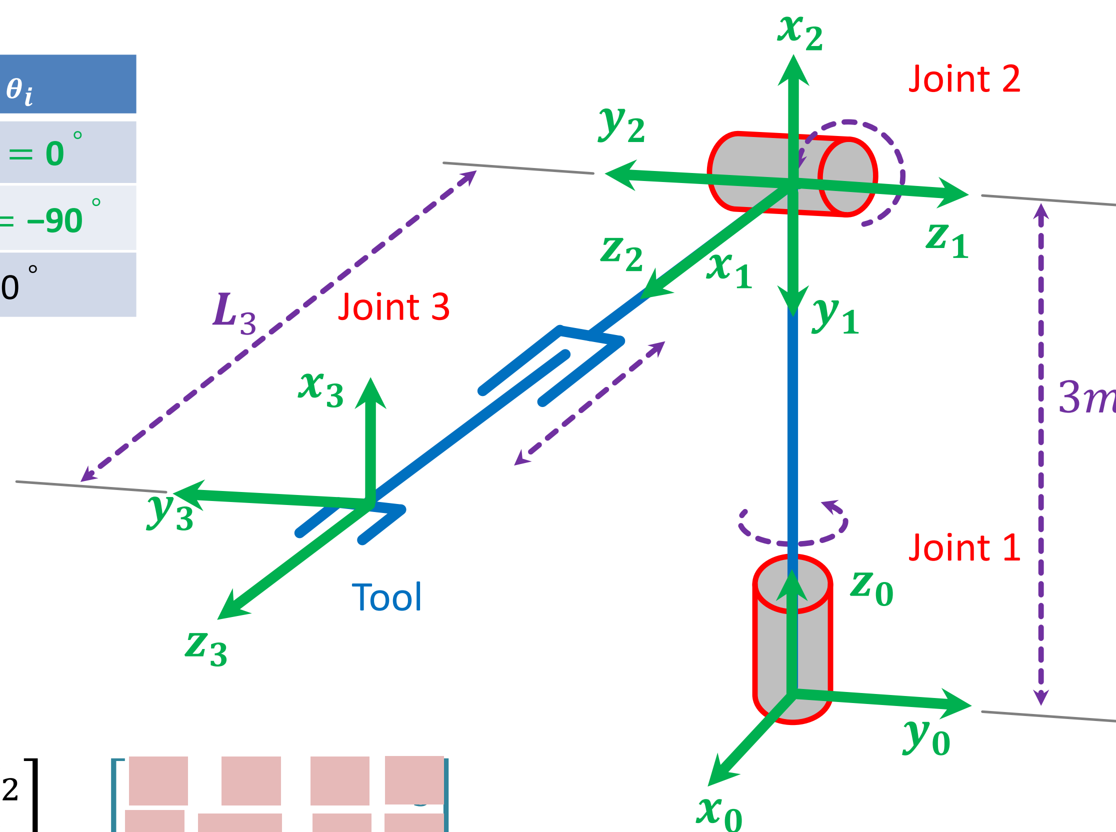
$$T_3^0 = \begin{bmatrix} c_1c_2 & s_1 & -c_1s_2 & -L_3c_1s_2 \\ s_1c_2 & -c_1 & -s_1s_2 & -L_3s_1s_2 \\ -s_2 & 0 & -c_2 & 3-L_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & 0 & ? & ? \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the current configuration

Example: RRP Robot

Find the A matrices

i	a_i	α_i	d_i	θ_i
1	0	–90 °	3m	$\theta_1 = 0^\circ$
2	0	–90 °	0	$\theta_2 = -90^\circ$
3	0	0 °	$d_3 = L_3$	0 °

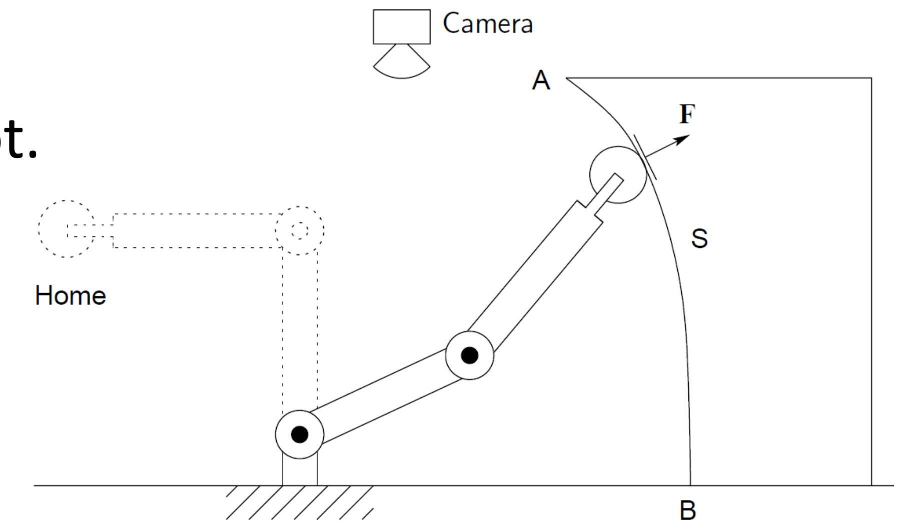


$$T_3^0 = \begin{bmatrix} c_1 c_2 & s_1 & -c_1 s_2 & -L_3 c_1 s_2 \\ s_1 c_2 & -c_1 & -s_1 s_2 & -L_3 s_1 s_2 \\ -s_2 & 0 & -c_2 & 3 - L_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \\ & 0 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the current configuration

Example: Two-Link Planar Robot

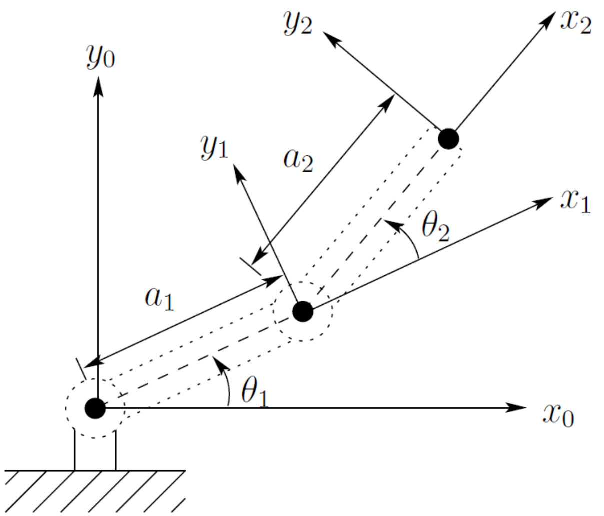
Assign coordinate frames so that we can find DH parameters for this robot.



Example: Two-Link Planar Robot

Find DH parameters for this robot.
Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1				
2				

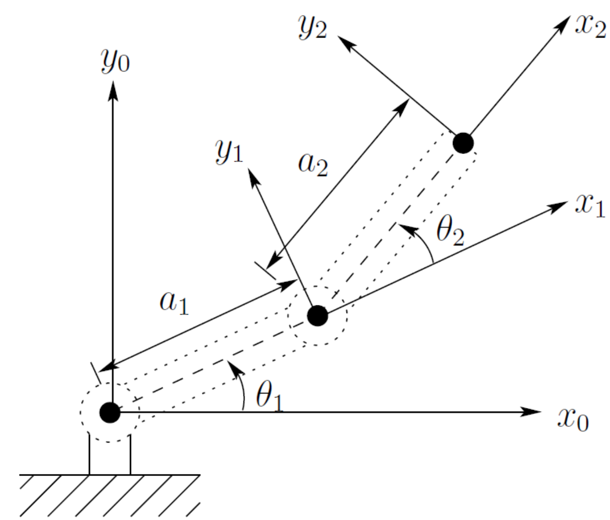


$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

Example: Two-Link Planar Robot

Find DH parameters for this robot.
Identify the joint variables.

i	a_i	α_i	d_i	θ_i
1	a_1	0°	0	$\boldsymbol{\theta_1}$
2	a_2	0°	0	$\boldsymbol{\theta_2}$



$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

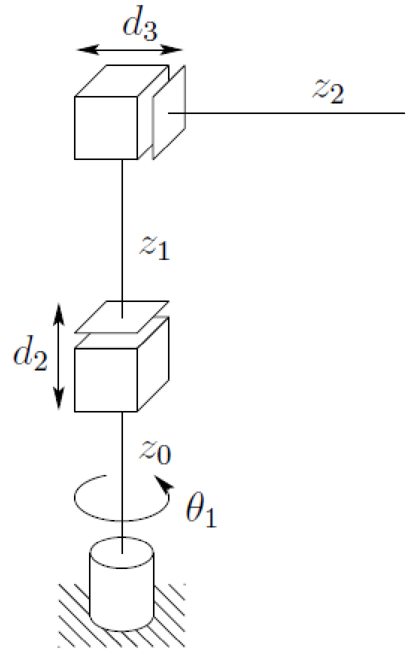
$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

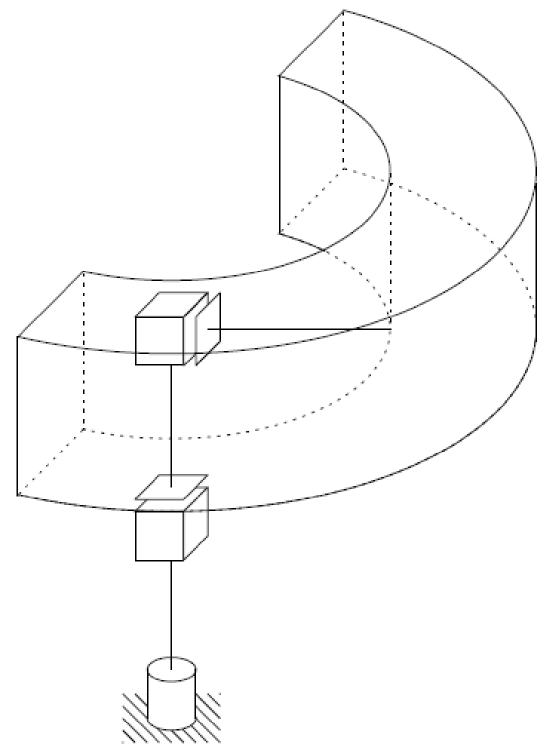
$$T_1^0 =$$

$$T_2^0 =$$

F.K. For Cylindrical Manipulator

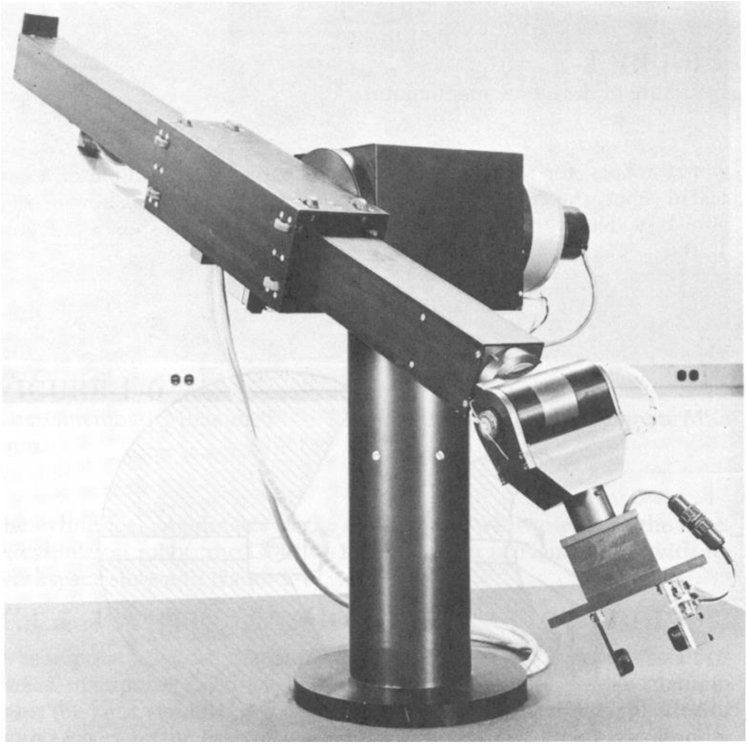
- Assign coordinate frames so that we can find DH parameters for this robot.
- Find DH parameters for this robot. Identify the joint variables.

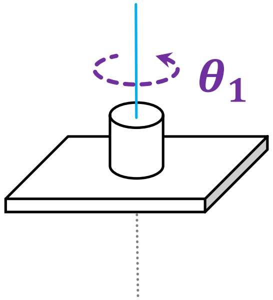




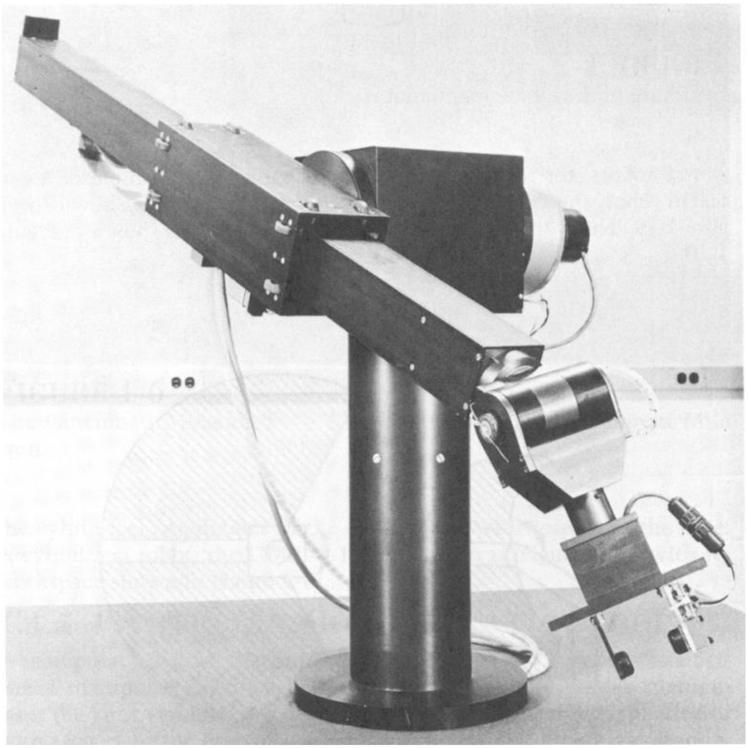
One rotational and two translational (RPP).
The axis of the second joint is parallel to the first axis.
The third joint axis is perpendicular to the second one.

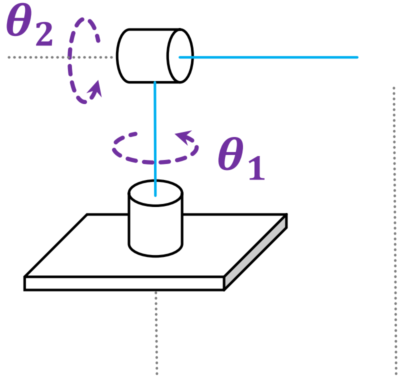
Stanford Arm



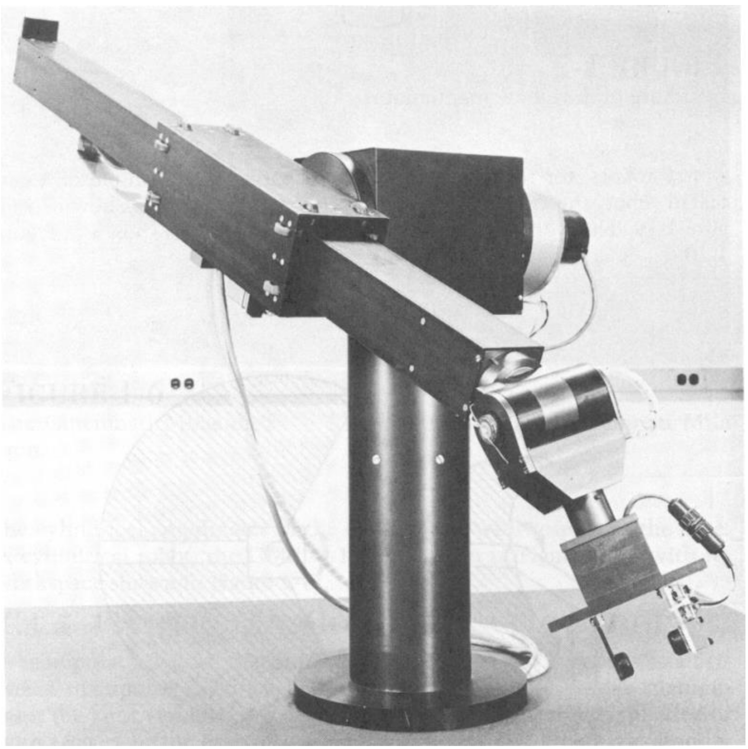


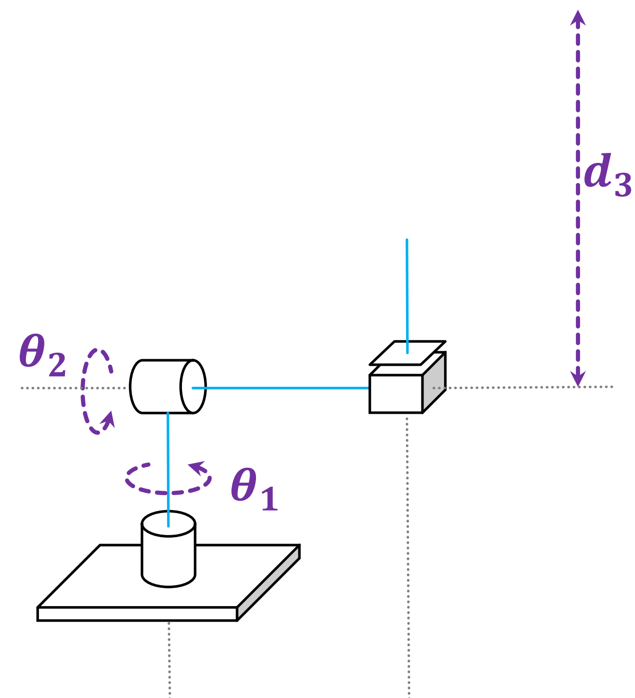
Stanford Arm



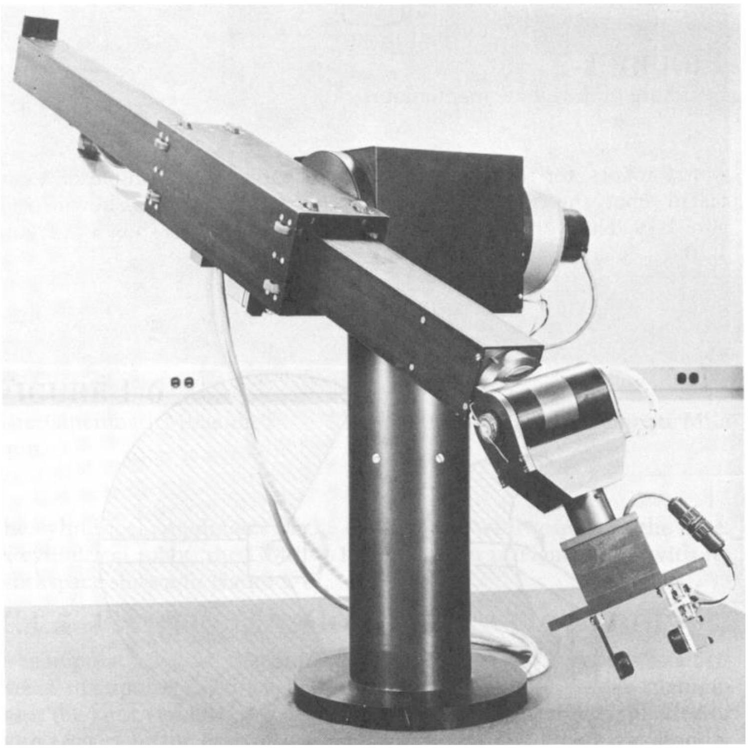


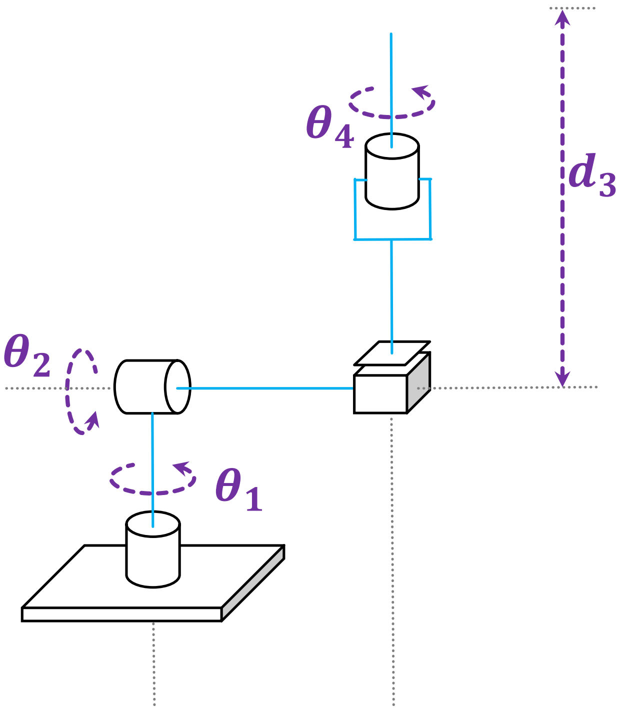
Stanford Arm



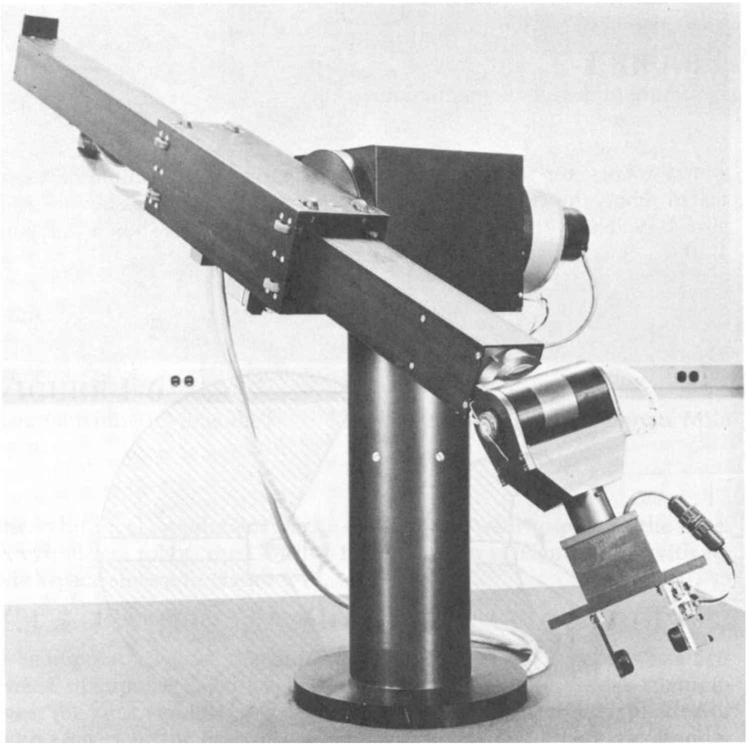


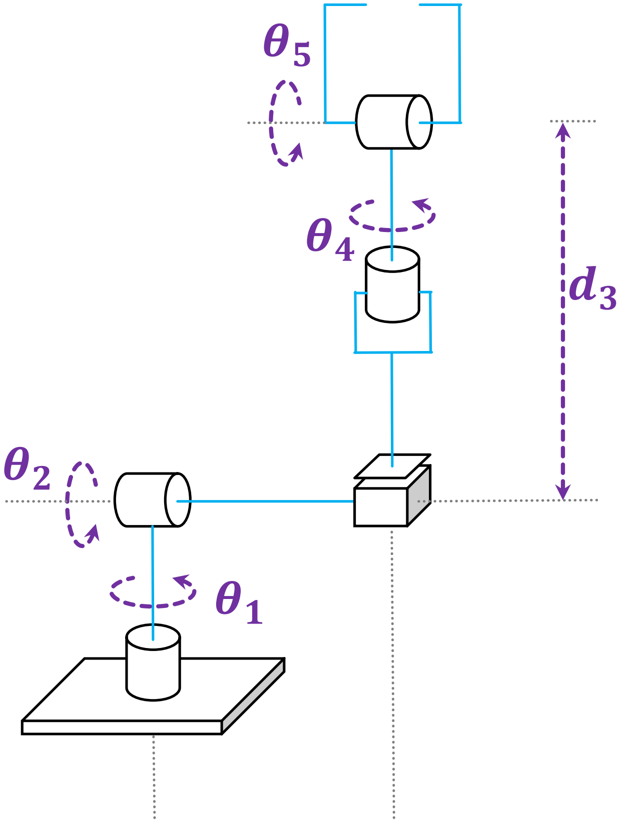
Stanford Arm



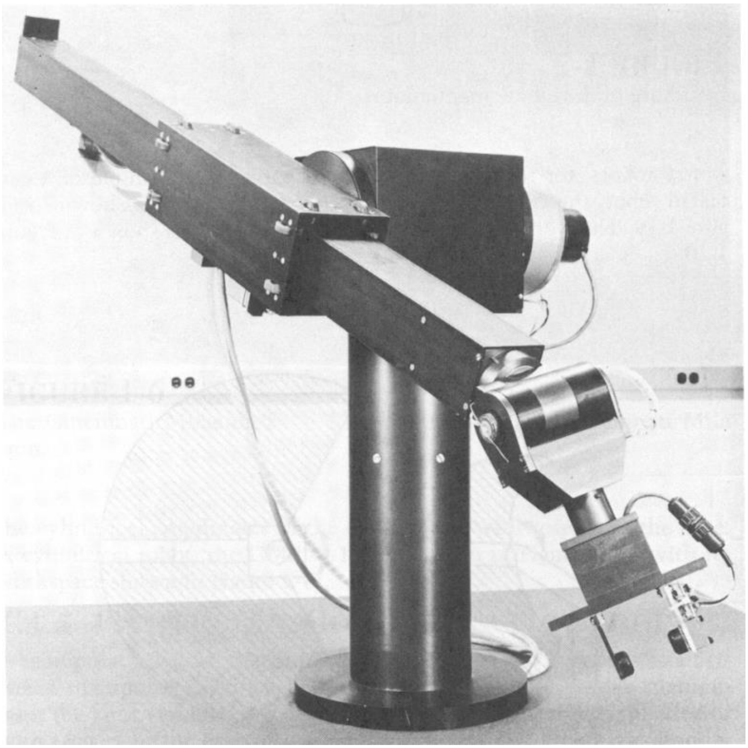


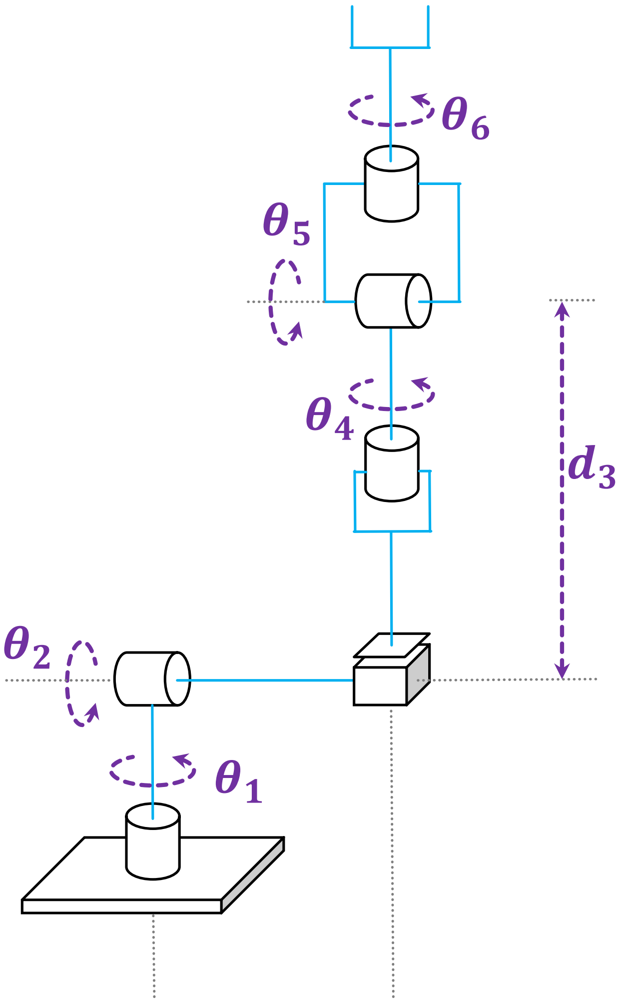
Stanford Arm



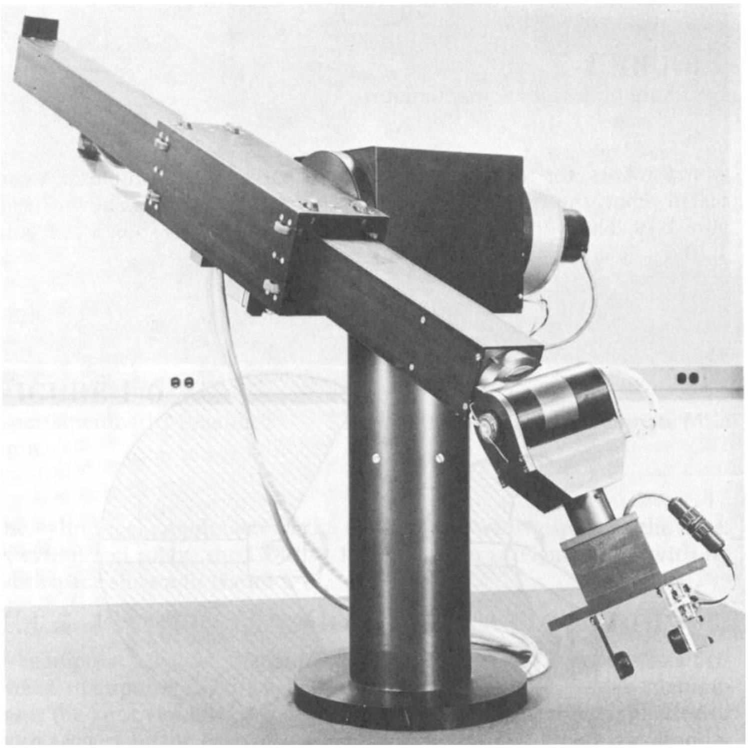


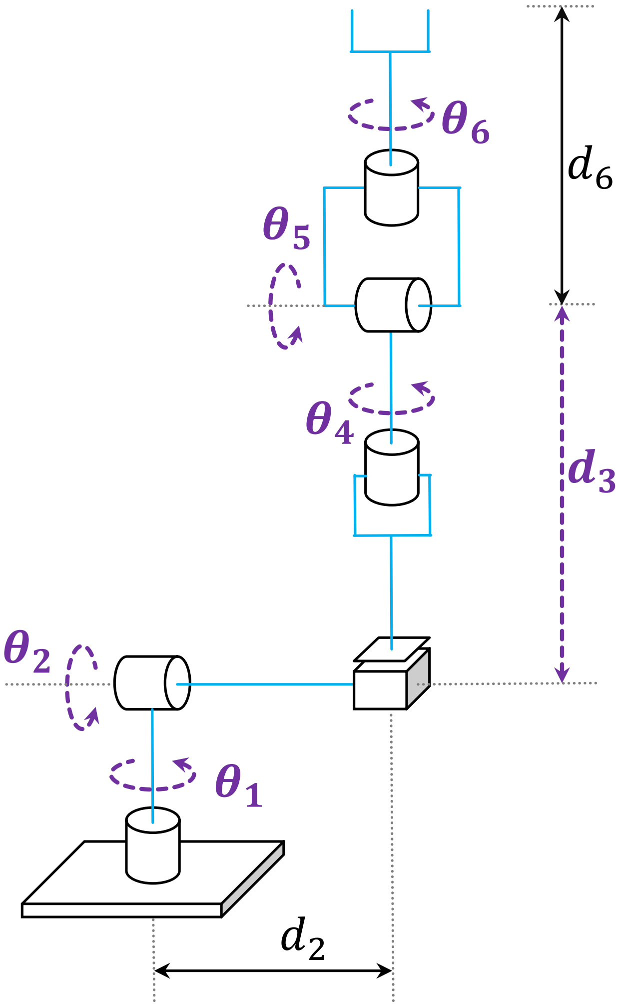
Stanford Arm



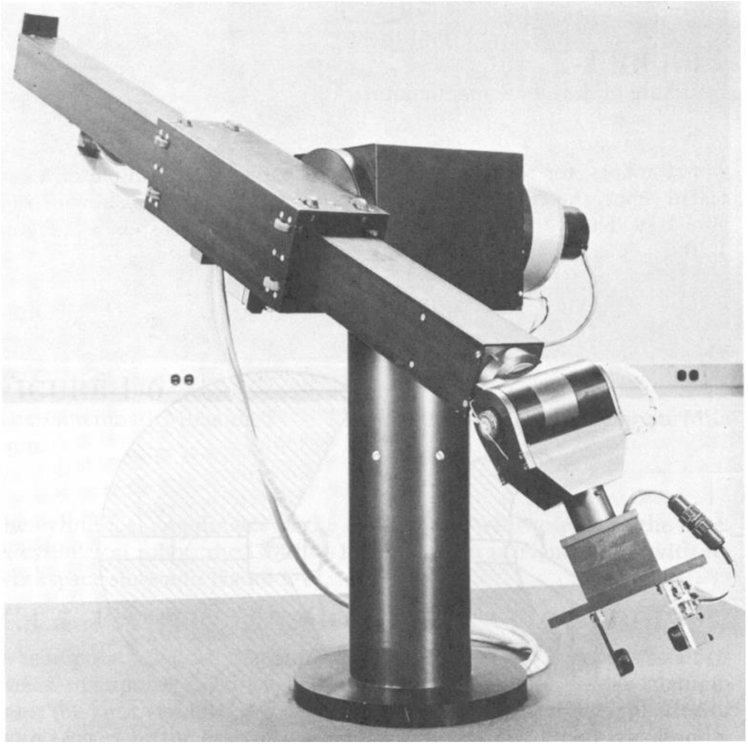


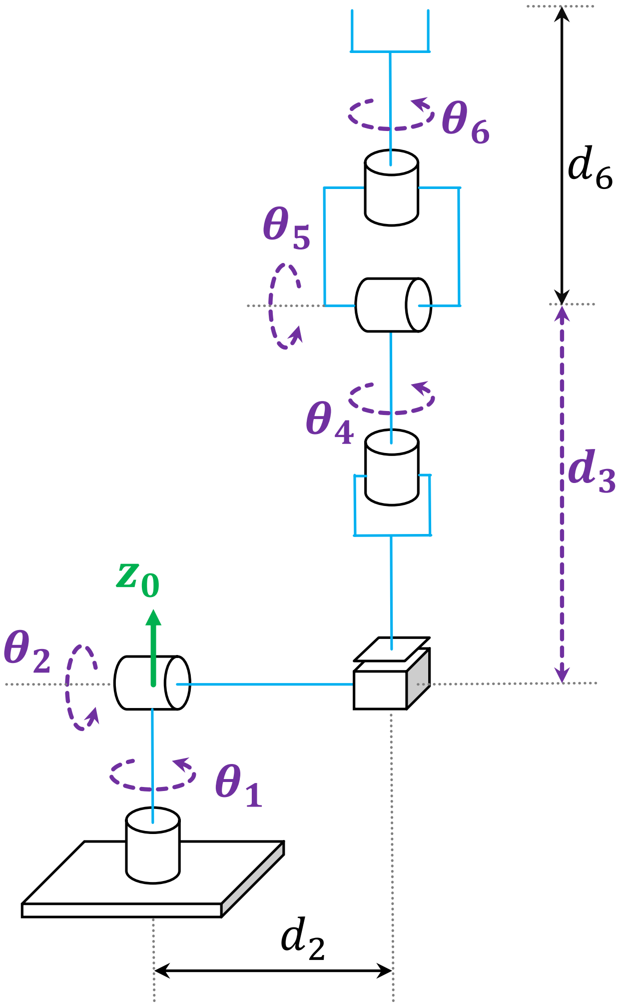
Stanford Arm



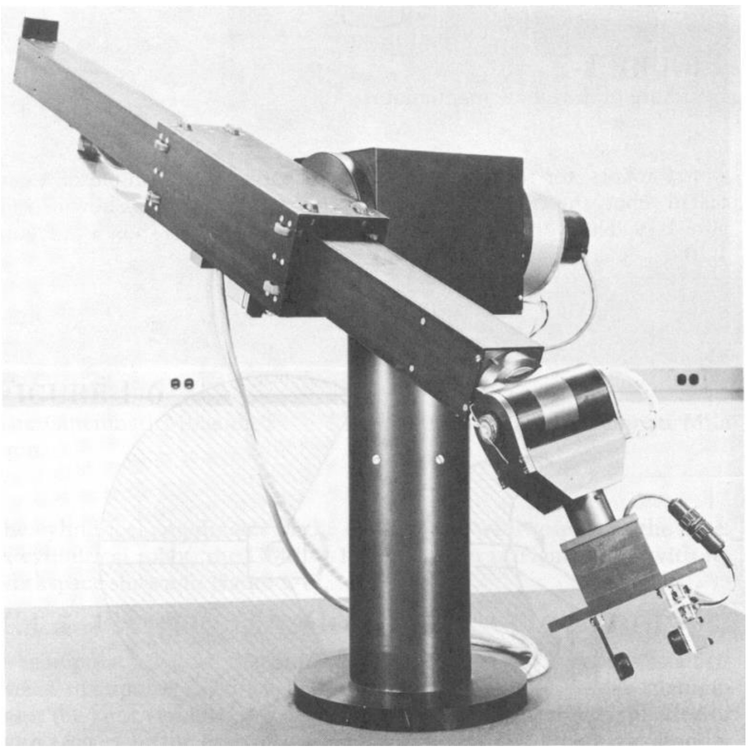


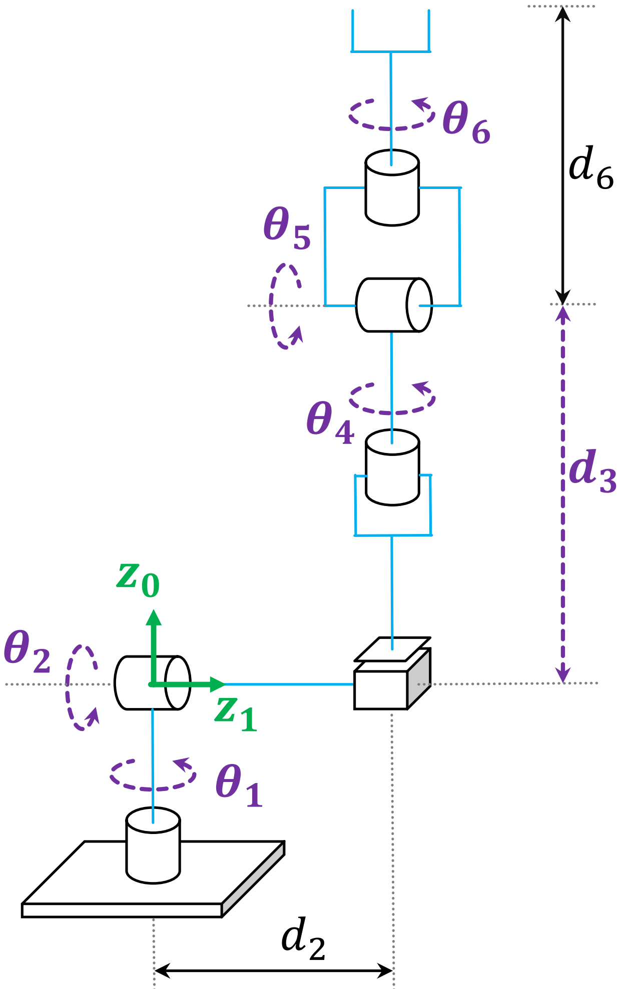
Stanford Arm



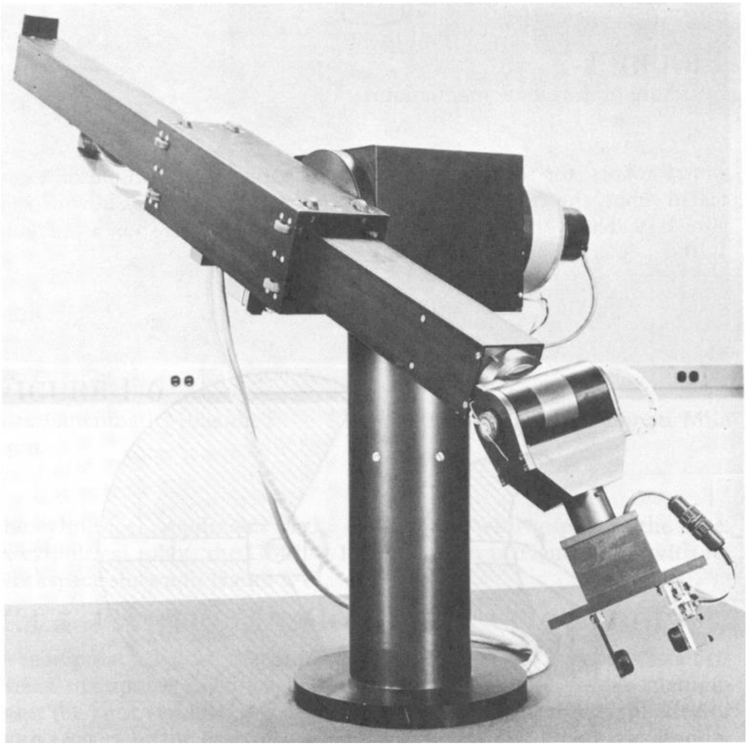


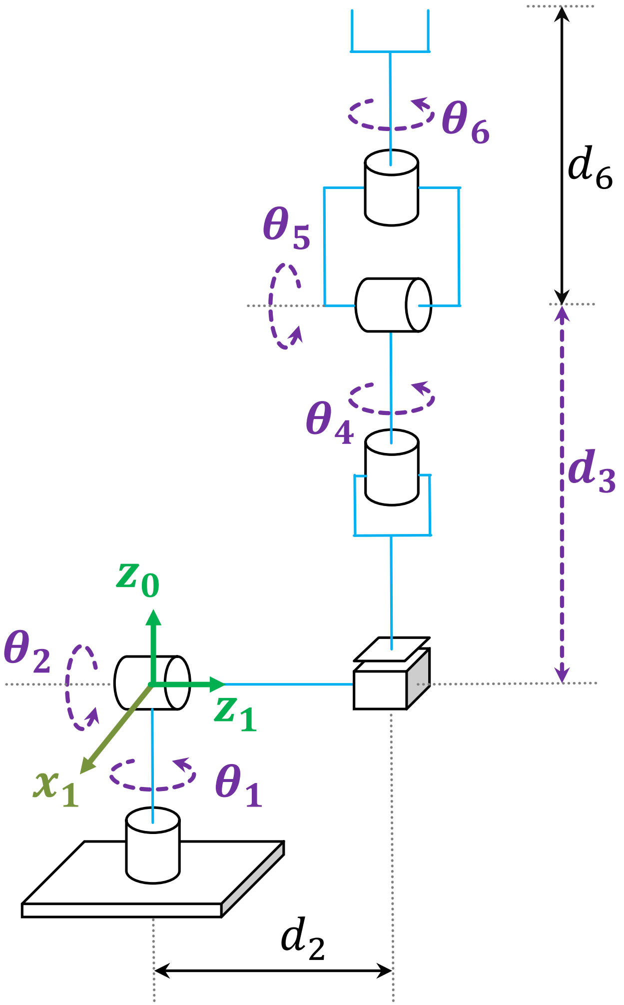
Stanford Arm



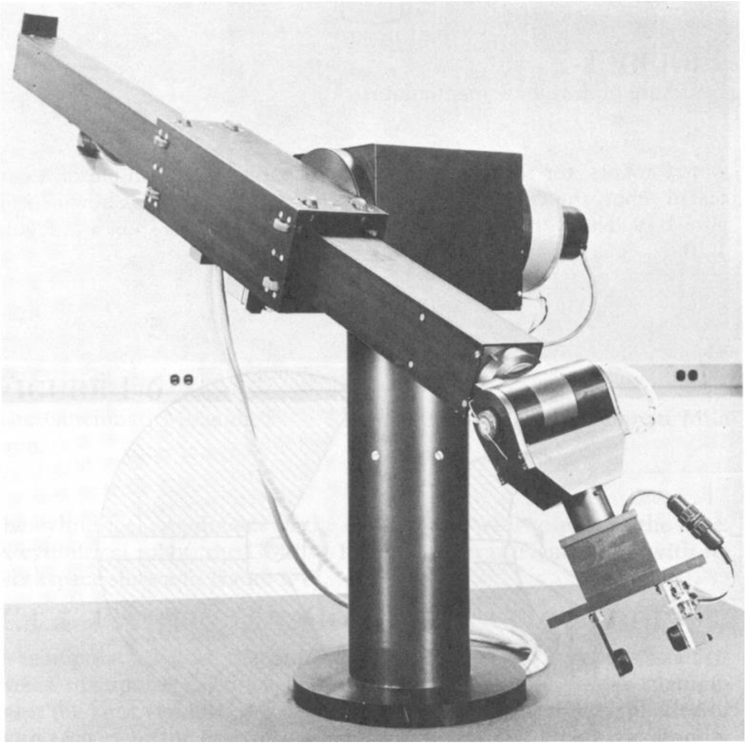


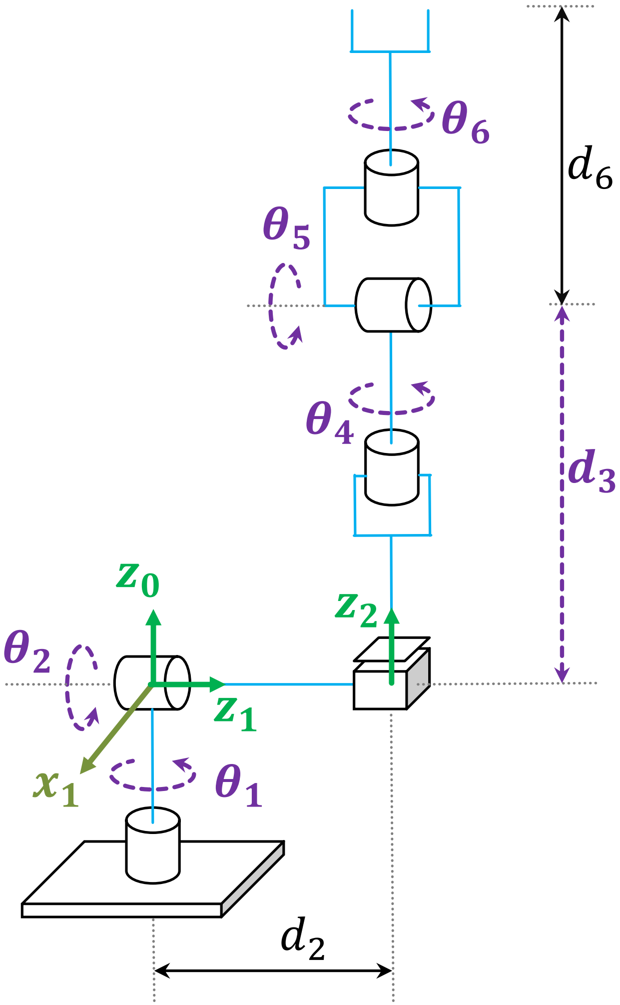
Stanford Arm



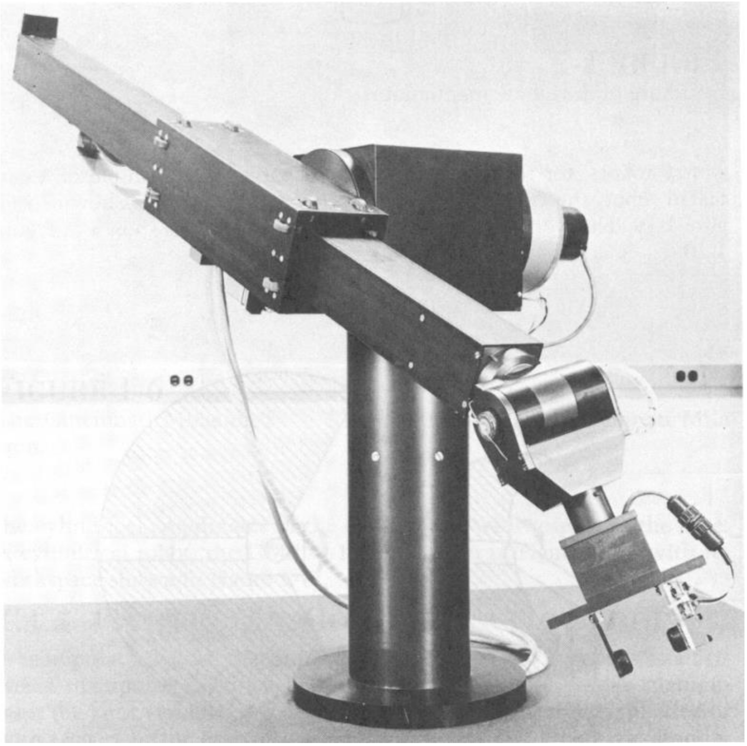


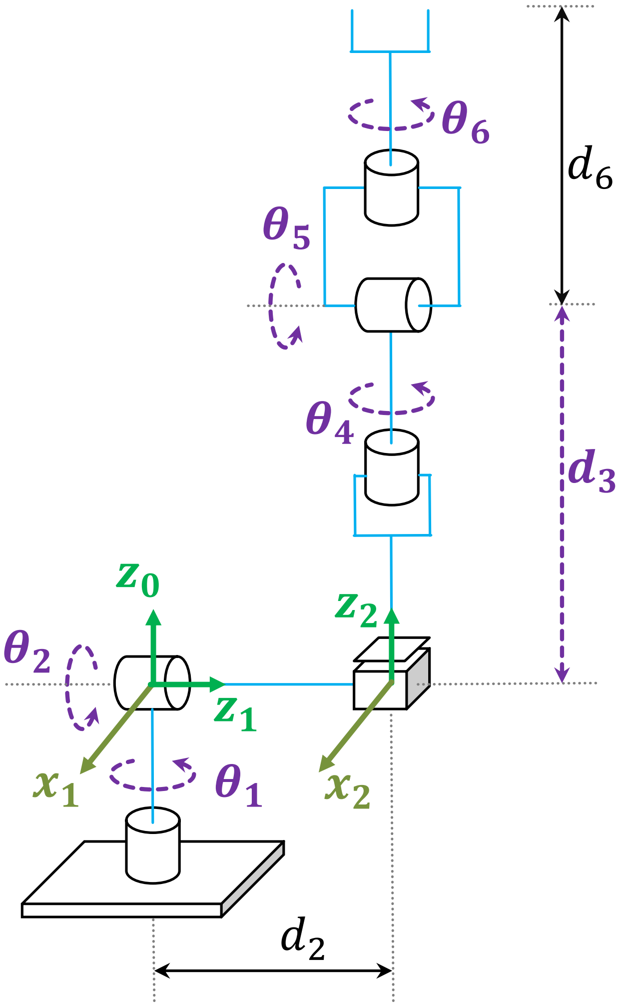
Stanford Arm



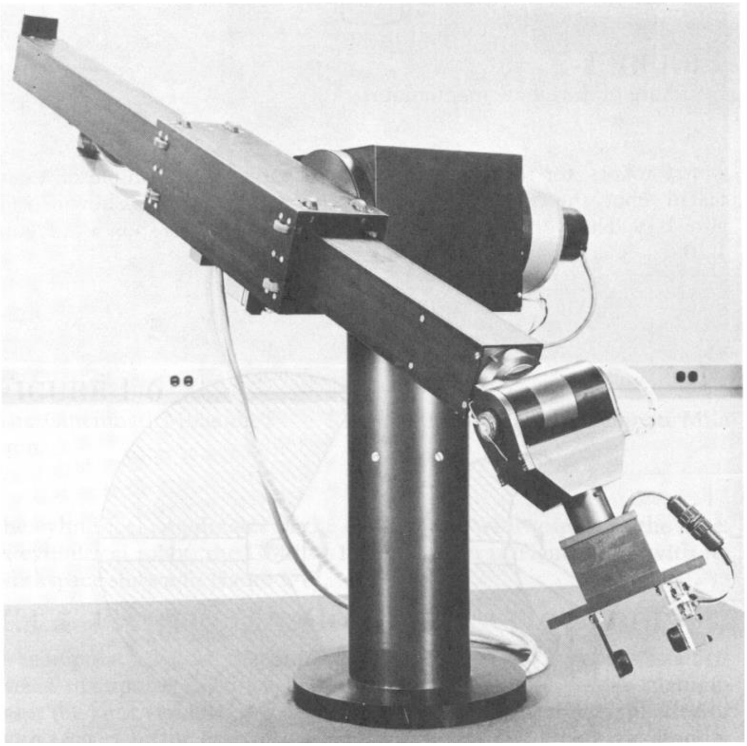


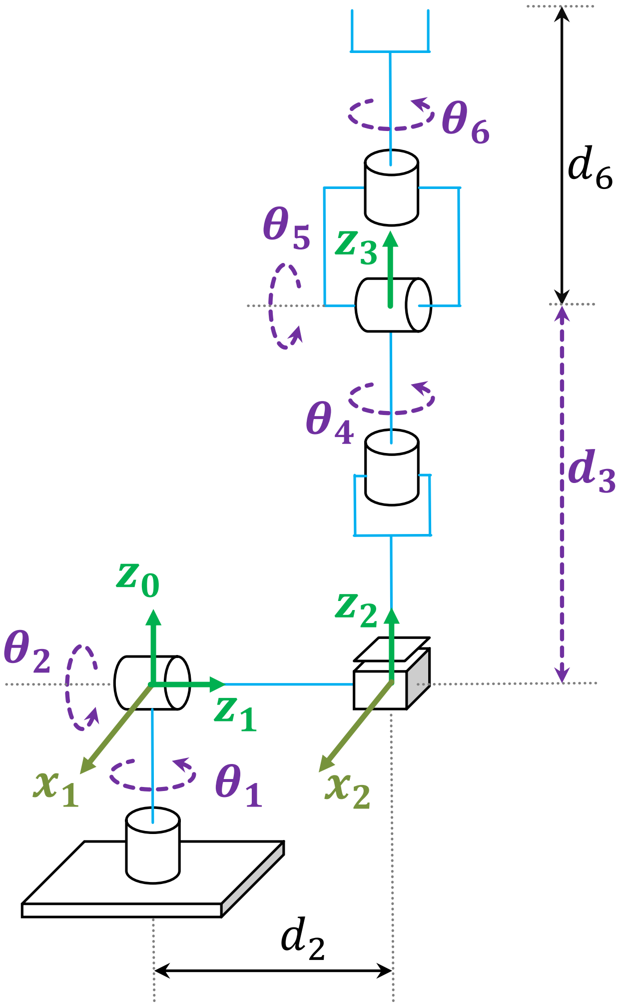
Stanford Arm



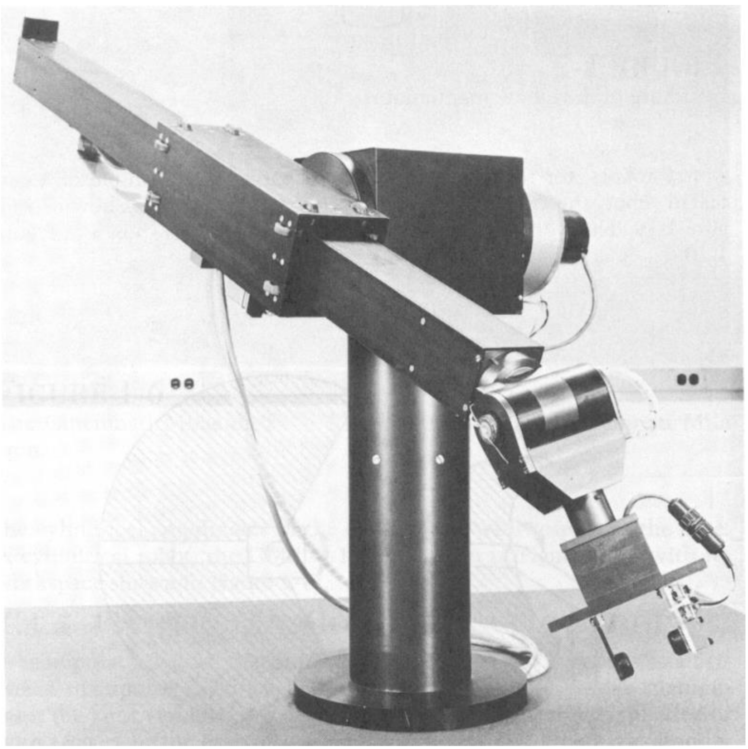


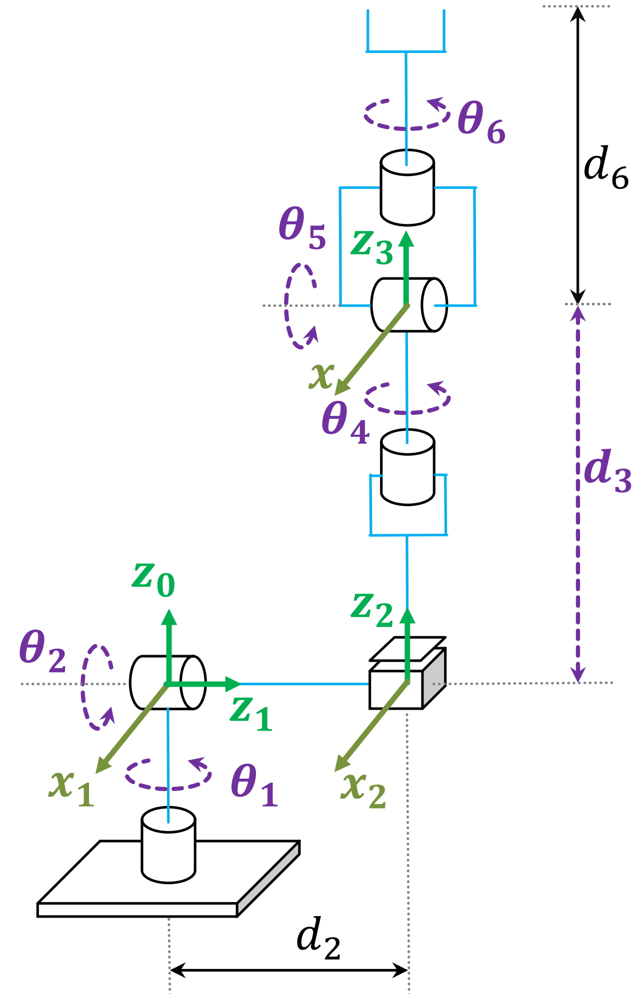
Stanford Arm



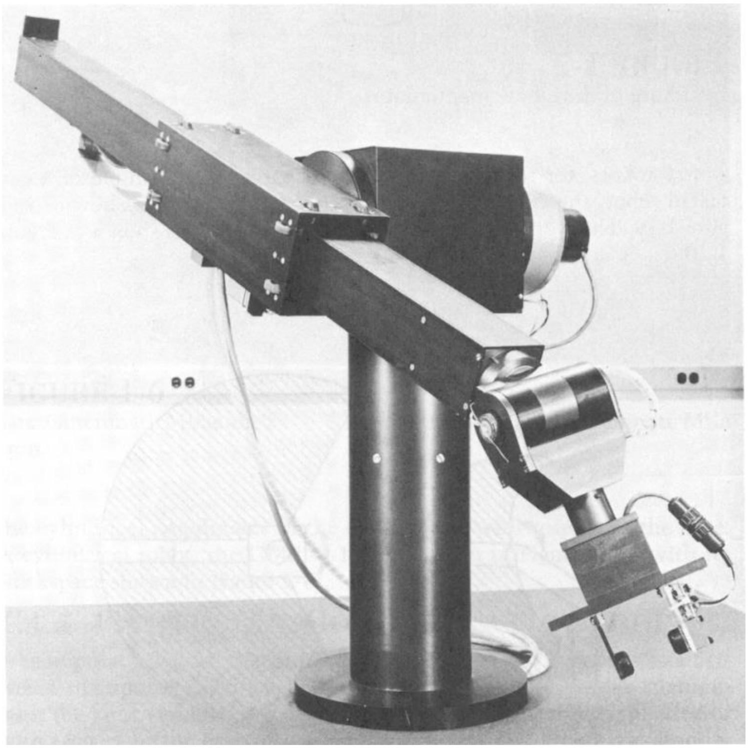


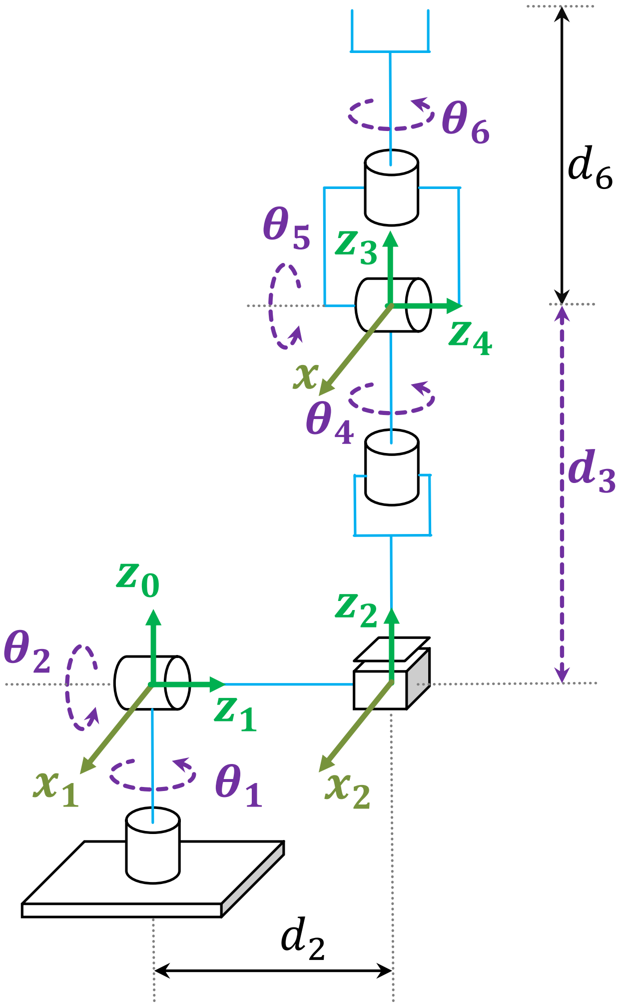
Stanford Arm



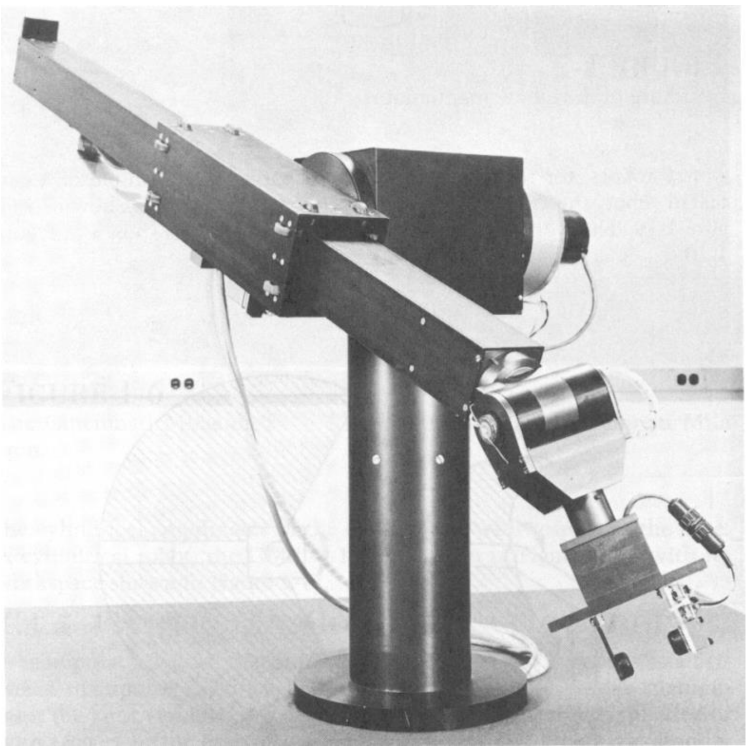


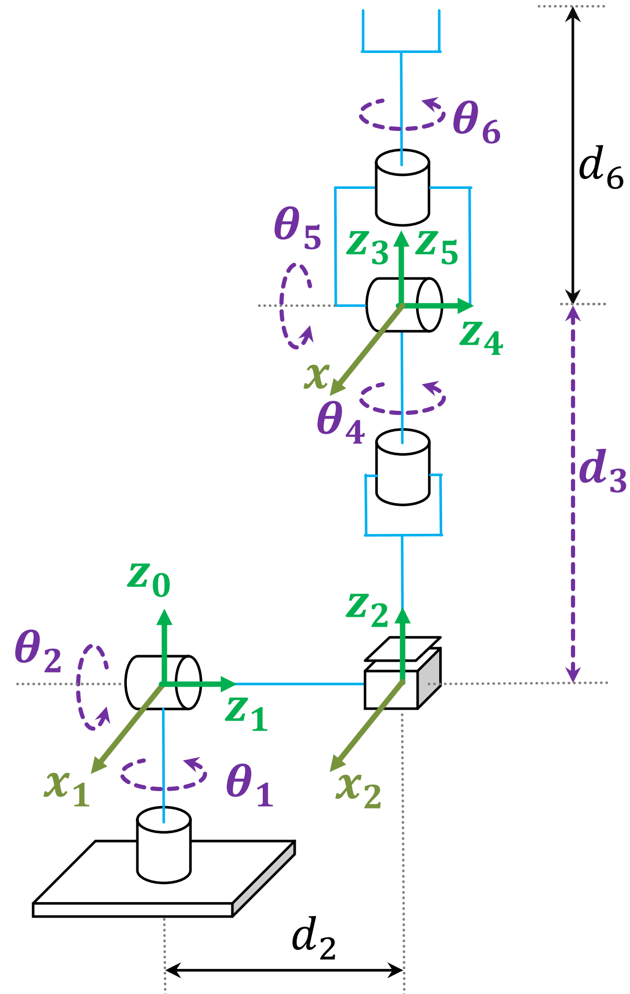
Stanford Arm



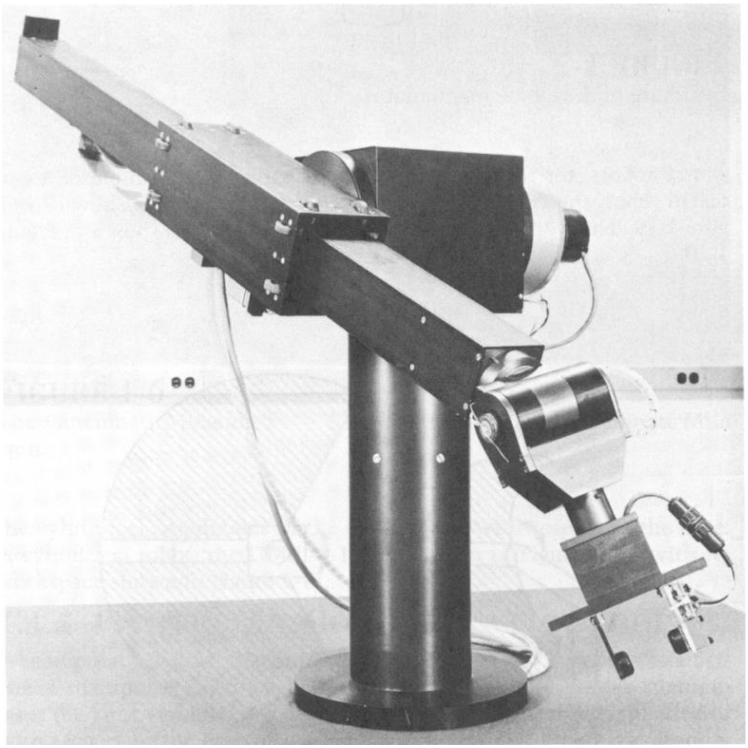


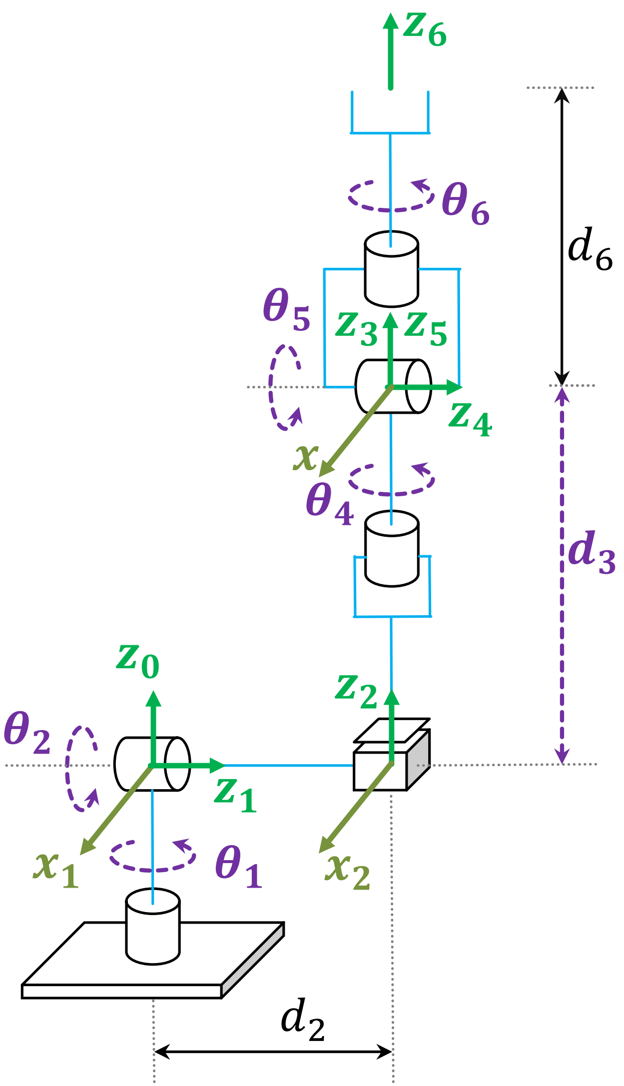
Stanford Arm



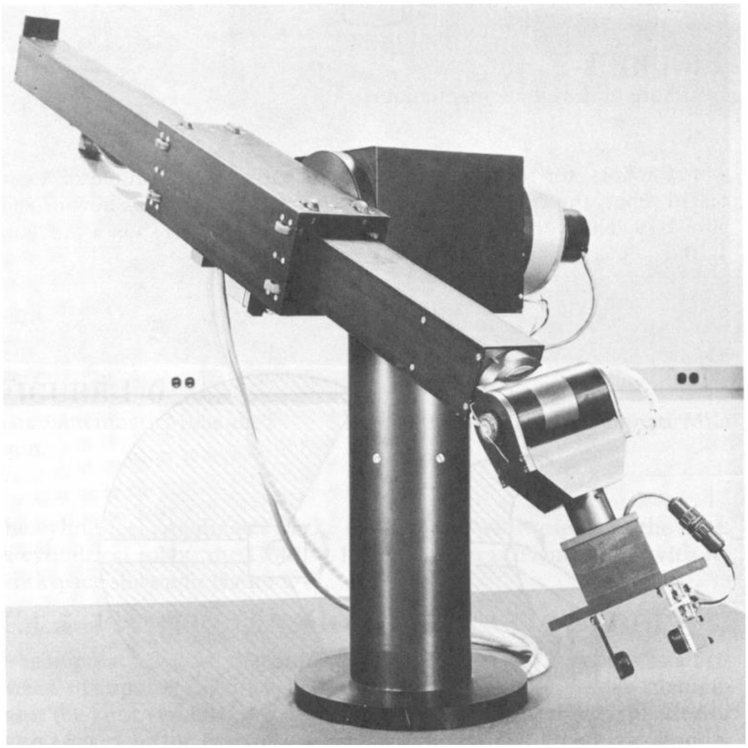


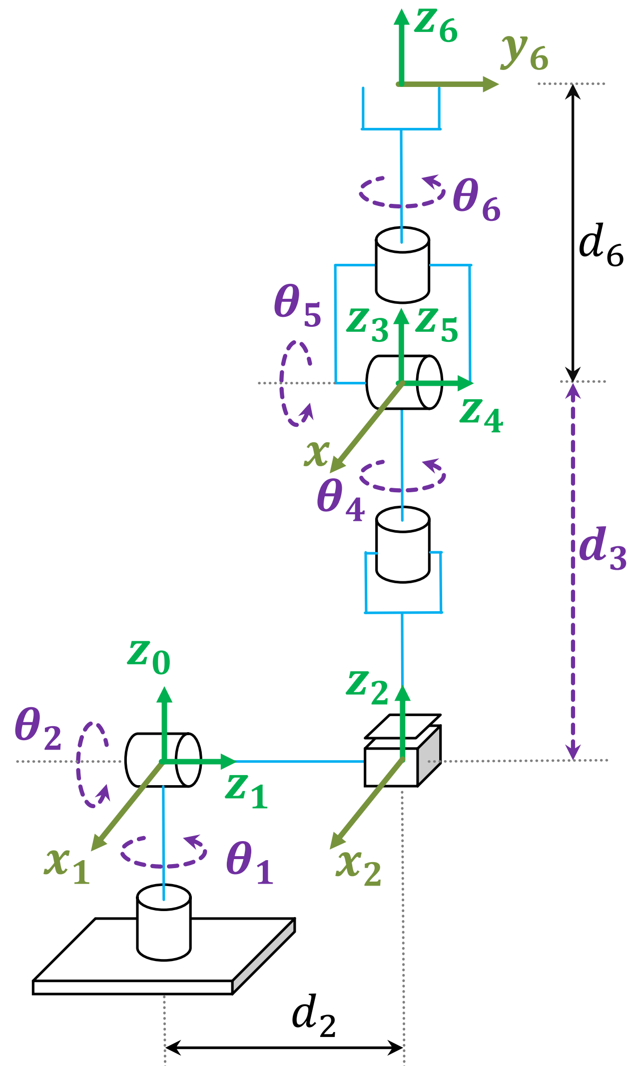
Stanford Arm



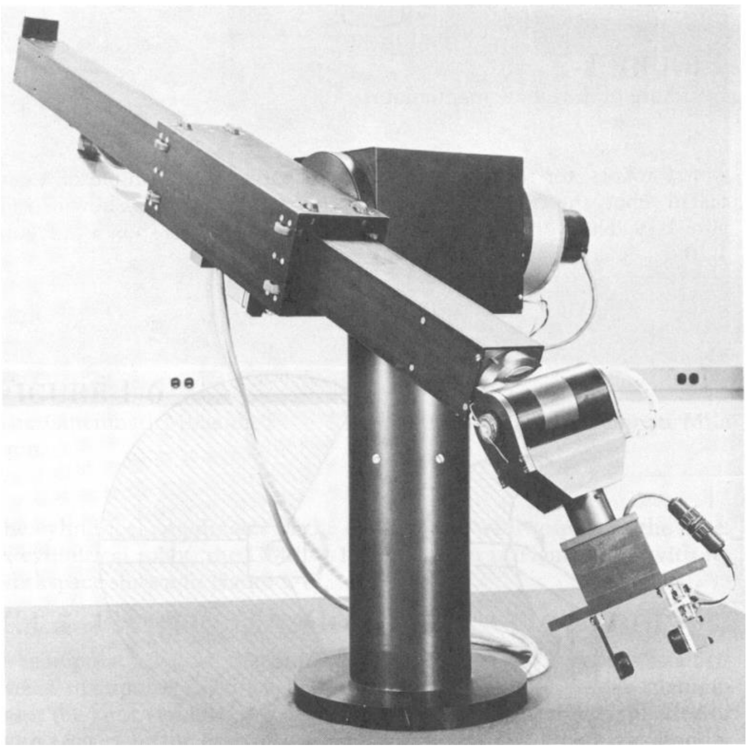


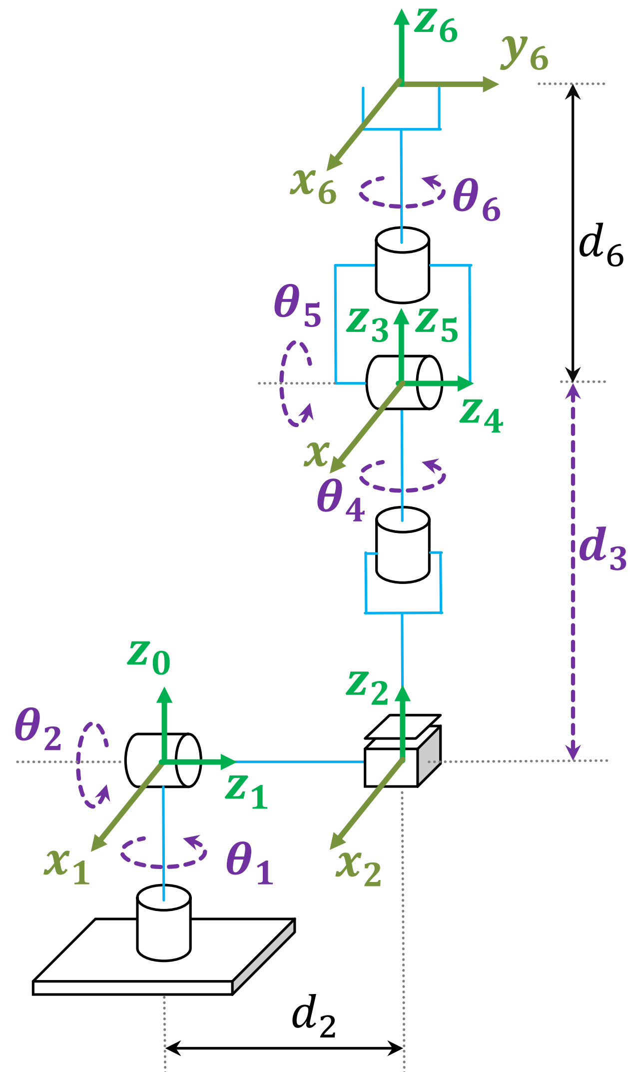
Stanford Arm





Stanford Arm

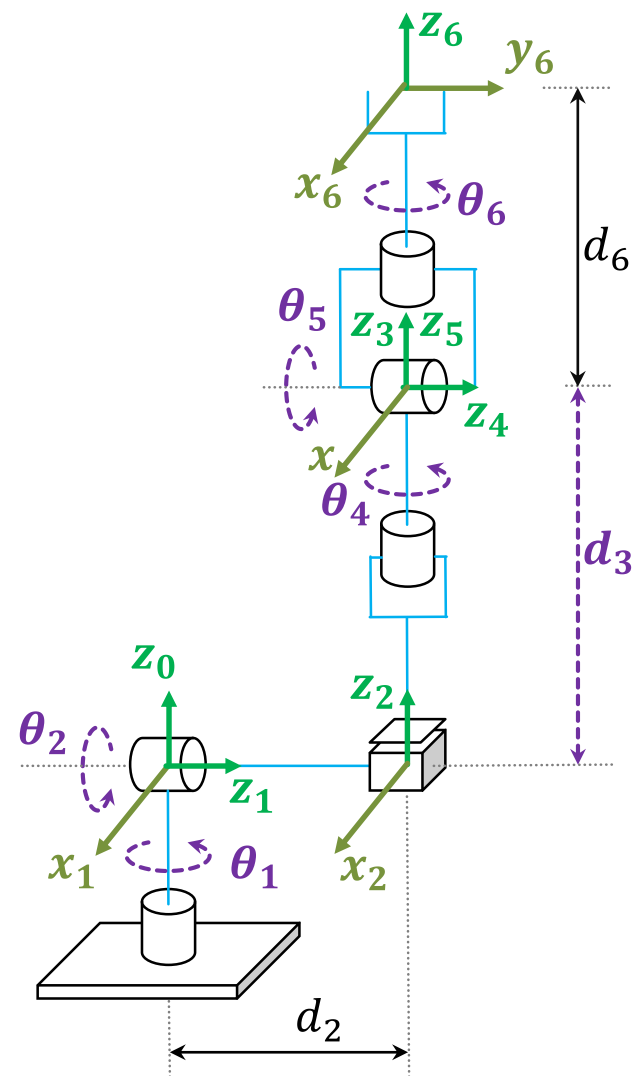




Stanford Arm

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

i	a_i	α_i	d_i	θ_i
1				
2				
3				
4				
5				
6				

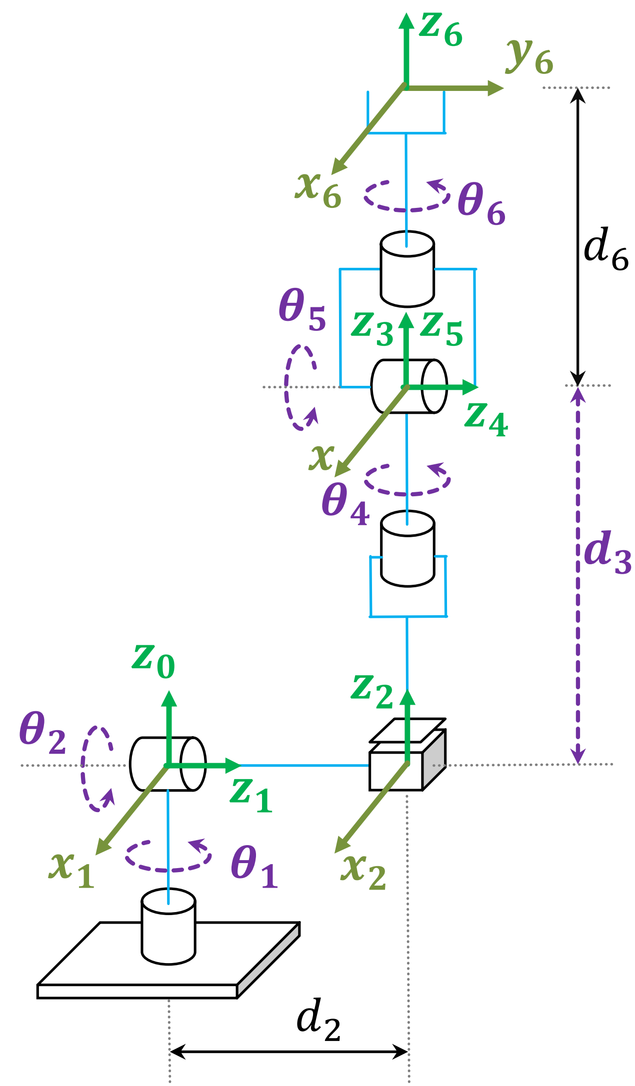


Stanford Arm

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

i	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1 *
2	0	$+90^\circ$	d_2	θ_2 *
3	0	0°	d_3 *	0
4	0	-90°	0	θ_4 *
5	0	$+90^\circ$	0	θ_5 *
6	0	0°	d_6	θ_6 *

$$T_6^0 = \quad = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



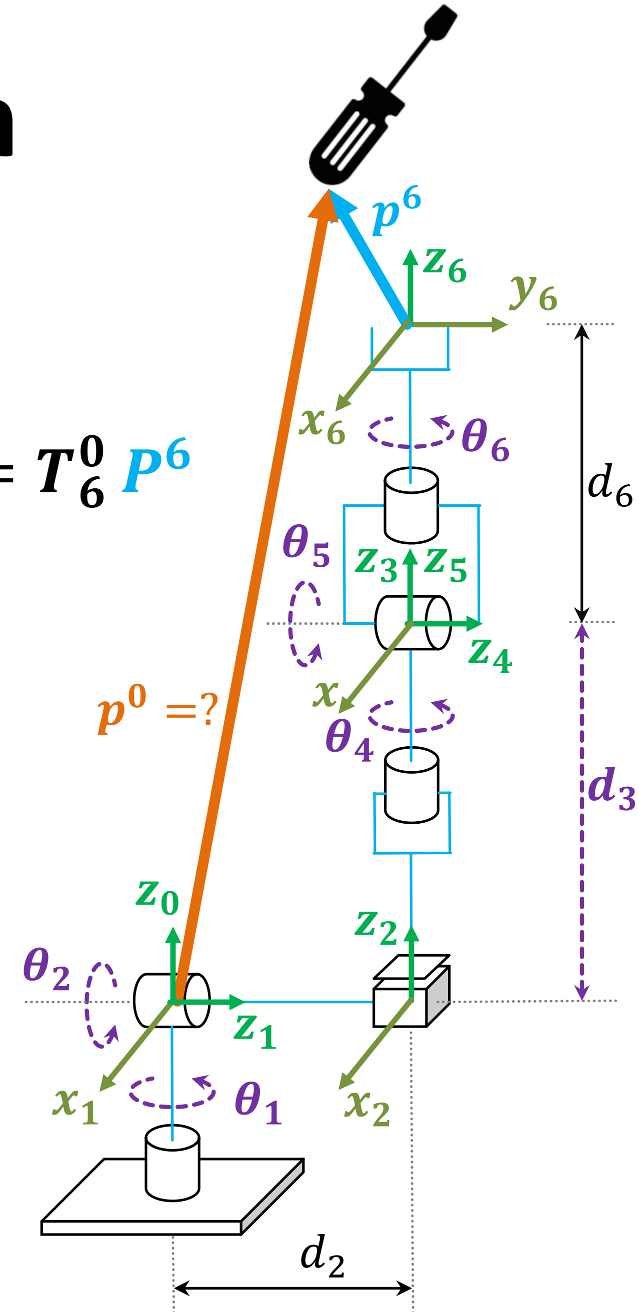
Stanford Arm

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

i	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1 *
2	0	$+90^\circ$	d_2	θ_2 *
3	0	0°	d_3 *	0
4	0	-90°	0	θ_4 *
5	0	$+90^\circ$	0	θ_5 *
6	0	0°	d_6	θ_6 *

$$T_6^0 = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^0 = T_6^0\, P^6$$



Stanford Arm

i	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1 *
2	0	$+90^\circ$	d_2	θ_2 *
3	0	0°	d_3 *	0
4	0	-90°	0	θ_4 *
5	0	$+90^\circ$	0	θ_5 *
6	0	0°	d_6	θ_6 *

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Arm

```

t1= sym('t1');t2= sym('t2');d3= sym('d3');t4= sym('t4');t5= sym('t5');
t6= sym('t6');d2= sym('d2');d6= sym('d6');r11= sym('r11');r12= sym('r12');
r13= sym('r13');r21= sym('r21');r22= sym('r22');r23= sym('r23');
r31= sym('r31');r32= sym('r32');r33= sym('r33');dx= sym('dx');
dy= sym('dy');dz= sym('dz');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A1=[    cos(t1),    0,  -sin(t1),    0;...
        sin(t1),    0,   cos(t1),    0;...
              0,   -1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A2=[    cos(t2),    0,   sin(t2),    0;...
        sin(t2),    0,  -cos(t2),    0;...
              0,    1,         0,   d2;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A3=[          1,    0,         0,    0;...
              0,    1,         0,    0;...
              0,    0,         1,   d3;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A4=[    cos(t4),    0,  -sin(t4),    0;...
        sin(t4),    0,   cos(t4),    0;...
              0,   -1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A5=[    cos(t5),    0,   sin(t5),    0;...
        sin(t5),    0,  -cos(t5),    0;...
              0,    -1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A6=[    cos(t6), -sin(t6),    0,    0;...
        sin(t6),  cos(t5),    0,    0;...
              0,        0,    1,   d6;...
              0,        0,    0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```


```
        sin(t6),  cos(t5),    0,    0;...
              0,        0,    1,   d6;...
              0,        0,    0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```


```
A12= A1*A2;
A123= A1*A2*A3;
A1234= A1*A2*A3*A4;
A12345= A1*A2*A3*A4*A5;
A123456= A1*A2*A3*A4*A5*A6;

```

Stanford Arm

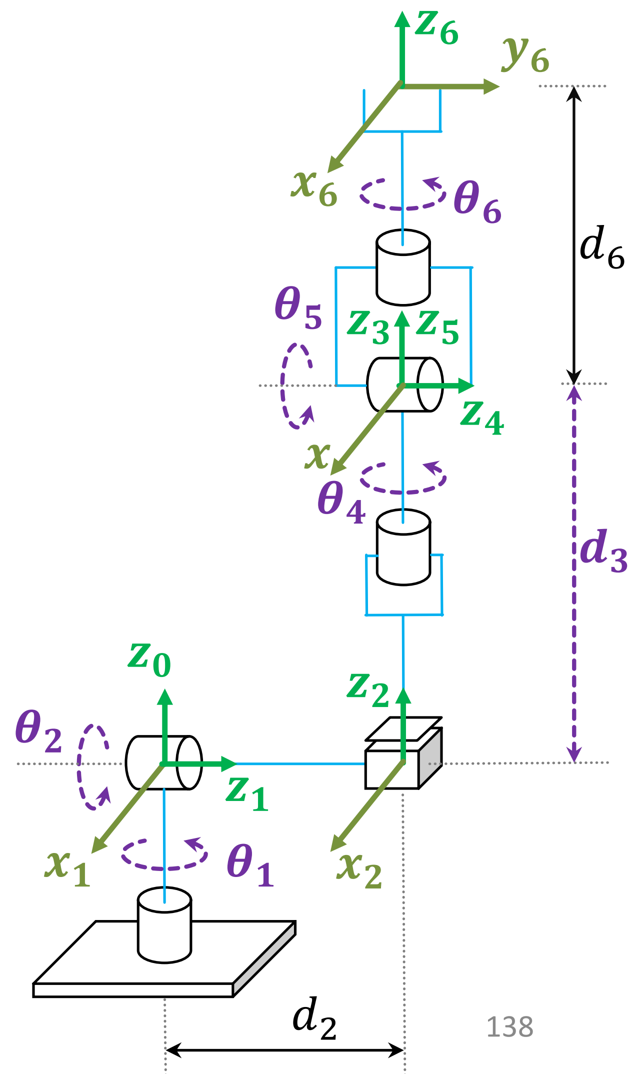
```
[ s6*(c4*s1 + c1*c2*s4) – c6*(c5*(s1*s4 – c1*c2*c4) + c1*s2*s5),   s6*(c5*(s1*s4 – c1*c2*c4) + c1*s2*s5) + c5*(c4*s1 + c1*c2*s4), c1*c5*s2 – s5*(s1*s4 – c1*c2*c4), d3*c1*s2 - d6*(s5*(s1*s4 – c1*c2*c4) – c1*c5*s2) - d2*s1 ]
[ c6*(c5*(c1*s4 + c2*c4*s1) – s1*s2*s5) – s6*(c1*c4 – c2*s1*s4), - s6*(c5*(c1*s4 + c2*c4*s1) – s1*s2*s5) – c5*(c1*c4 – c2*s1*s4), s5*(c1*s4 + c2*c4*s1) + c5*s1*s2, d2*c1 + d6*(s5*(c1*s4 + c2*c4*s1) + c5*s1*s2) + d3*s1*s2]
[                              -c6*(c2*s5 + c4*c5*s2) – s2*s4*s6,                              s6*(c2*s5 + c4*c5*s2) – c5*s2*s4,                c2*c5 – c4*s2*s5,                                   d6*(c2*c5 – c4*s2*s5) + d3*c2]
[                                                              0,                                                              0,                               0,                                                               1]
```

$$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$$
$$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{31} = -c_6.(c_2.s_5 + c_4.c_5.s_2) - s_2.s_4.s_6$$

$$r_{12} = s_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5) + c_5.(c_4.s_1 + c_1.c_2.s_4)$$
$$r_{22} = -s_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - c_5.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{32} = s_6.(c_2.s_5 + c_4.c_5.s_2) - c_5.s_2.s_4$$

$$r_{31} = c_1.c_5.s_2 - s_5.(s_1.s_4 - c_1.c_2.c_4)$$
$$r_{32} = s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2$$
$$r_{33} = c_2.c_5 - c_4.s_2.s_5$$

$$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$$
$$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$$
$$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$$



Stanford Arm

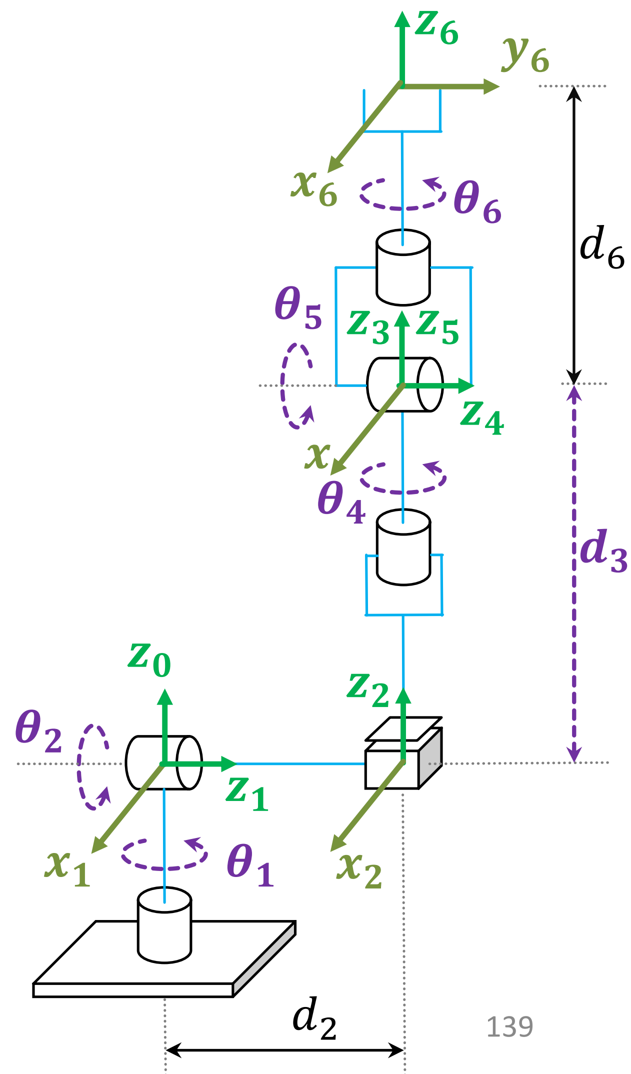
$$T_6^0 = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$$
$$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{31} = -c_6.(c_2.s_5 + c_4.c_5.s_2) - s_2.s_4.s_6$$

$$r_{12} = s_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5) + c_5.(c_4.s_1 + c_1.c_2.s_4)$$
$$r_{22} = -s_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - c_5.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{32} = s_6.(c_2.s_5 + c_4.c_5.s_2) - c_5.s_2.s_4$$

$$r_{31} = c_1.c_5.s_2 - s_5.(s_1.s_4 - c_1.c_2.c_4)$$
$$r_{32} = s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2$$
$$r_{33} = c_2.c_5 - c_4.s_2.s_5$$

$$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$$
$$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$$
$$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$$



Stanford Arm

In the configuration shown, find:

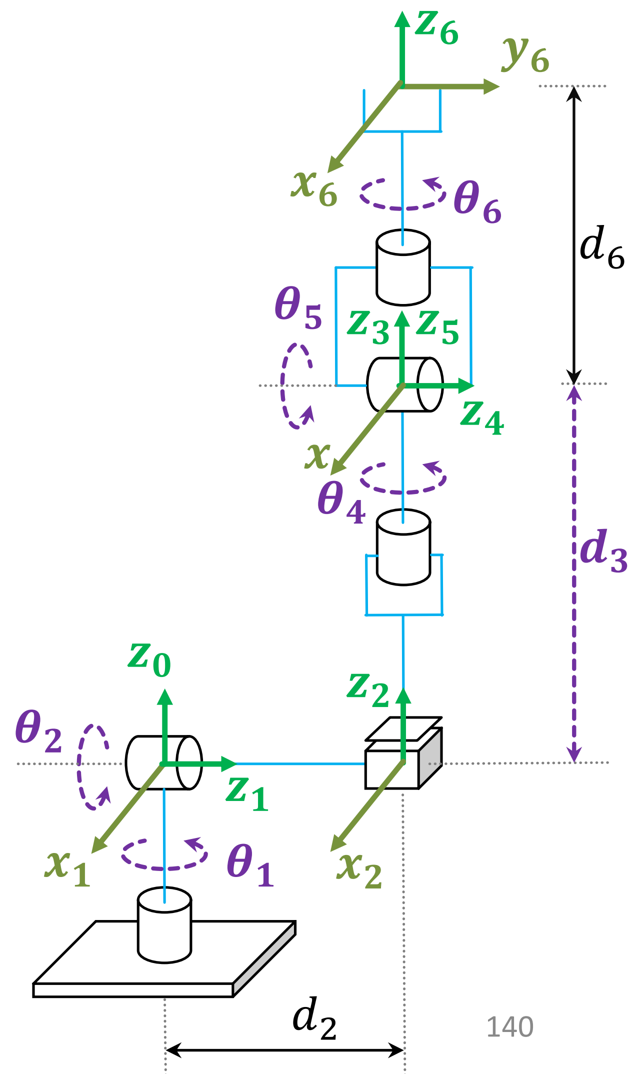
$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$$
$$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{31} = -c_6.(c_2.s_5 + c_4.c_5.s_2) - s_2.s_4.s_6$$

$$r_{12} = s_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5) + c_5.(c_4.s_1 + c_1.c_2.s_4)$$
$$r_{22} = -s_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - c_5.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{32} = s_6.(c_2.s_5 + c_4.c_5.s_2) - c_5.s_2.s_4$$

$$r_{31} = c_1.c_5.s_2 - s_5.(s_1.s_4 - c_1.c_2.c_4)$$
$$r_{32} = s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2$$
$$r_{33} = c_2.c_5 - c_4.s_2.s_5$$

$$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$$
$$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$$
$$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$$



Stanford Arm

In the configuration shown, find:

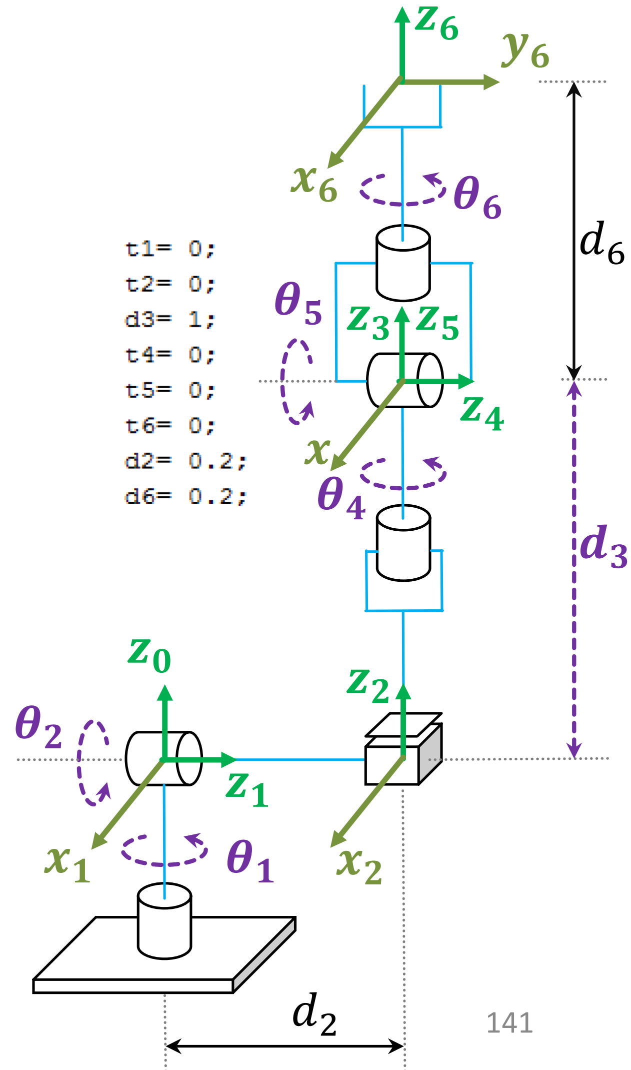
$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$$
$$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{31} = -c_6.(c_2.s_5 + c_4.c_5.s_2) - s_2.s_4.s_6$$

$$r_{12} = s_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5) + c_5.(c_4.s_1 + c_1.c_2.s_4)$$
$$r_{22} = -s_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - c_5.(c_1.c_4 - c_2.s_1.s_4)$$
$$r_{32} = s_6.(c_2.s_5 + c_4.c_5.s_2) - c_5.s_2.s_4$$

$$r_{31} = c_1.c_5.s_2 - s_5.(s_1.s_4 - c_1.c_2.c_4)$$
$$r_{32} = s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2$$
$$r_{33} = c_2.c_5 - c_4.s_2.s_5$$

$$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$$
$$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$$
$$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$$



Stanford Arm

In the configuration shown, find:

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$

$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$

$r_{31} = -c_6.($ …

$r_{12} = s_6.(c$ … $c_1.c_2.s_4)$

$r_{22} = -s_6.$ … $- c_2.s_1.s_4)$

$r_{32} = s_6.(c$ …

$r_{31} = c_1.c_5.$ …

$r_{32} = s_5.(c$ …

$r_{33} = c_2.c_5$ …

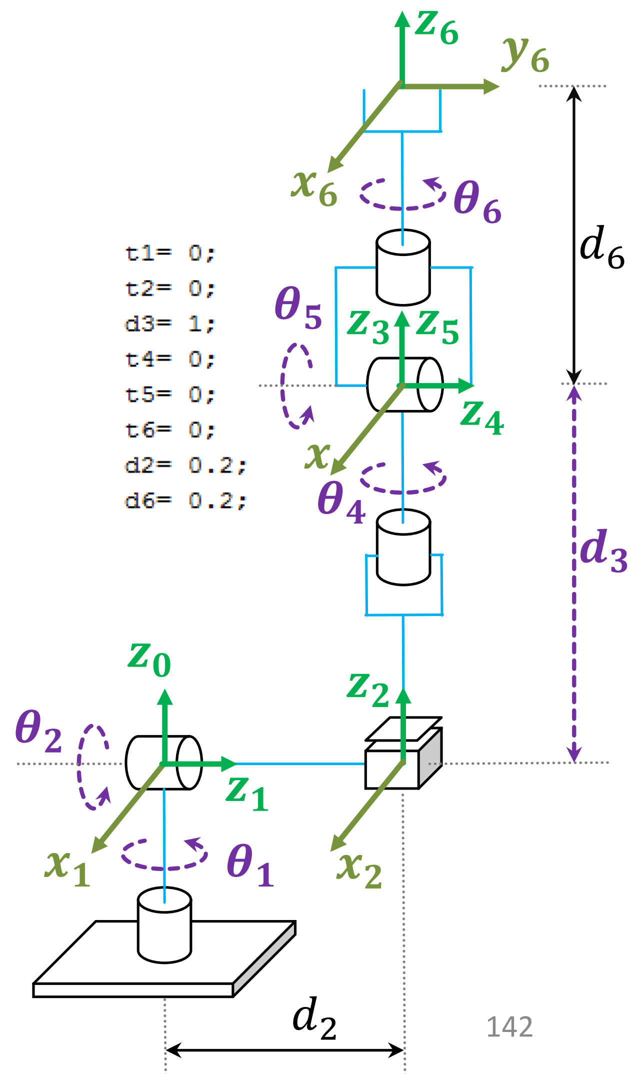
```
>> A123456

A123456 =

    1.0000         0         0         0
         0   -1.0000         0    0.2000
         0         0    1.0000    1.2000
         0         0         0    1.0000

>>
```

$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$

$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$

$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$

```
t1= 0;
t2= 0;
d3= 1;
t4= 0;
t5= 0;
t6= 0;
d2= 0.2;
d6= 0.2;
```



Stanford Arm

In the configuration shown, find:

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$

$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$

$r_{31} = -c_6.($ …

$r_{12} = s_6.(c$ … $c_1.c_2.s_4)$

$r_{22} = -s_6.$ … $- c_2.s_1.s_4)$

$r_{32} = s_6.(c$ …

$r_{31} = c_1.c_5.$ …

$r_{32} = s_5.(c$ …

$r_{33} = c_2.c_5$ …

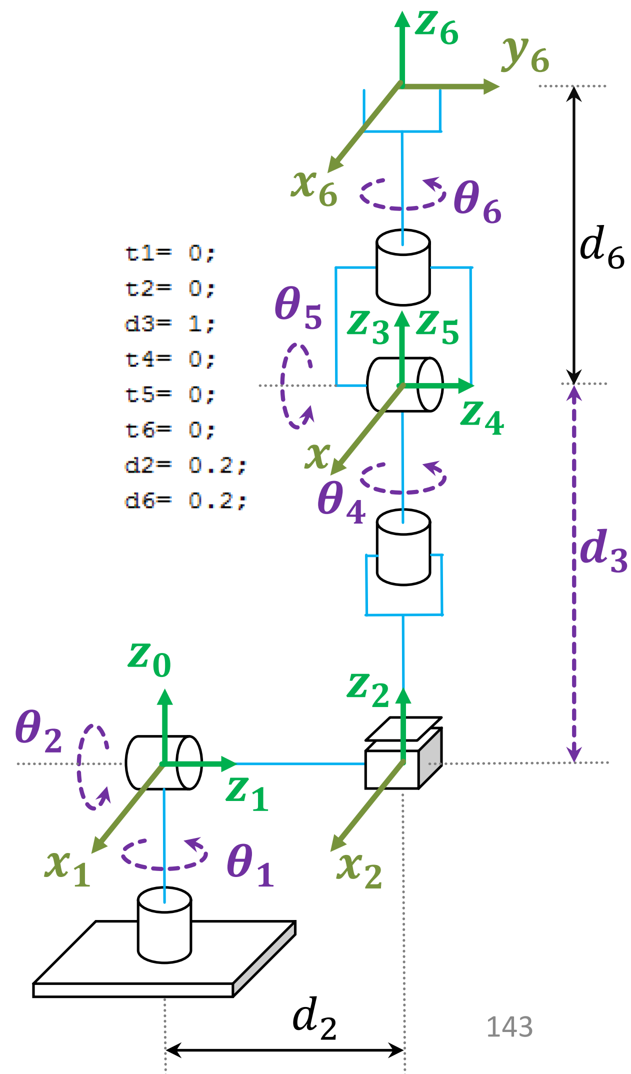
```
>> A123456

A123456 =

    1.0000         0         0         0
         0   -1.0000         0    0.2000
         0         0    1.0000    1.2000
         0         0         0    1.0000

>>
```

$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$

$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$

$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$

```
t1= 0;
t2= 0;
d3= 1;
t4= 0;
t5= 0;
t6= 0;
d2= 0.2;
d6= 0.2;
```



Stanford Arm

In the configuration shown, find:

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$r_{11} = s_6.(c_4.s_1 + c_1.c_2.s_4) - c_6.(c_5.(s_1.s_4 - c_1.c_2.c_4) + c_1.s_2.s_5)$

$r_{21} = c_6.(c_5.(c_1.s_4 + c_2.c_4.s_1) - s_1.s_2.s_5) - s_6.(c_1.c_4 - c_2.s_1.s_4)$

$r_{31} = -c_6.(\ldots$

$r_{12} = s_6.(c \ldots c_1.c_2.s_4)$

$r_{22} = -s_6. \ldots - c_2.s_1.s_4)$

$r_{32} = s_6.(c \ldots$

$r_{31} = c_1.c_5. \ldots$

$r_{32} = s_5.(c \ldots$

$r_{33} = c_2.c_5 \ldots$

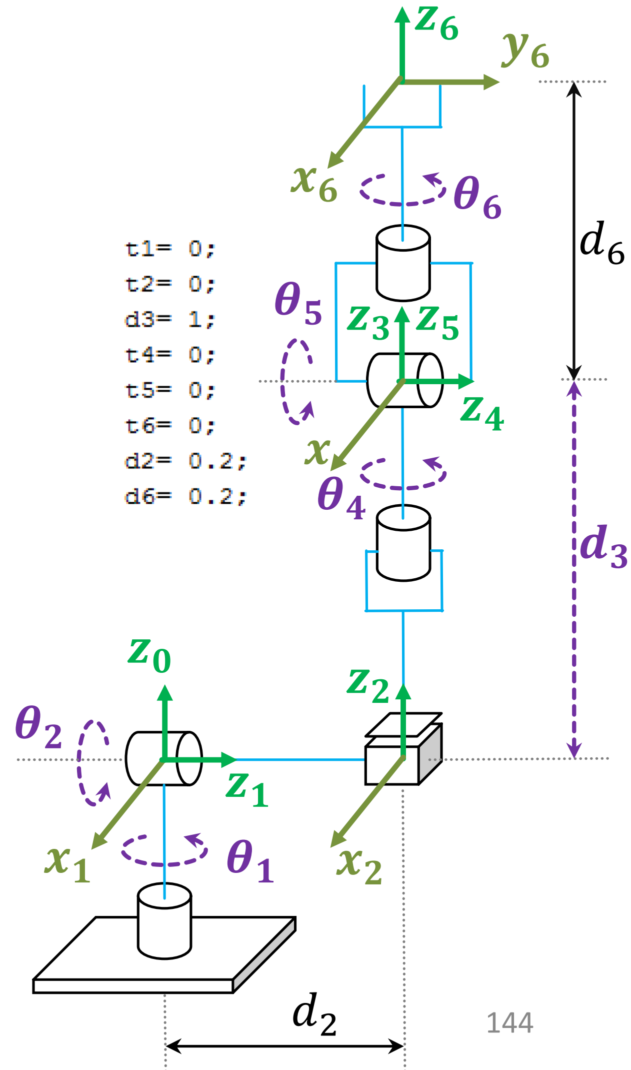
```
>> A123456

A123456 =

    1.0000         0         0         0
         0   -1.0000         0    0.2000
         0         0    1.0000    1.2000
         0         0         0    1.0000

>>
```

$d_x = d_3.c_1.s_2 - d_6.(s_5.(s_1.s_4 - c_1.c_2.c_4) - c_1.c_5.s_2) - d_2.s_1$

$d_y = d_2.c_1 + d_6.(s_5.(c_1.s_4 + c_2.c_4.s_1) + c_5.s_1.s_2) + d_3.s_1.s_2$

$d_z = d_6.(c_2.c_5 - c_4.s_2.s_5) + d_3.c_2$

What's wrong?




```
t1= 0;
t2= 0;
d3= 1;
t4= 0;
t5= 0;
t6= 0;
d2= 0.2;
d6= 0.2;
```

Stanford Arm

i	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1 *
2	0	$+90^\circ$	d_2	θ_2 *
3	0	0°	d_3 *	0
4	0	-90°	0	θ_4 *
5	0	$+90^\circ$	0	θ_5 *
6	0	0°	d_6	θ_6 *

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
t1= sym('t1');t2= sym('t2');d3= sym('d3');t4= sym('t4');t5= sym('t5');
t6= sym('t6');d2= sym('d2');d6= sym('d6');r11= sym('r11');r12= sym('r12');
r13= sym('r13');r21= sym('r21');r22= sym('r22');r23= sym('r23');
r31= sym('r31');r32= sym('r32');r33= sym('r33');dx= sym('dx');
dy= sym('dy');dz= sym('dz');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A1=[    cos(t1),    0,  -sin(t1),    0;...
        sin(t1),    0,   cos(t1),    0;...
              0,   -1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A2=[    cos(t2),    0,   sin(t2),    0;...
        sin(t2),    0,  -cos(t2),    0;...
              0,    1,         0,   d2;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A3=[          1,    0,         0,    0;...
              0,    1,         0,    0;...
              0,    0,         1,   d3;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A4=[    cos(t4),    0,  -sin(t4),    0;...
        sin(t4),    0,   cos(t4),    0;...
              0,   -1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A5=[    cos(t5),    0,   sin(t5),    0;...
        sin(t5),    0,  -cos(t5),    0;...
              0,    1,         0,    0;...
              0,    0,         0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A6=[    cos(t6), -sin(t6),   0,    0;...
        sin(t6),  cos(t6),   0,    0;...
              0,        0,   1,   d6;...
              0,        0,   0,    1 ];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A12= A1*A2;
A123= A1*A2*A3;
A1234= A1*A2*A3*A4;
A12345= A1*A2*A3*A4*A5;
A123456= A1*A2*A3*A4*A5*A6;
```

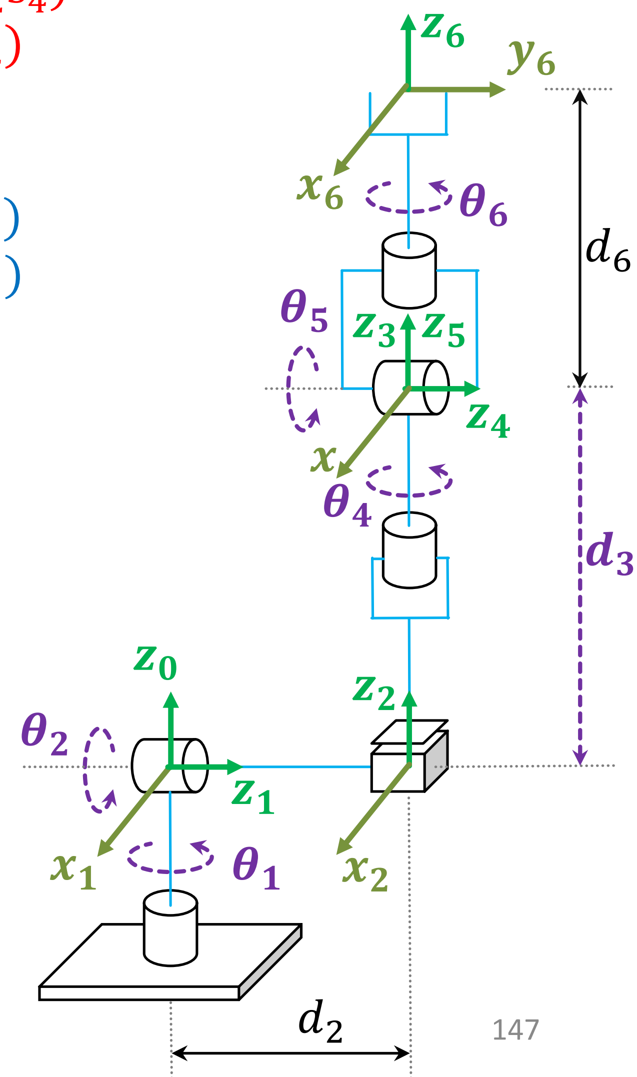
Stanford Arm

$$r_{11} = -c_6(c_5(s_1 s_4 - c_1 c_2 c_4) + c_1 s_2 s_5) - s_6(c_4 s_1 + c_1 c_2 s_4)$$
$$r_{21} = c_6(c_5(c_1 s_4 + c_2 c_4 s_1) - s_1 s_2 s_5) + s_6(c_1 c_4 - c_2 s_1 s_4)$$
$$r_{31} = s_2 s_4 s_6 - c_6(c_2 s_5 + c_4 c_5 s_2)$$

$$r_{12} = s_6(c_5(s_1 s_4 - c_1 c_2 c_4) + c_1 s_2 s_5) - c_6(c_4 s_1 + c_1 c_2 s_4)$$
$$r_{22} = c_6(c_1 c_4 - c_2 s_1 s_4) - s_6(c_5(c_1 s_4 + c_2 c_4 s_1) - s_1 s_2 s_5)$$
$$r_{32} = s_6(c_2 s_5 + c_4 c_5 s_2) + c_6 s_2 s_4$$

$$r_{13} = c_1 c_5 s_2 - s_5(s_1 s_4 - c_1 c_2 c_4)$$
$$r_{23} = s_5(c_1 s_4 + c_2 c_4 s_1) + c_5 s_1 s_2$$
$$r_{33} = c_2 c_5 - c_4 s_2 s_5$$

$$d_x = d_3 c_1 s_2 - d_6(s_5(s_1 s_4 - c_1 c_2 c_4) - c_1 c_5 s_2) - d_2 s_1$$
$$d_y = d_2 c_1 + d_6(s_5(c_1 s_4 + c_2 c_4 s_1) + c_5 s_1 s_2) + d_3 s_1 s_2$$
$$d_z = d_6(c_2 c_5 - c_4 s_2 s_5) + d_3 c_2$$



Stanford Arm

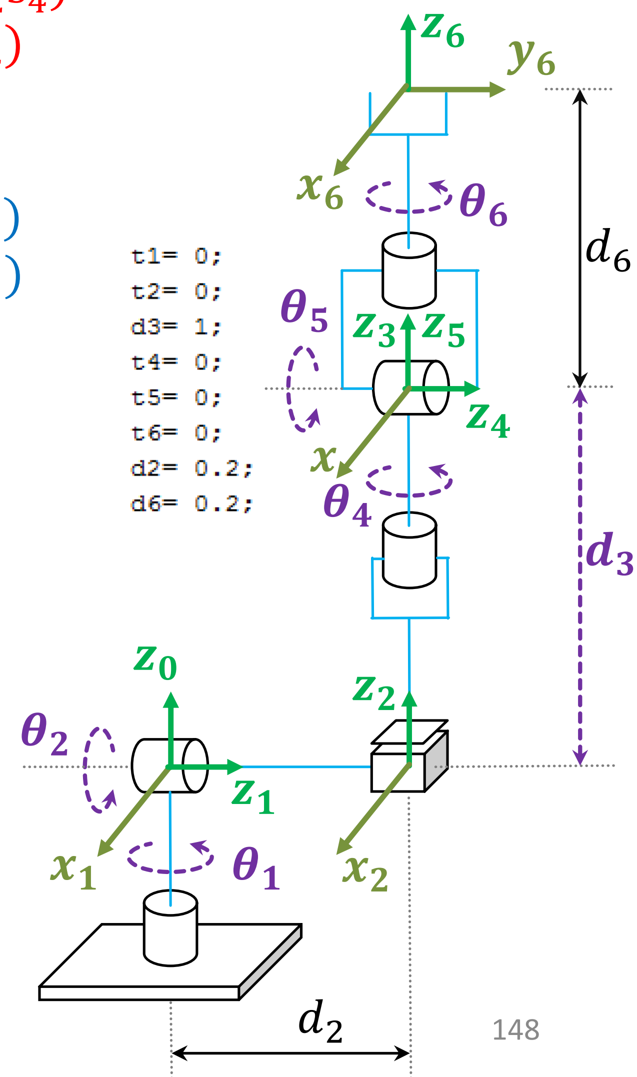
$r_{11} = -c_6(c_5(s_1 s_4 - c_1 c_2 c_4) + c_1 s_2 s_5) - s_6(c_4 s_1 + c_1 c_2 s_4)$
$r_{21} = c_6(c_5(c_1 s_4 + c_2 c_4 s_1) - s_1 s_2 s_5) + s_6(c_1 c_4 - c_2 s_1 s_4)$
$r_{31} = s_2 s_4 s_6 - c_6(c_2 s_5 + c_4 c_5 s_2)$

```
A123456 =

    1.0000         0         0         0
         0    1.0000         0    0.2000
         0         0    1.0000    1.2000
         0         0         0    1.0000

fx >>
```

$r_{12} =$
$r_{22} =$
$r_{32} =$

$r_{13} =$
$r_{23} =$
$r_{33} =$

$d_x = d_3 c_1 s_2 - d_6(s_5(s_1 s_4 - c_1 c_2 c_4) - c_1 c_5 s_2) - d_2 s_1$
$d_y = d_2 c_1 + d_6(s_5(c_1 s_4 + c_2 c_4 s_1) + c_5 s_1 s_2) + d_3 s_1 s_2$
$d_z = d_6(c_2 c_5 - c_4 s_2 s_5) + d_3 c_2$



Stanford Arm

$r_{11} = -c_6(c_5(s_1 s_4 - c_1 \ldots \; s_6(c_4 s_1 + c_1 c_2 s_4)$

$r_{21} = c_6(c_5(c_1 s_4 + c_2 c_4 \ldots \; {}_6(c_1 c_4 - c_2 s_1 s_4)$

$r_{31} = s_2 s_4 s_6 - c_6(c_2 s_5 \ldots$

$r_{12} = s_6(c_5(s_1 s_4 - c_1 c_2 \ldots \; {}_6(c_4 s_1 + c_1 c_2 s_4)$

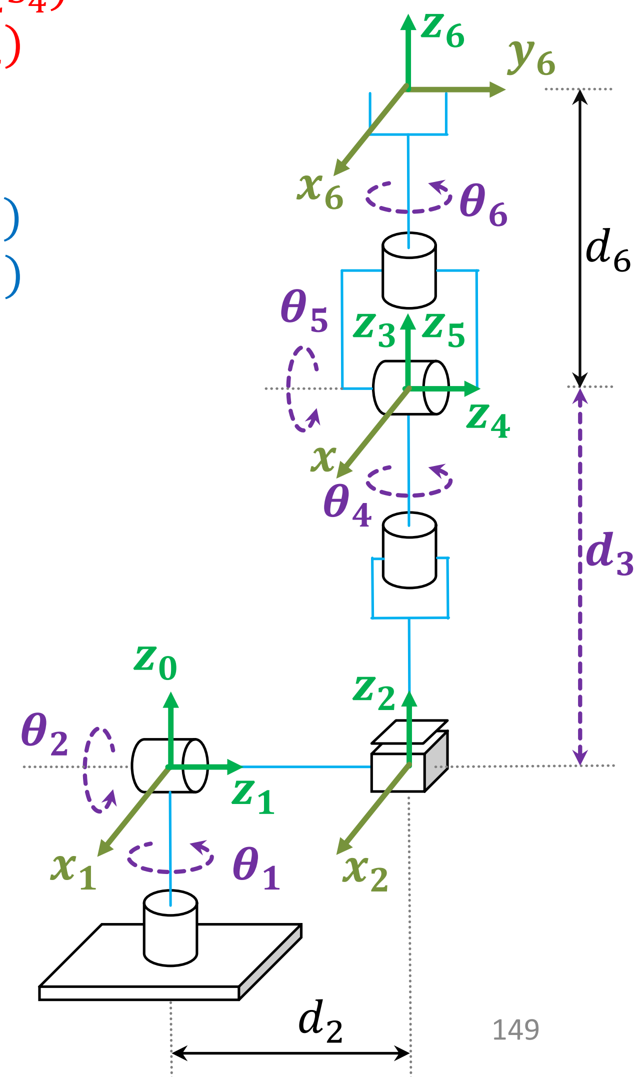
$r_{22} = c_6(c_1 c_4 - c_2 s_1 s_4) \ldots \; c_2 c_4 s_1) - s_1 s_2 s_5)$

$r_{32} = s_6(c_2 s_5 + c_4 c_5 s_2) \ldots$

```
t1= 0;
t2= 0;
d3= 1;
t4= 0;
t5= pi/2.0;
t6= 0;
d2= 0.2;
d6= 0.2;
```

$r_{13} = c_1 c_5 s_2 - s_5(s_1 s_4 - c_1 c_2 c_4)$

```
A123456 =

    0.0000         0    1.0000    0.2000
         0    0.0000         0    0.2000
   -1.0000         0    0.0000    1.0000
         0         0         0    1.0000

>>
```



Stanford Arm

$r_{11} = -c_6(c_5(s_1 s_4 - c$ … $s_6(c_4 s_1 + c_1 c_2 s_4)$
$r_{21} = c_6(c_5(c_1 s_4 + c_2 c$ … $(c_1 c_4 - c_2 s_1 s_4)$
$r_{31} = s_2 s_4 s_6 - c_6(c_2 s_5$ …

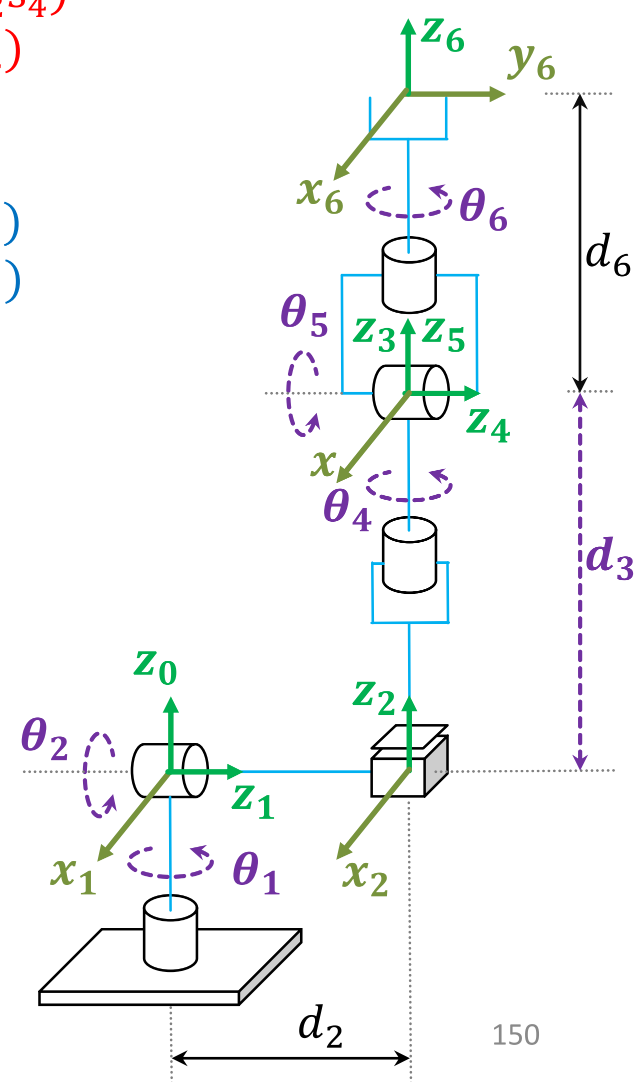
$r_{12} = s_6(c_5(s_1 s_4 - c_1 c$ … $(c_4 s_1 + c_1 c_2 s_4)$
$r_{22} = c_6(c_1 c_4 - c_2 s_1 s_4$ … $c_4 s_1) - s_1 s_2 s_5)$
$r_{32} = s_6(c_2 s_5 + c_4 c_5 s_2$ …

$r_{13} = c_1 c_5 s_2 - s_5(s_1 s_4$ …

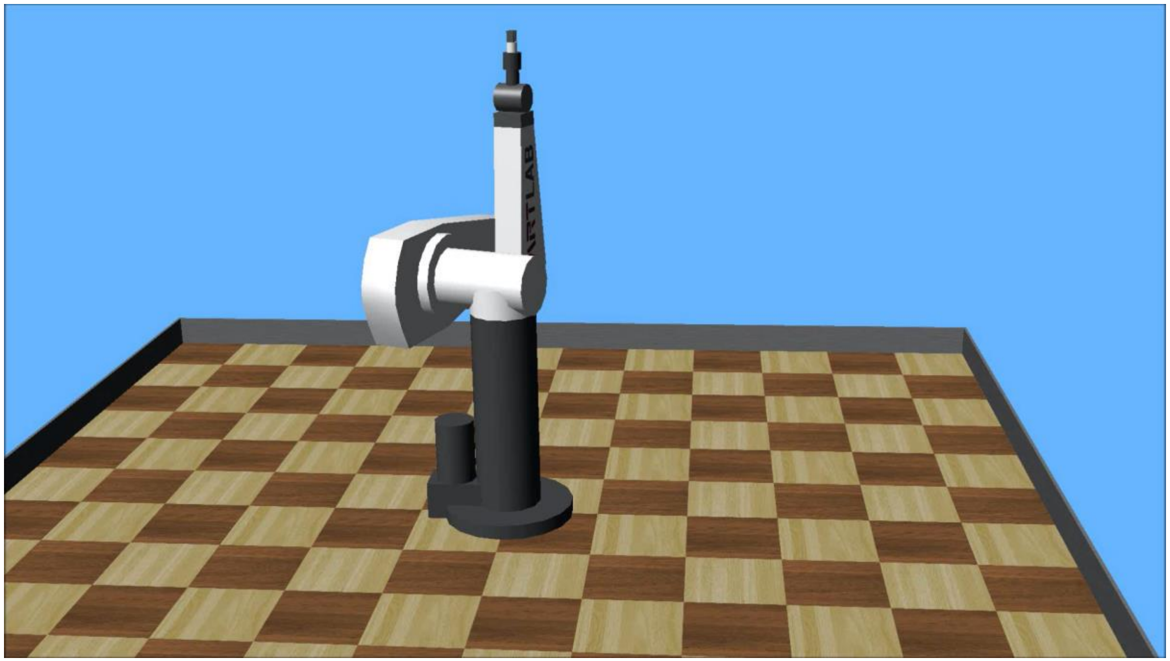
```
t1= 0;
t2= 0;
d3= 1;
t4= pi/2.0;
t5= pi/2.0;
t6= 0;
d2= 0.2;
d6= 0.2;
```

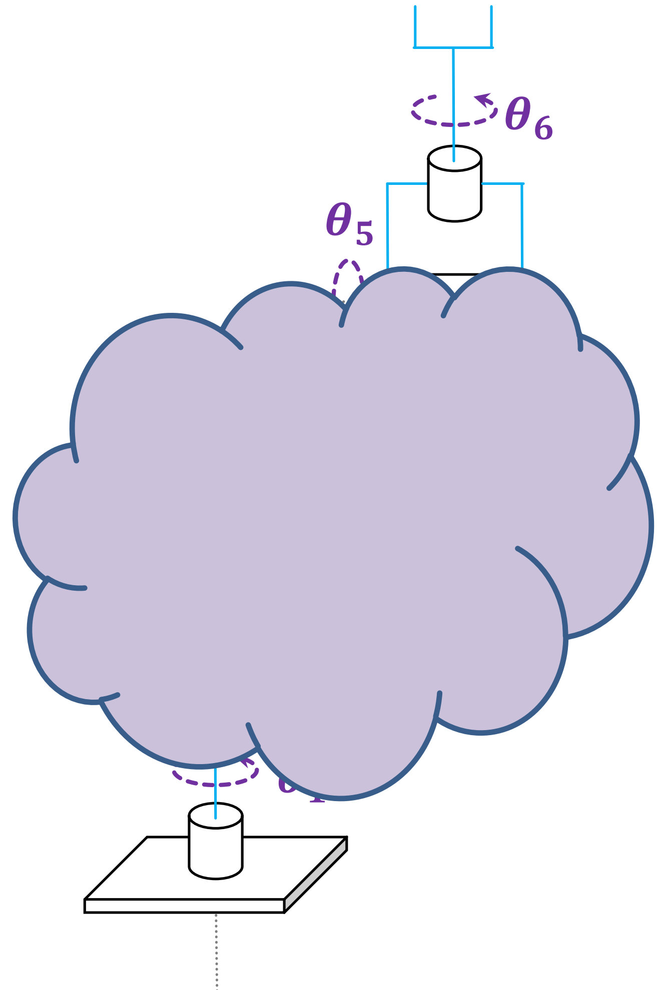
```
A123456 =

    0.0000   -1.0000    0.0000    0.0000
    0.0000    0.0000    1.0000    0.4000
   -1.0000         0    0.0000    1.0000
         0         0         0    1.0000
```

1
2

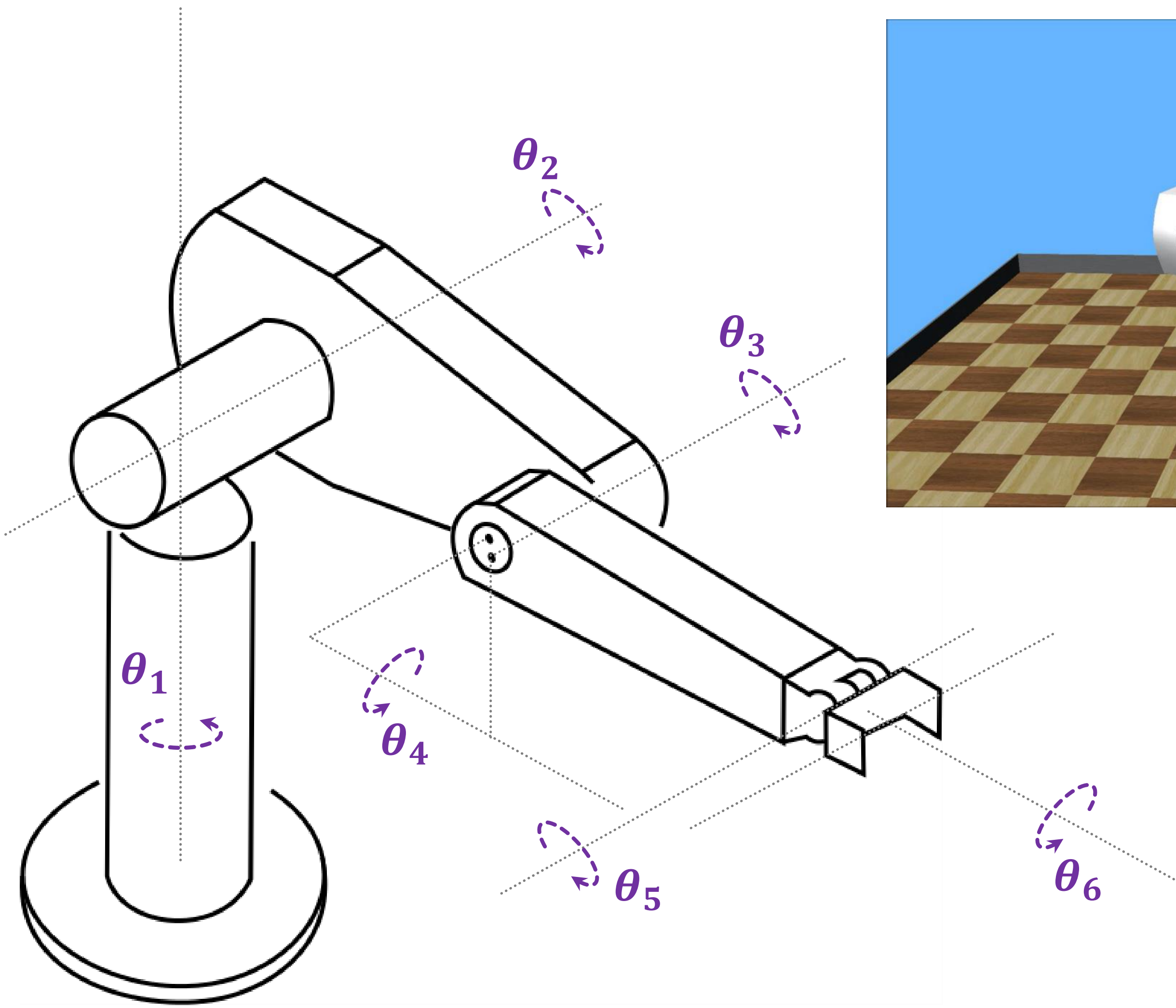


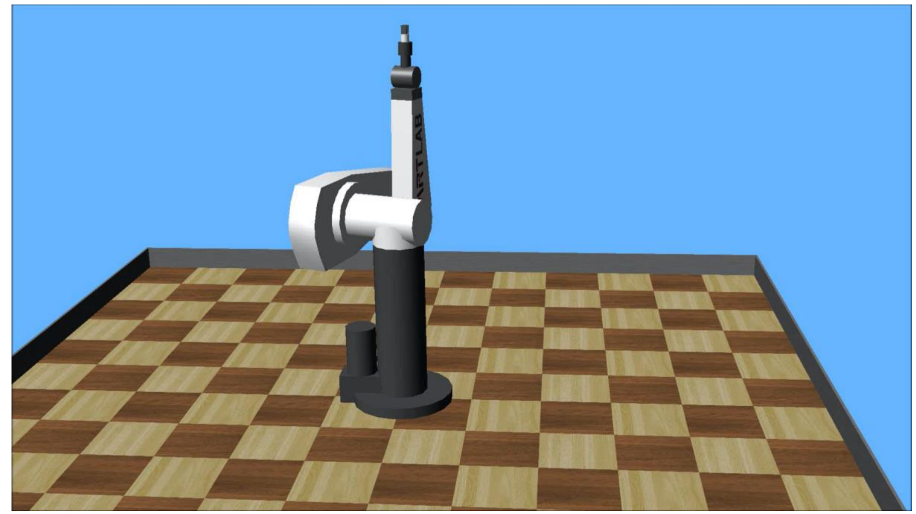
PUMA 260



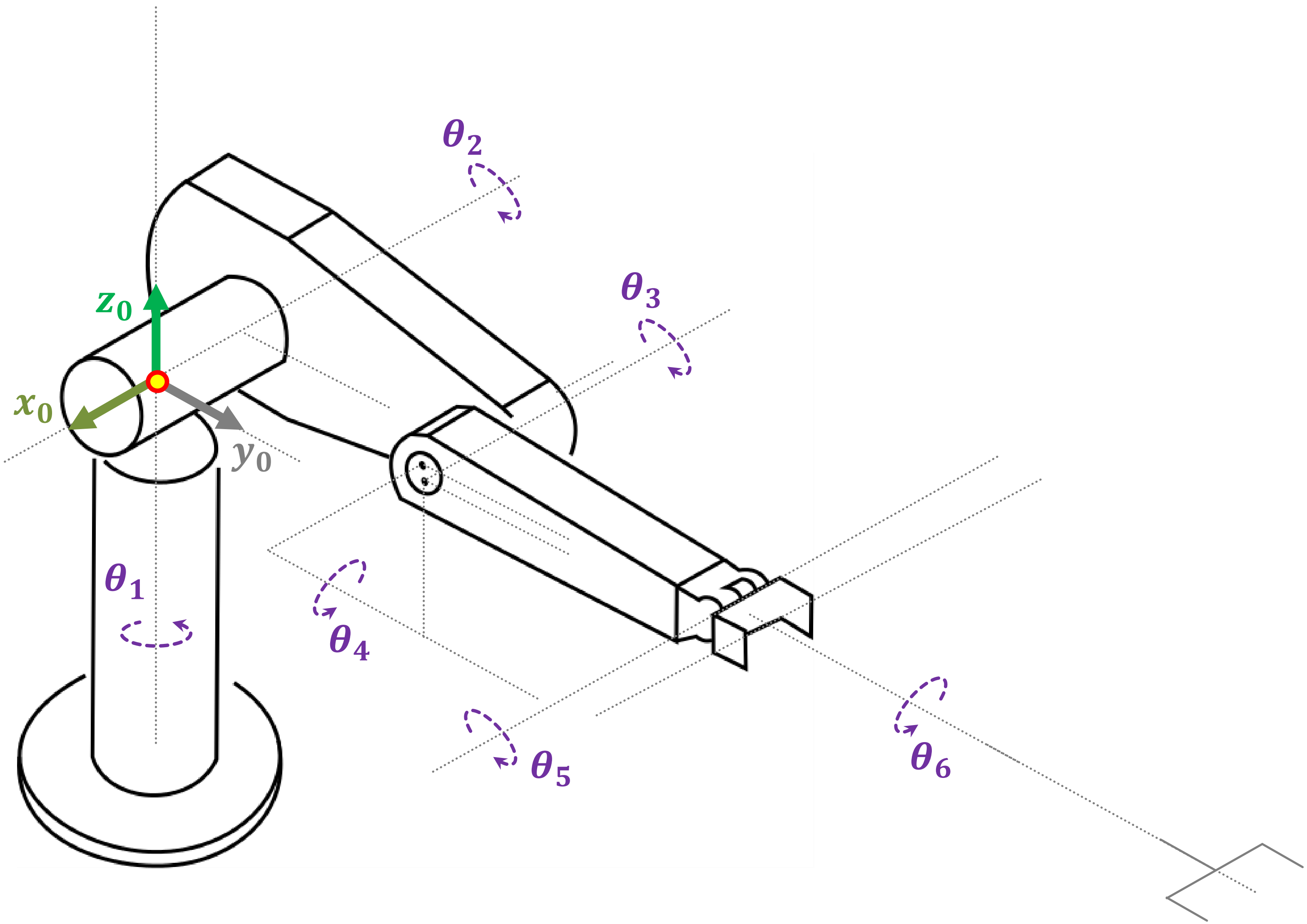


PUMA 260

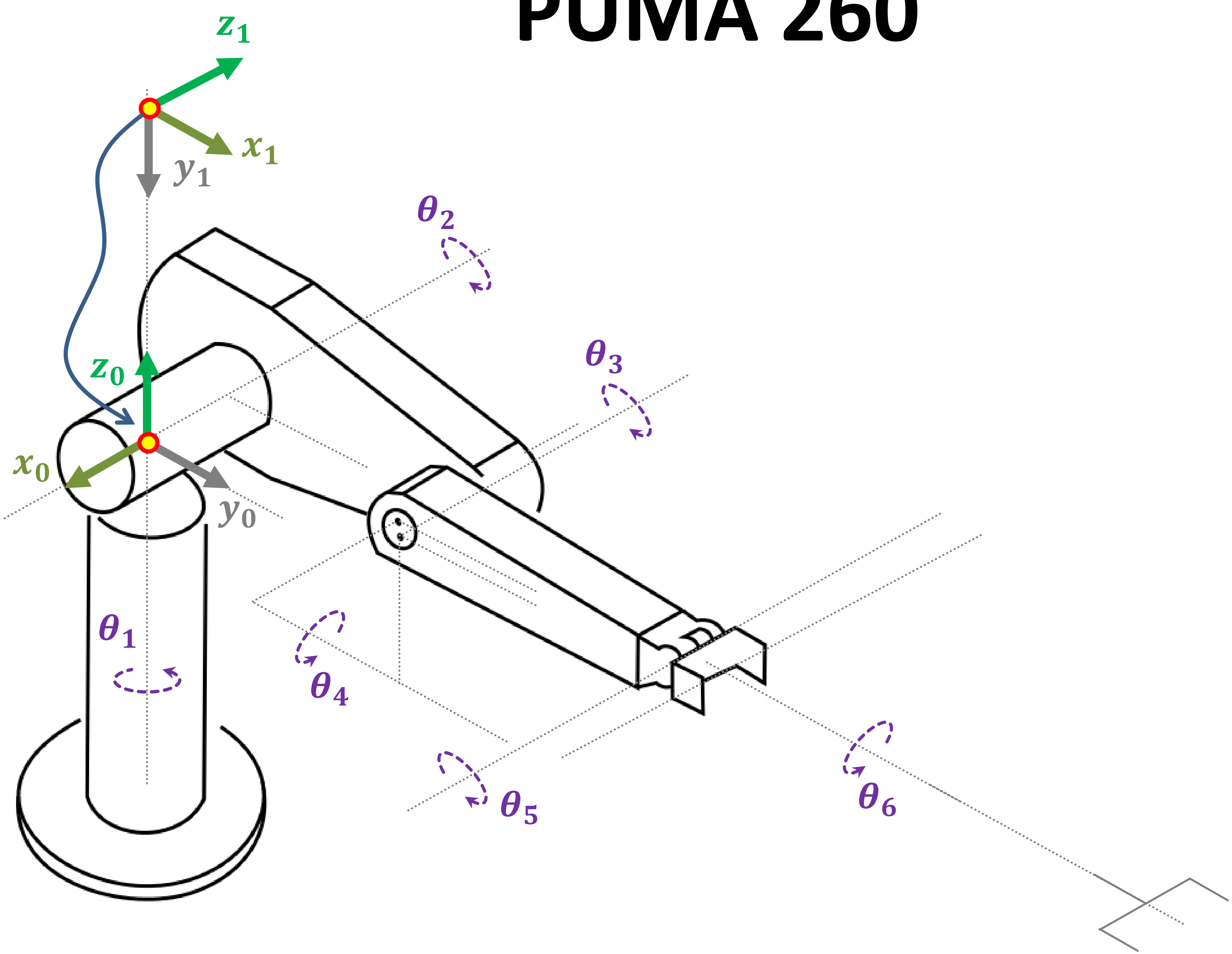




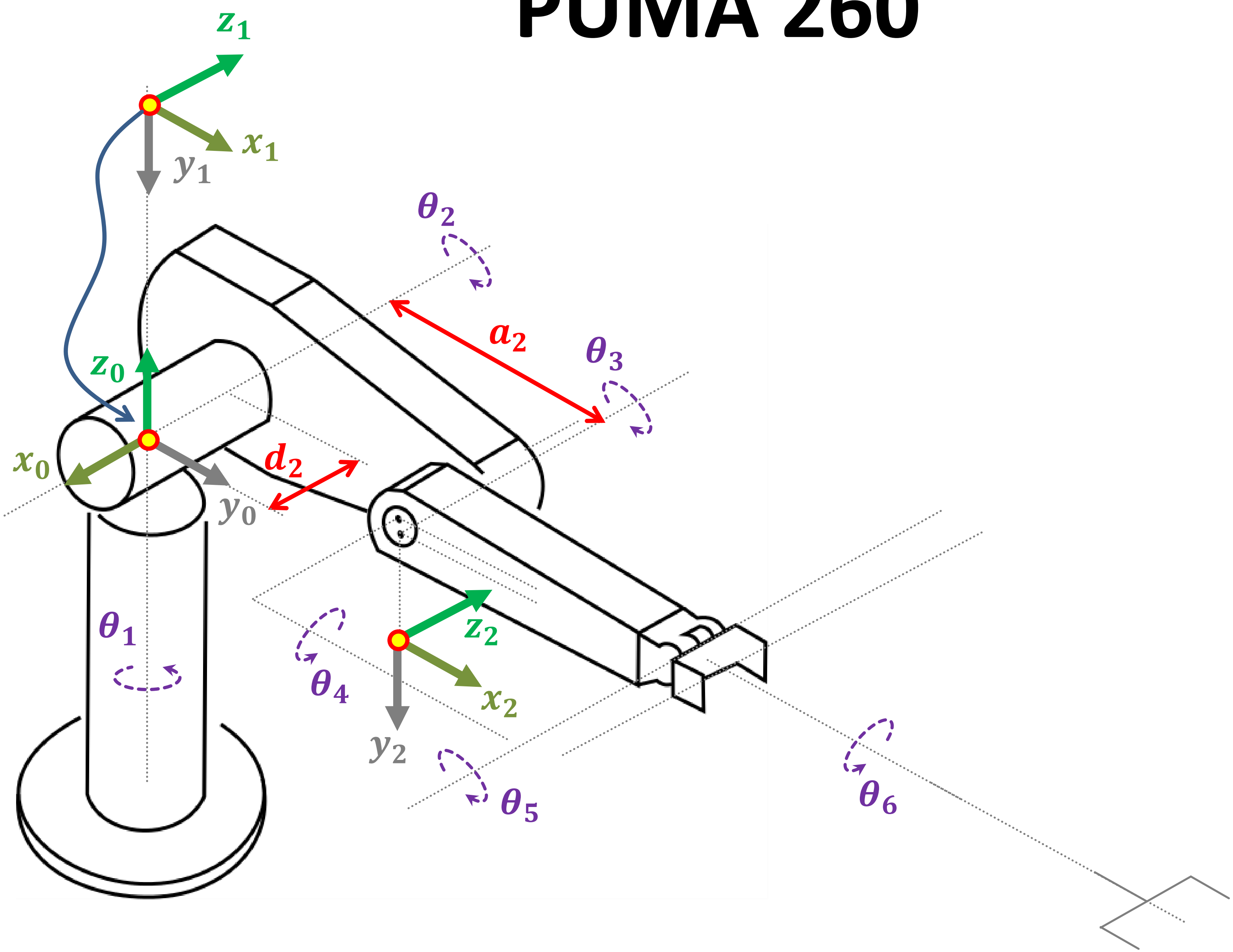
PUMA 260



PUMA 260

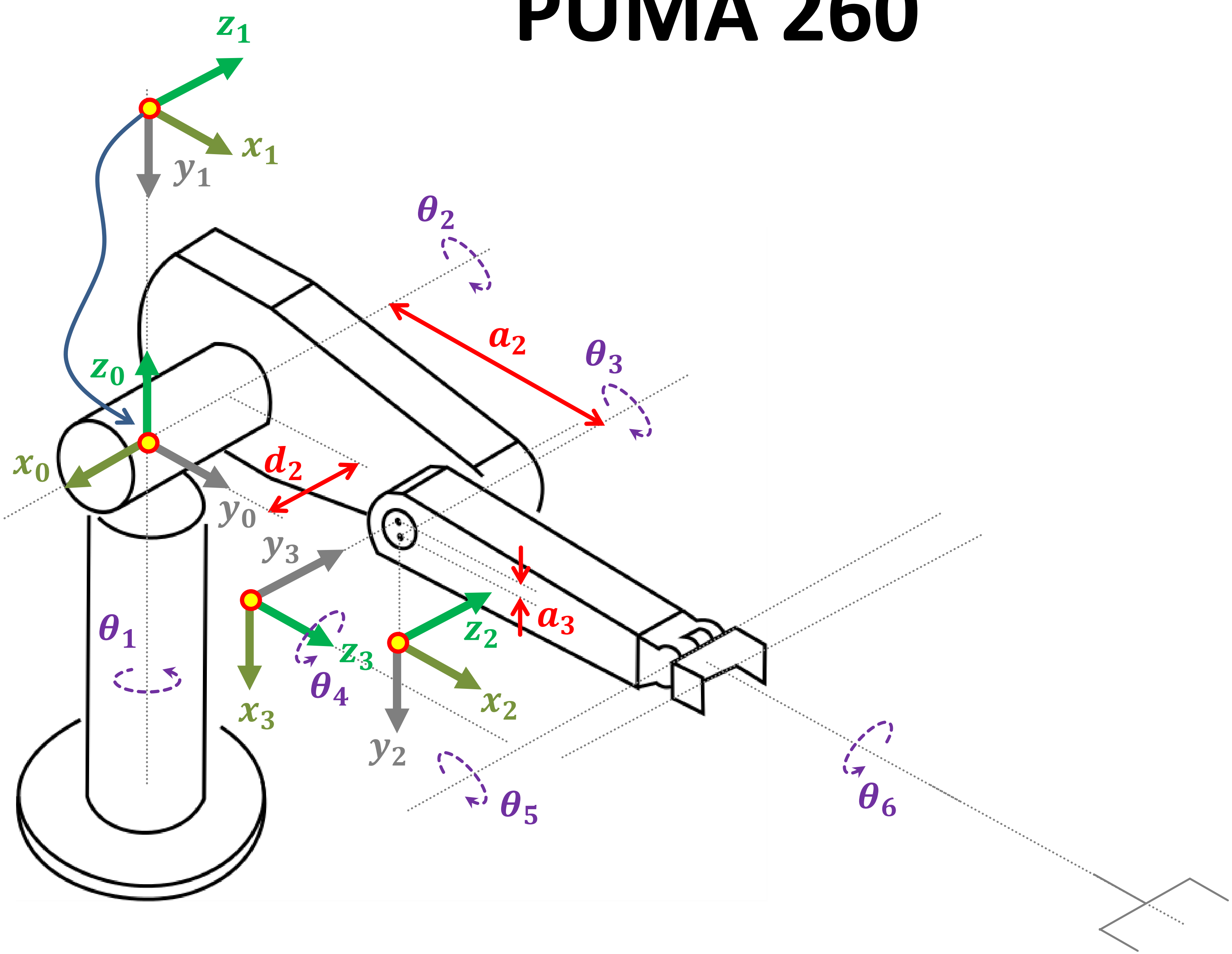


PUMA 260



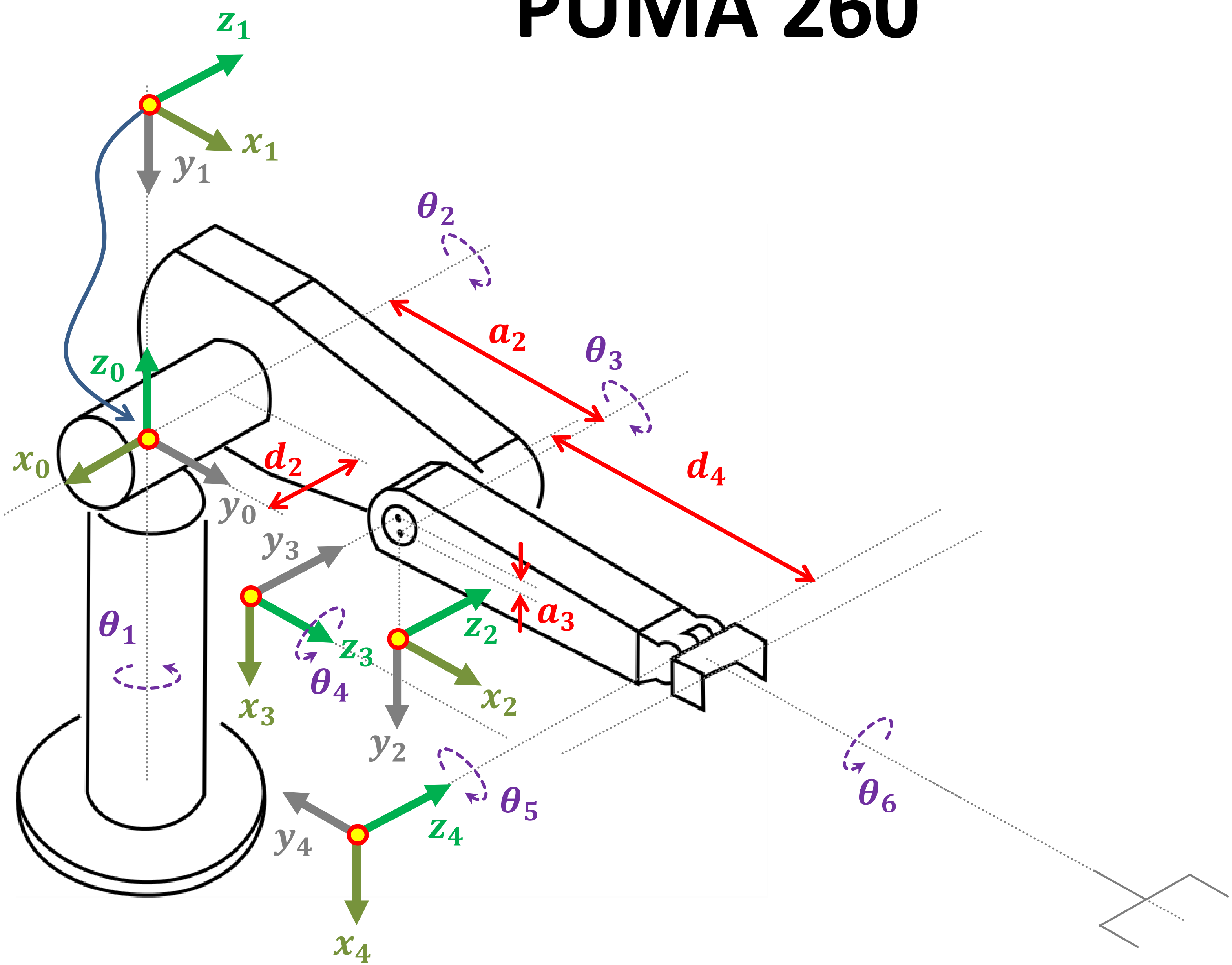
z_1
x_1
y_1
θ_2
a_2
θ_3
z_0
x_0
y_0
d_2
θ_1
θ_4
z_2
x_2
y_2
θ_5
θ_6

PUMA 260



z_1
x_1
y_1
θ_2
a_2
θ_3
z_0
x_0
d_2
y_0
y_3
θ_1
z_2
a_3
z_3
θ_4
x_3
x_2
y_2
θ_5
θ_6

PUMA 260



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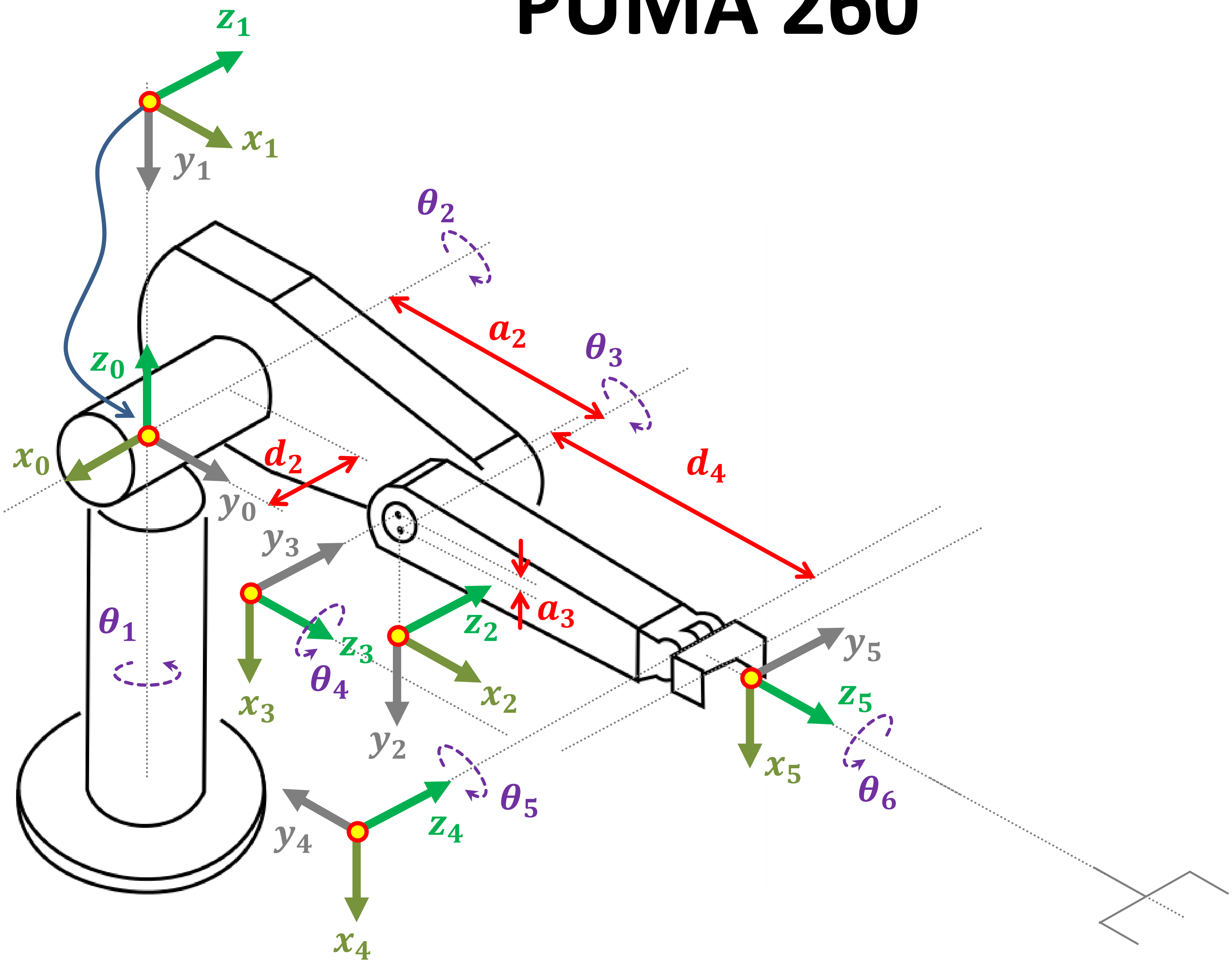
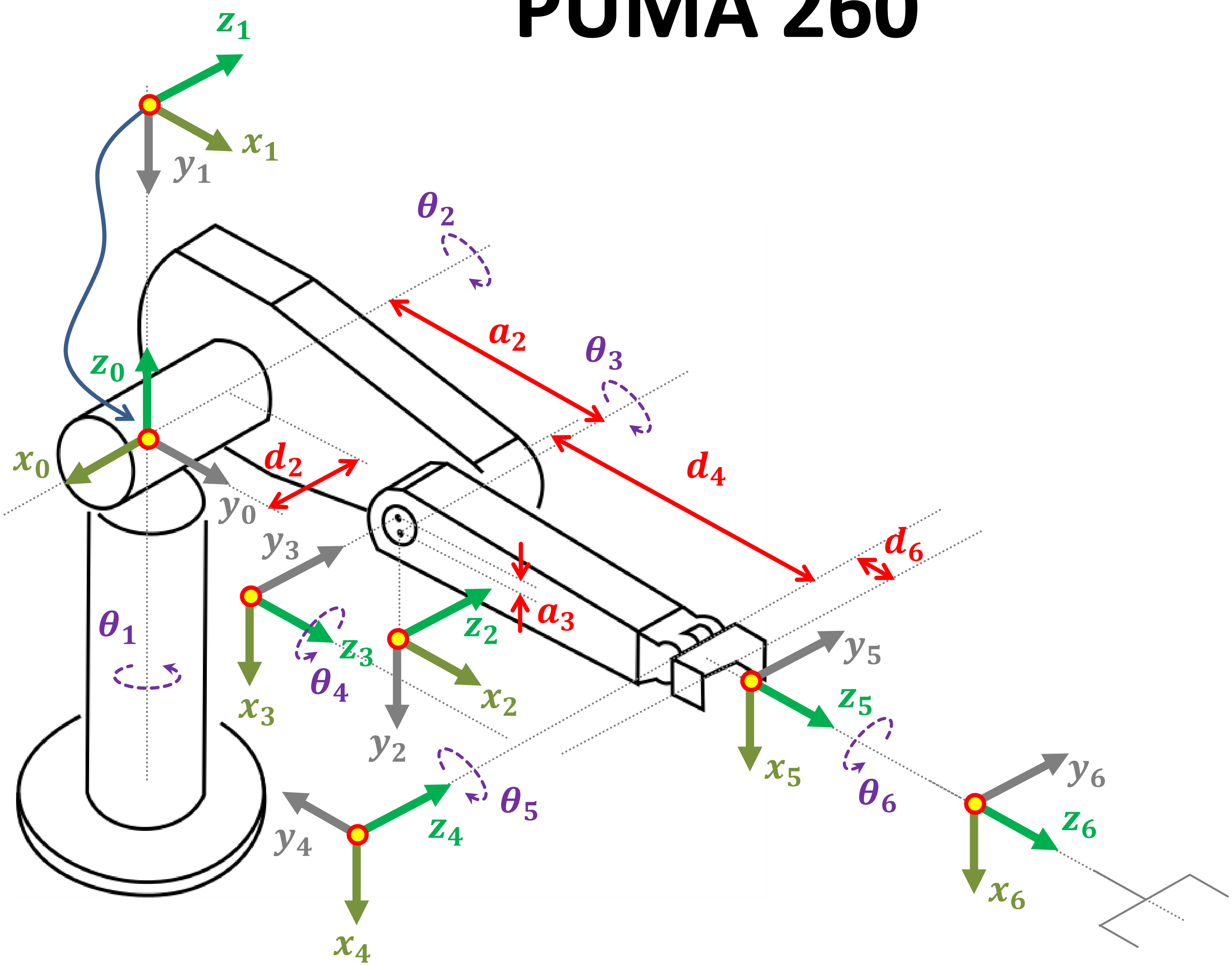
x_1
y_1
θ_2
a_2
θ_3
z_0
x_0
y_0
d_2
d_4
y_3
a_3
θ_1
z_3
z_2
θ_4
x_3
x_2
y_2
θ_5
θ_6
y_4
z_4
x_4

PUMA 260



z_1
x_1
y_1
θ_2
a_2
θ_3
z_0
x_0
y_0
d_2
d_4
y_3
θ_1
a_3
z_2
y_5
z_3
θ_4
x_3
x_2
z_5
y_2
x_5
θ_5
θ_6
y_4
z_4
x_4

PUMA 260

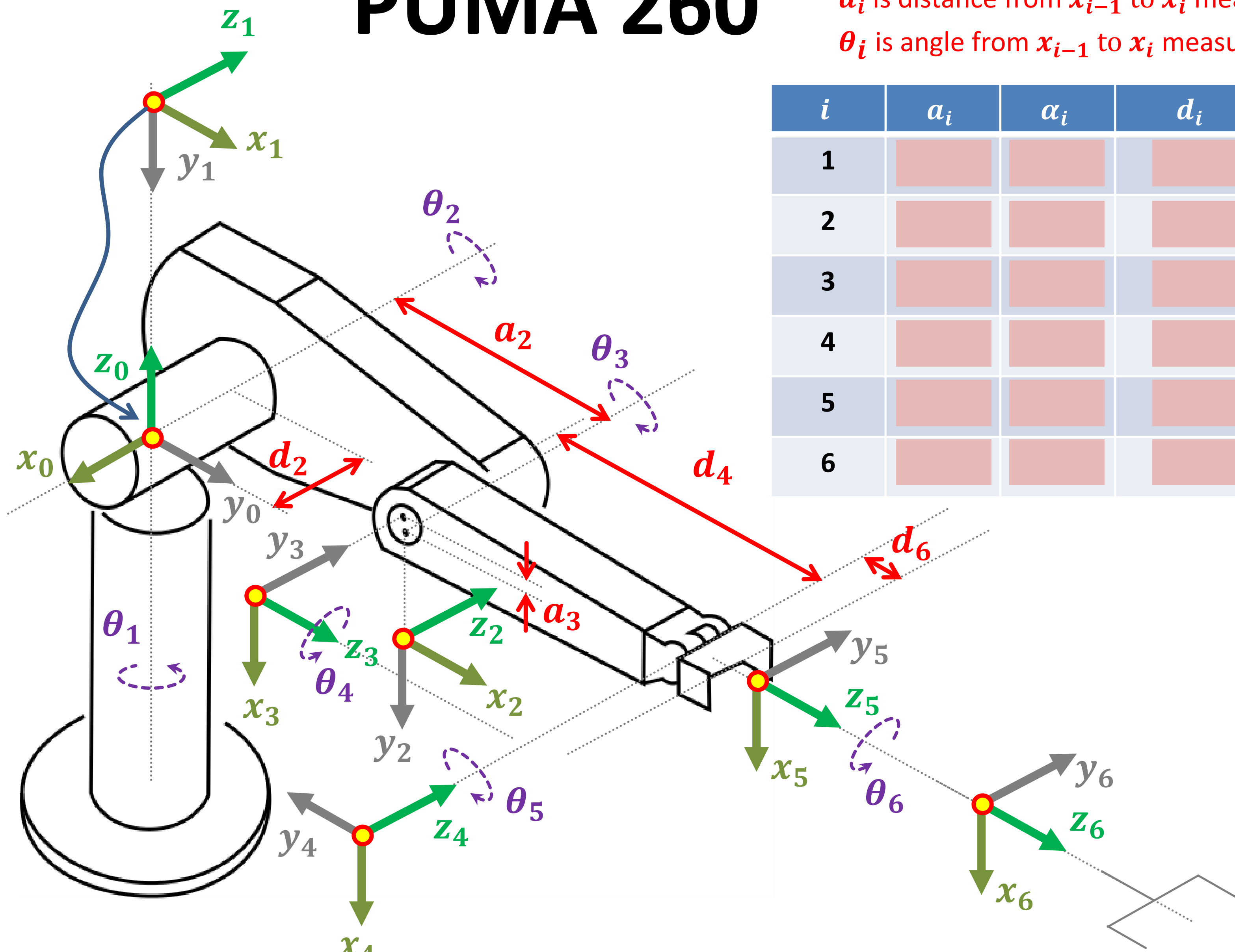


z_1
x_1
y_1
θ_2
a_2
θ_3
z_0
x_0
y_0
d_2
d_4
d_6
y_3
θ_1
a_3
z_3
x_3
θ_4
z_2
x_2
y_2
y_5
z_5
x_5
θ_6
y_6
z_6
x_6
y_4
z_4
θ_5
x_4

PUMA 260

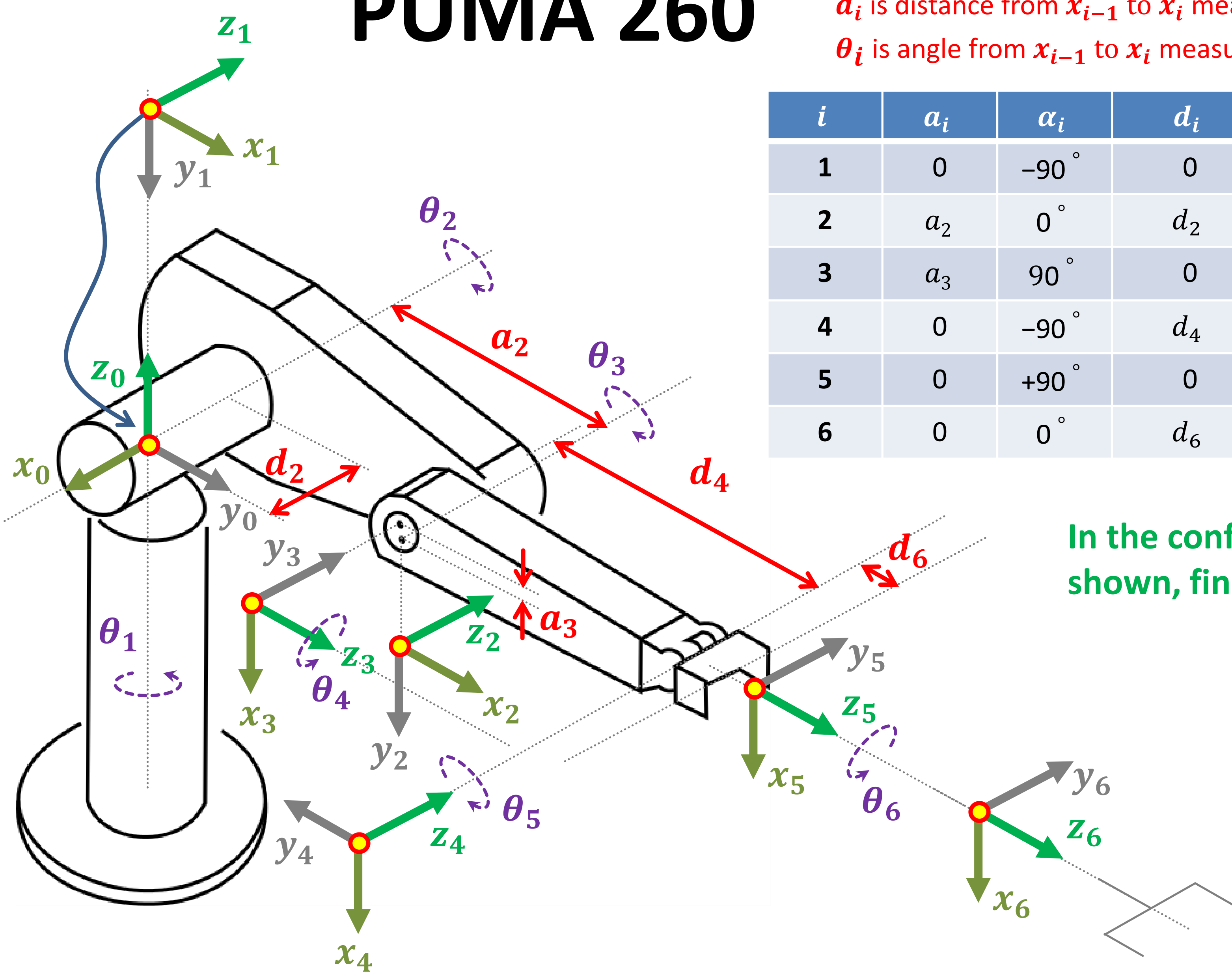
a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

i	a_i	α_i	d_i	θ_i
1				
2				
3				
4				
5				
6				



PUMA 260

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.



i	a_i	α_i	d_i	θ_i
1	0	-90°	0	
2	a_2	0°	d_2	
3	a_3	90°	0	
4	0	-90°	d_4	
5	0	$+90^\circ$	0	
6	0	0°	d_6	

In the configuration shown, find θ_i?

PUMA 260

Reminder: A_i

$$\begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	a_i	α_i	d_i	θ_i
1	0	-90°	0	θ_1 *
2	a_2	0°	d_2	θ_2 *
3	a_3	90°	0	θ_3 *
4	0	-90°	d_4	θ_4 *
5	0	$+90^\circ$	0	θ_5 *
6	0	0°	d_6	θ_6 *

Coordinate Transformation Matrices

$$\mathbf{A}_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_2^3 = \begin{bmatrix} C_3 & 0 & S_3 & a_3C_3 \\ S_3 & 0 & -C_3 & a_3S_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_3^4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_4^5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A}_5^6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PUMA 260

$$r_{11} = -s_6(c_4 s_1 - s_4(c_1 s_2 s_3 - c_1 c_2 c_3)) - c_6\left(c_5(s_1 s_4 + c_4(c_1 s_2 s_3 - c_1 c_2 c_3)) + s_5(c_1 c_2 s_3 + c_1 c_3 s_2)\right)$$
$$r_{12} = s_6\left(c_5(s_1 s_4 + c_4(c_1 s_2 s_3 - c_1 c_2 c_3)) + s_5(c_1 c_2 s_3 + c_1 c_3 s_2)\right) - c_6(c_4 s_1 - s_4(c_1 s_2 s_3 - c_1 c_2 c_3))$$
$$r_{13} = c_5(c_1 c_2 s_3 + c_1 c_3 s_2) - s_5(s_1 s_4 + c_4(c_1 s_2 s_3 - c_1 c_2 c_3))$$
$$d_x = d_4(c_1 c_2 s_3 + c_1 c_3 s_2) - d_2 s_1 - d_6\left(s_5(s_1 s_4 + c_4(c_1 s_2 s_3 - c_1 c_2 c_3)) - c_5(c_1 c_2 s_3 + c_1 c_3 s_2)\right) + a_2 c_1 c_2 + a_3 c_1 c_2 c_3 - a_3 c_1 s_2 s_3$$

$$r_{21} = s_6(c_1 c_4 + s_4(s_1 s_2 s_3 - c_2 c_3 s_1)) + c_6\left(c_5(c_1 s_4 - c_4(s_1 s_2 s_3 - c_2 c_3 s_1)) - s_5(c_2 s_1 s_3 + c_3 s_1 s_2)\right)$$
$$r_{22} = c_6(c_1 c_4 + s_4(s_1 s_2 s_3 - c_2 c_3 s_1)) - s_6\left(c_5(c_1 s_4 - c_4(s_1 s_2 s_3 - c_2 c_3 s_1)) - s_5(c_2 s_1 s_3 + c_3 s_1 s_2)\right)$$
$$r_{23} = s_5(c_1 s_4 - c_4(s_1 s_2 s_3 - c_2 c_3 s_1)) + c_5(c_2 s_1 s_3 + c_3 s_1 s_2)$$
$$d_y = d_6\left(s_5(c_1 s_4 - c_4(s_1 s_2 s_3 - c_2 c_3 s_1)) + c_5(c_2 s_1 s_3 + c_3 s_1 s_2)\right) + d_4(c_2 s_1 s_3 + c_3 s_1 s_2) + d_2 c_1 + a_2 c_2 s_1 + a_3 c_2 c_3 s_1 - a_3 s_1 s_2 s_3$$

$$r_{31} = s_4 s_6(c_2 s_3 + c_3 s_2) - c_6(s_5(c_2 c_3 - s_2 s_3) + c_4 c_5(c_2 s_3 + c_3 s_2))$$
$$r_{32} = s_6(s_5(c_2 c_3 - s_2 s_3) + c_4 c_5(c_2 s_3 + c_3 s_2)) + c_6 s_4(c_2 s_3 + c_3 s_2)$$
$$r_{33} = c_5(c_2 c_3 - s_2 s_3) - c_4 s_5(c_2 s_3 + c_3 s_2)$$
$$d_z = d_4(c_2 c_3 - s_2 s_3) - a_2 s_2 + d_6(c_5(c_2 c_3 - s_2 s_3) - c_4 s_5(c_2 s_3 + c_3 s_2)) - a_3 c_2 s_3 - a_3 c_3 s_2$$

PUMA 260

$$r_{11} = -s_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3)) - c_6\big(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\big)$$

$$r_{12} = s_6\big(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\big) - c_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3))$$

$$r_{13} = c_5(c_1c_2s_3 + c_1c_3s_2) - s_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3))$$

$d \ \dots \ c_5(c_1c_2s_3 + c_1c_3s_2)\big) + a_2c_1c_2 + a_3c_1c_2c_3 - a_3c_1s_2s_3$

$r \ \dots \ \big) - s_5(c_2s_1s_3 + c_3s_1s_2)\big)$

$r \ \dots \ \big) - s_5(c_2s_1s_3 + c_3s_1s_2)\big)$

r

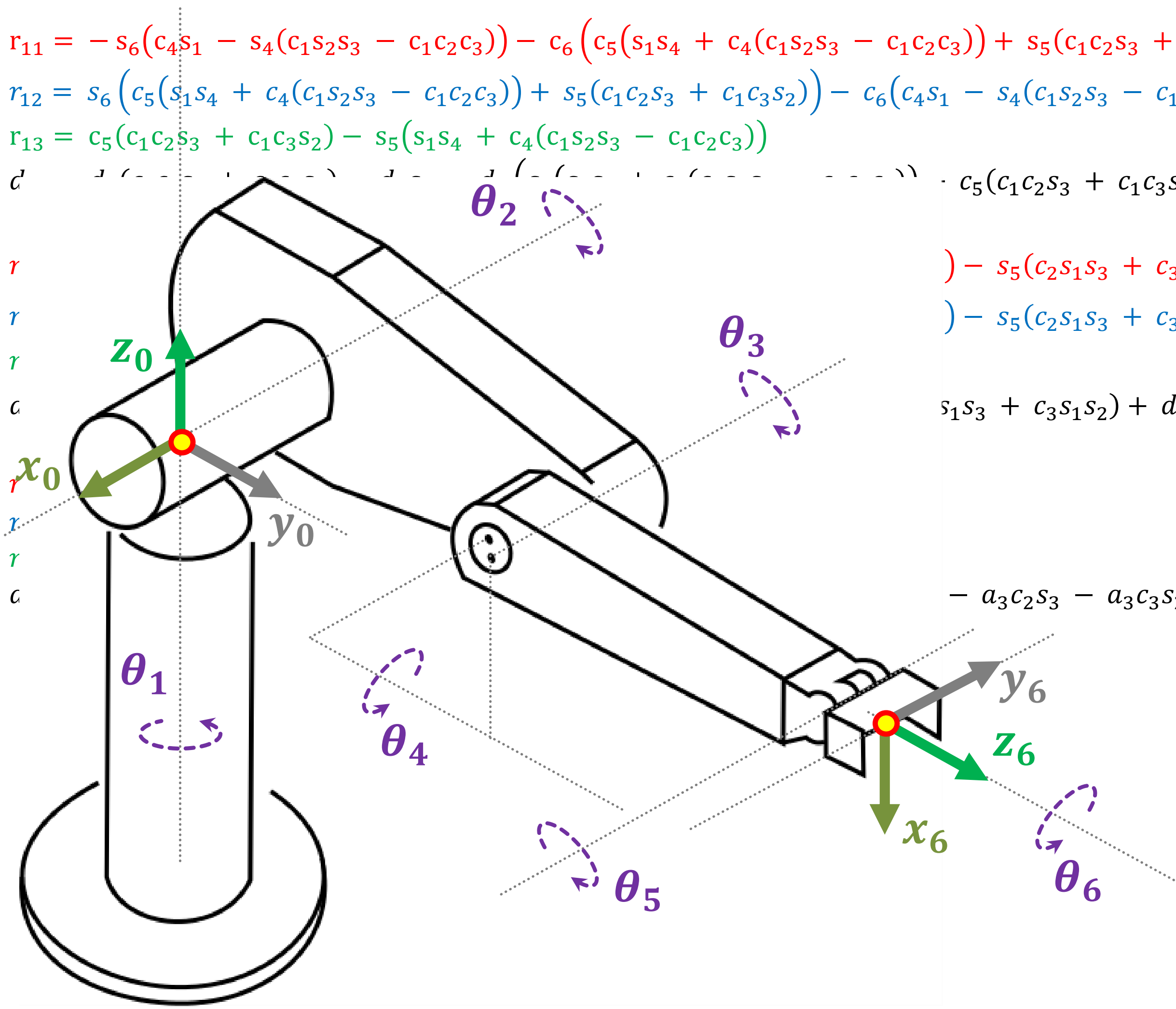
$d \ \dots \ s_1s_3 + c_3s_1s_2) + d_2c_1 + a_2c_2s_1 + a_3c_2c_3s_1 - a_3s_1s_2s_3$

r

r

r

$d \ \dots \ - a_3c_2s_3 - a_3c_3s_2$

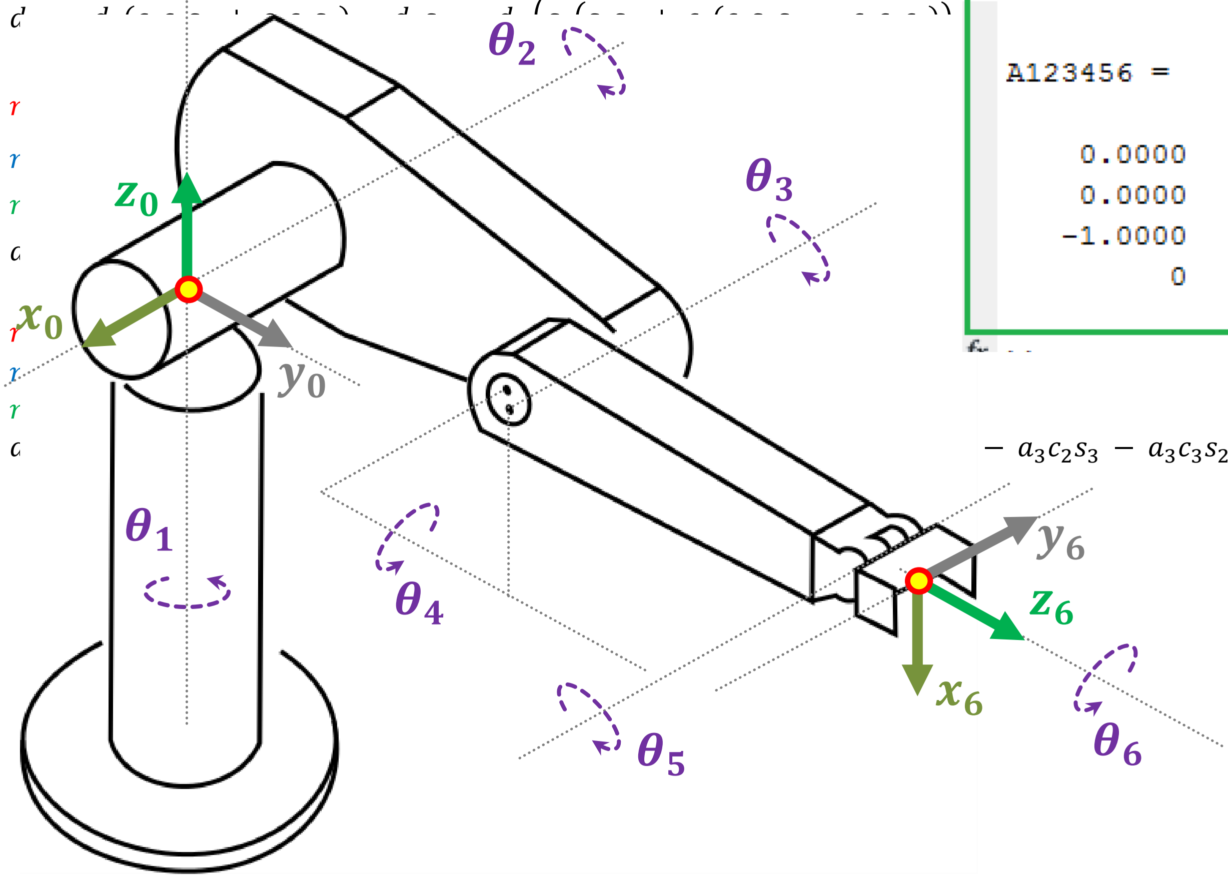


PUMA 260

$r_{11} = -s_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3)) - c_6\left(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\right)$

$r_{12} = s_6\left(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\right) - c_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3))$

$r_{13} = c_5(c_1c_2s_3 + c_1c_3s_2) - s_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3))$



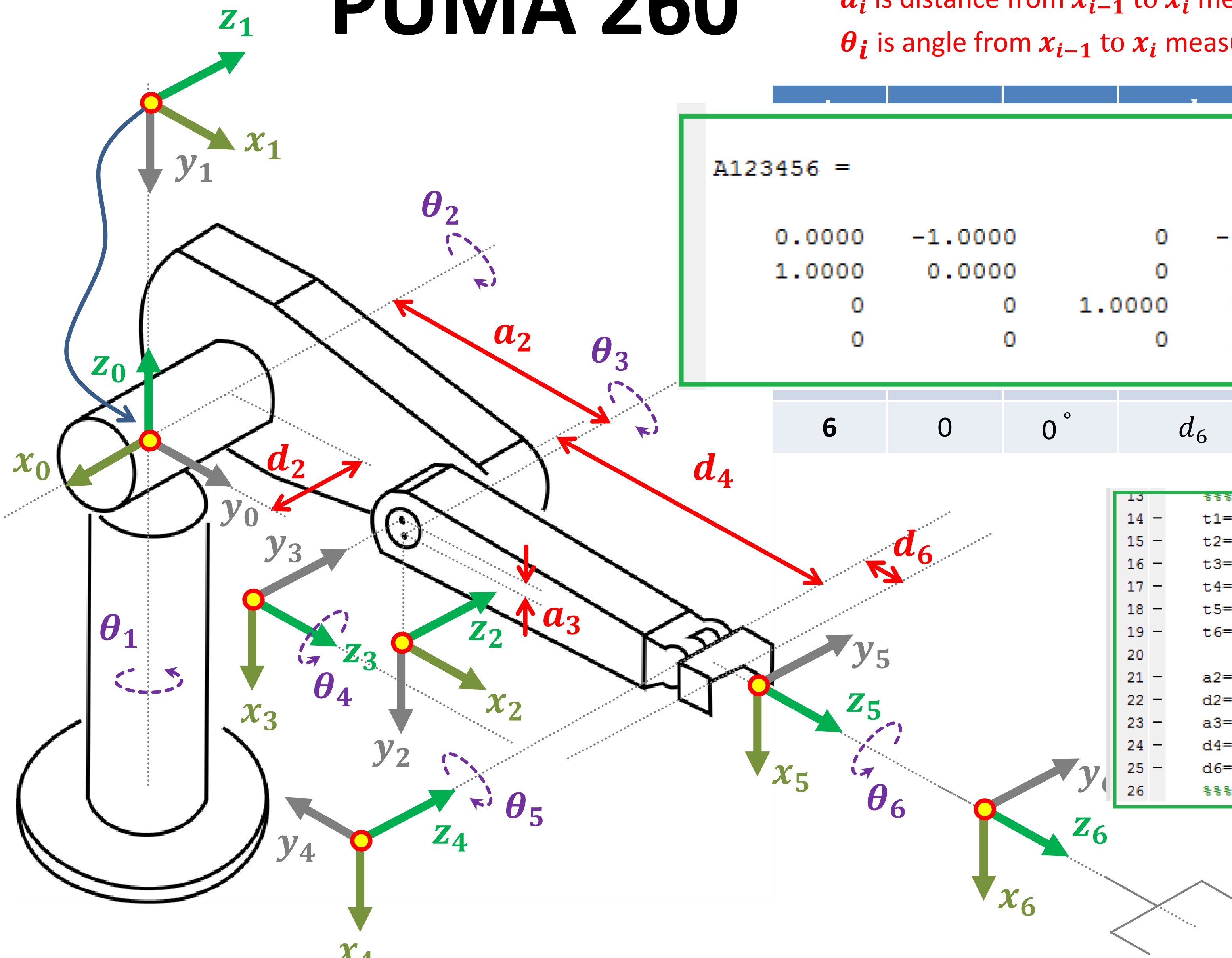
$- a_3c_2s_3 - a_3c_3s_2$

```
A123456 =

    0.0000   -1.0000    0.0000   -0.1491
    0.0000    0.0000    1.0000    0.9211
   -1.0000         0    0.0000   -0.0203
         0         0         0    1.0000
```

```
%%%%%%%%%%%%%%%
t1= pi/2.0;
t2= 0;
t3= pi/2.0;
t4= 0;
t5= 0;
t6= 0;

a2= 0.4318;
d2= 0.14909;
a3= 0.02032;
d4= 0.43307;
d6= 0.05625;
%%%%%%%%%%%%%%%
```

PUMA 260

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.



				θ_i
6	0	0°	d_6	0

```
A123456 =

    0.0000   -1.0000         0   -0.1491
    1.0000    0.0000         0    0.4521
         0         0    1.0000    0.4893
         0         0         0    1.0000
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t1= pi/2.0;
t2= 0;
t3= 0;
t4= 0;
t5= 0;
t6= 0;

a2= 0.4318;
d2= 0.14909;
a3= 0.02032;
d4= 0.43307;
d6= 0.05625;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

PUMA 260

$$r_{11} = -s_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3)) - c_6\big(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\big)$$

$$r_{12} = s_6\big(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2)\big) - c_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3))$$

$$r_{13} = c_5(c_1c_2s_3 + c_1c_3s_2) - s_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3))$$

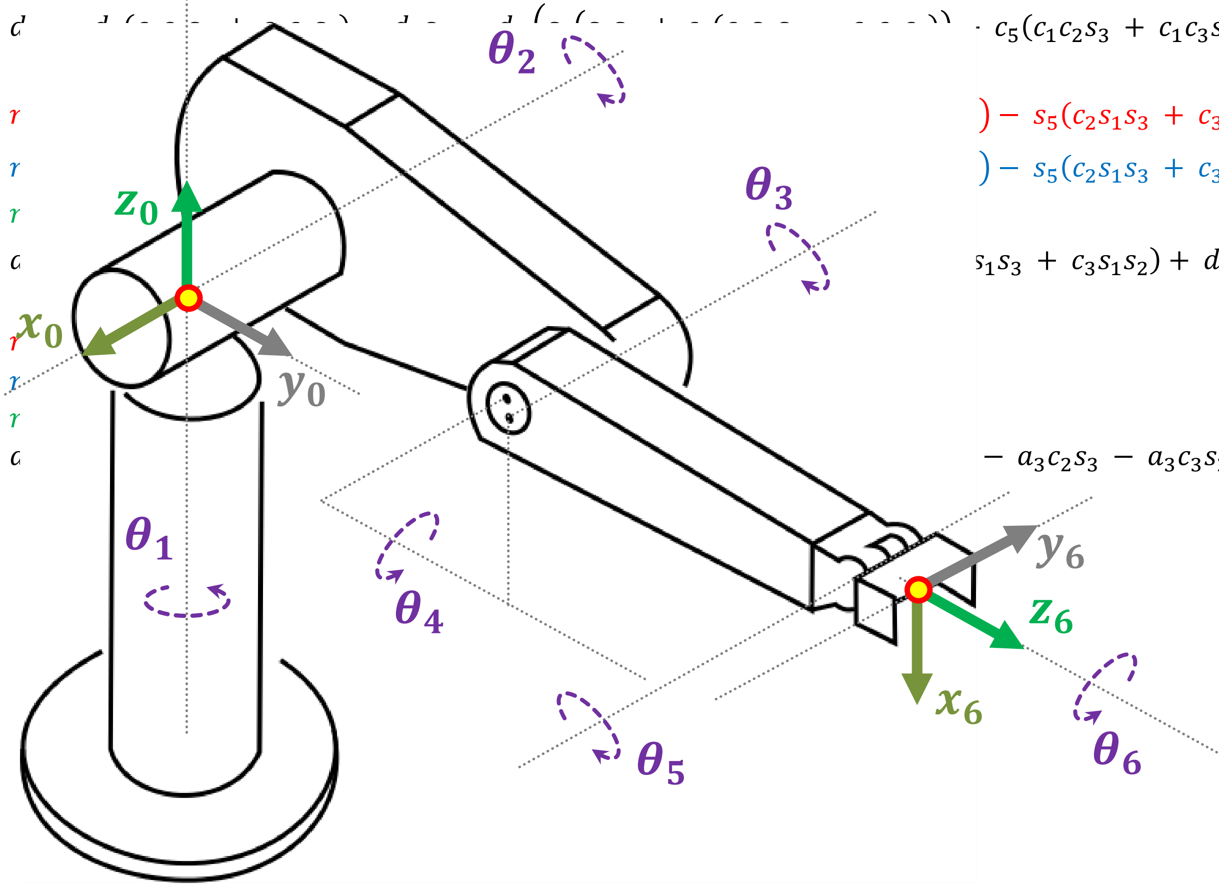
$\cdot\, c_5(c_1c_2s_3 + c_1c_3s_2)\big) + a_2c_1c_2 + a_3c_1c_2c_3 - a_3c_1s_2s_3$

$\big) - s_5(c_2s_1s_3 + c_3s_1s_2)\big)$

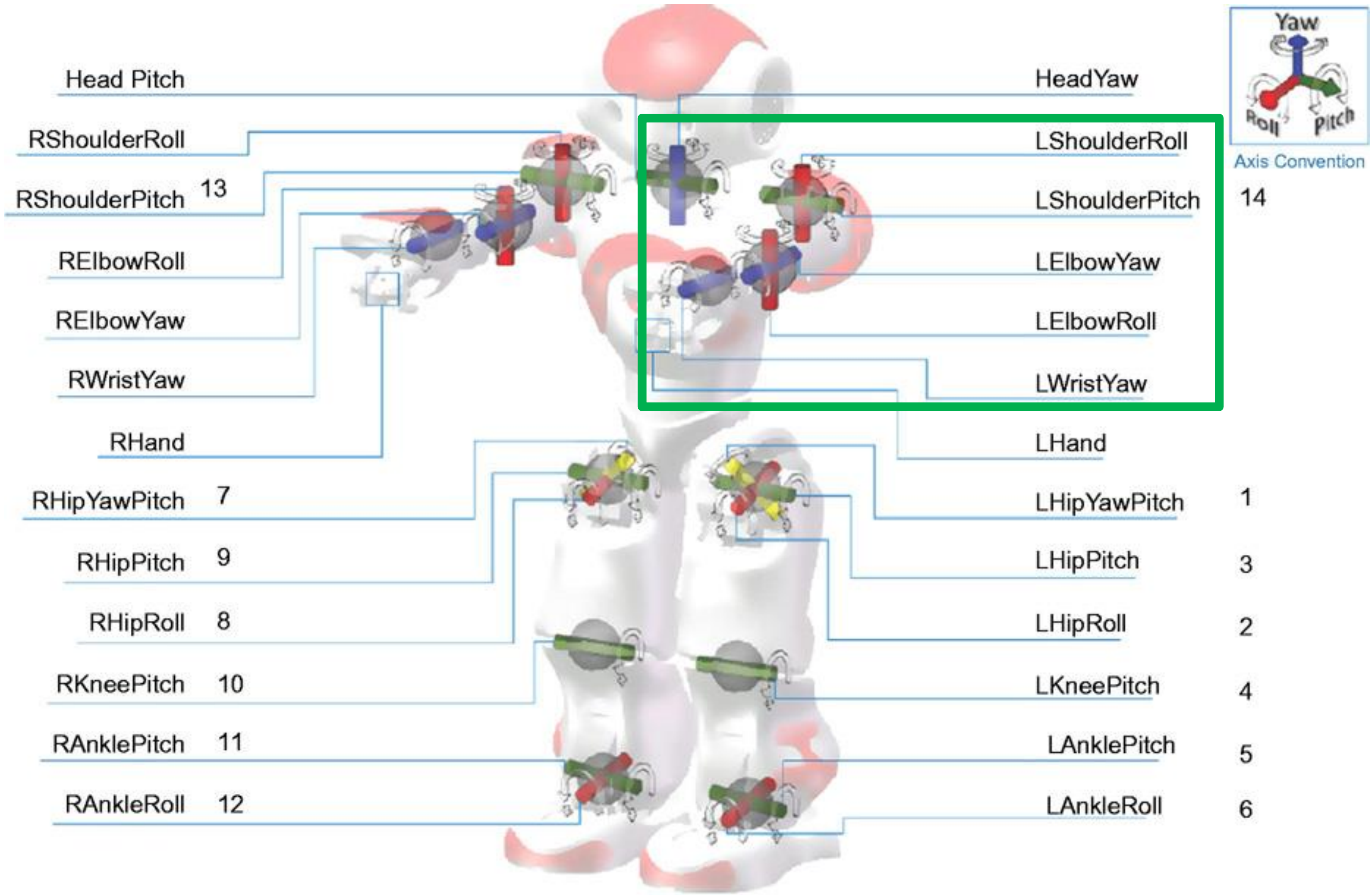
$\big) - s_5(c_2s_1s_3 + c_3s_1s_2)\big)$

$s_1s_3 + c_3s_1s_2) + d_2c_1 + a_2c_2s_1 + a_3c_2c_3s_1 - a_3s_1s_2s_3$

$- a_3c_2s_3 - a_3c_3s_2$



NAO Left Arm

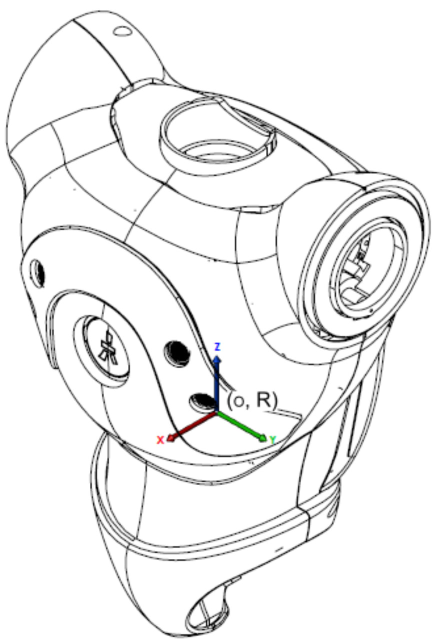


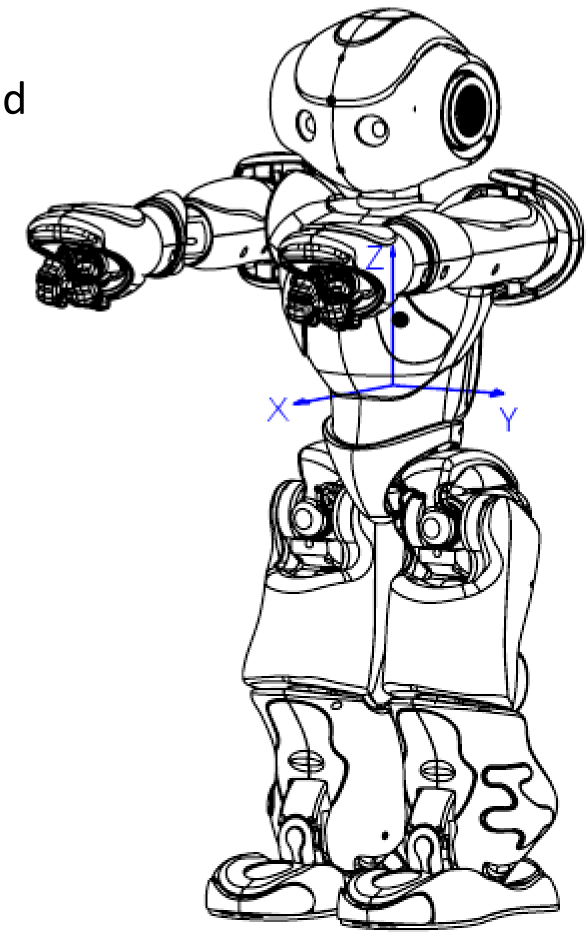
Head Pitch
RShoulderRoll
RShoulderPitch
13
RElbowRoll
RElbowYaw
RWristYaw
RHand
RHipYawPitch
7
RHipPitch
9
RHipRoll
8
RKneePitch
10
RAnklePitch
11
RAnkleRoll
12
HeadYaw
LShoulderRoll
LShoulderPitch
14
LElbowYaw
LElbowRoll
LWristYaw
LHand
LHipYawPitch
1
LHipPitch
3
LHipRoll
2
LKneePitch
4
LAnklePitch
5
LAnkleRoll
6
Yaw
Roll
Pitch
Axis Convention

NAO Zero Position

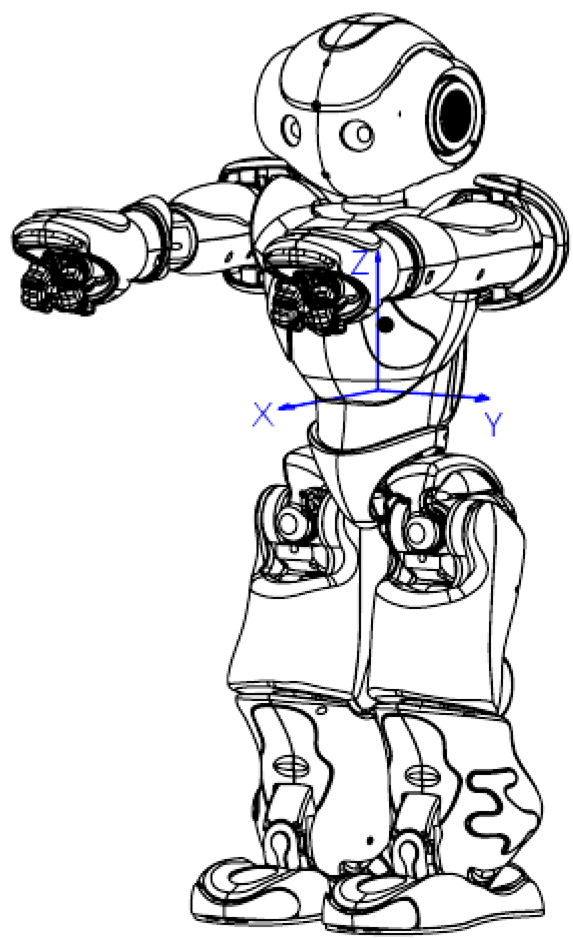
Provided by Aldebaran Robotics

The torso is the point where all the kinematic chains begin and is located at the center of the NAO body.

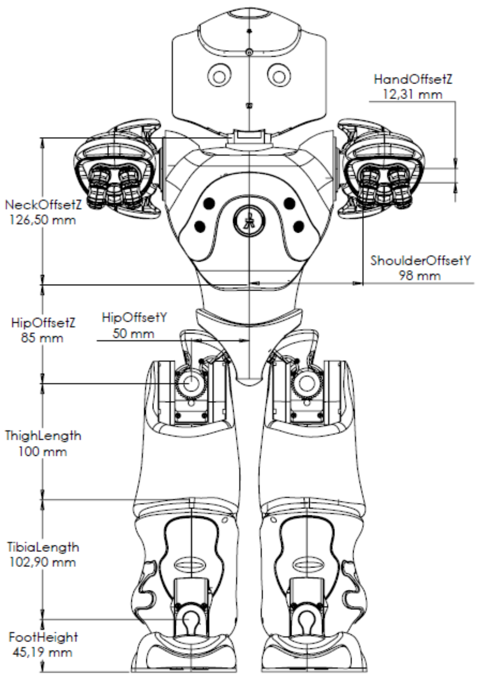


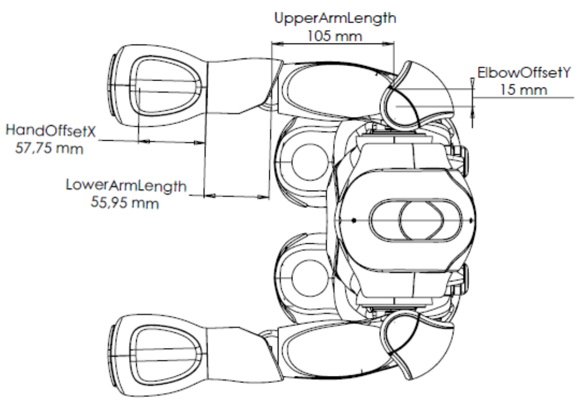


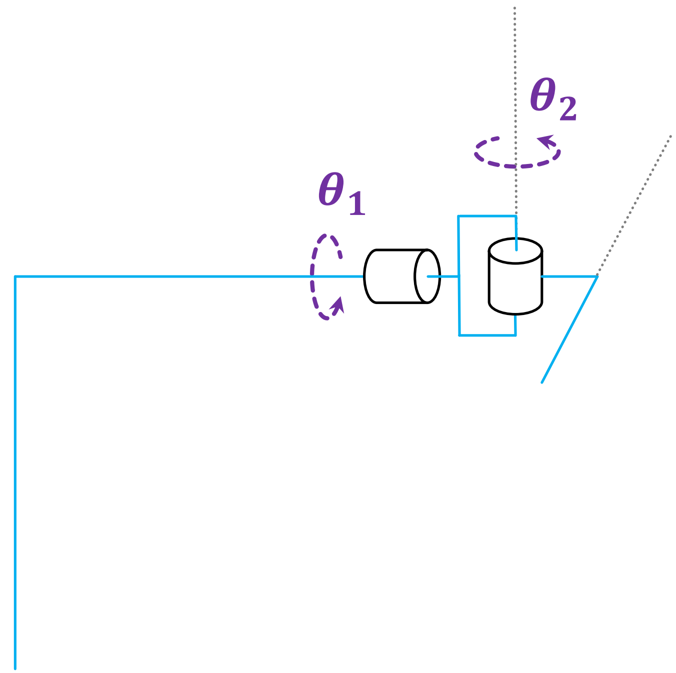
NAO Zero Position



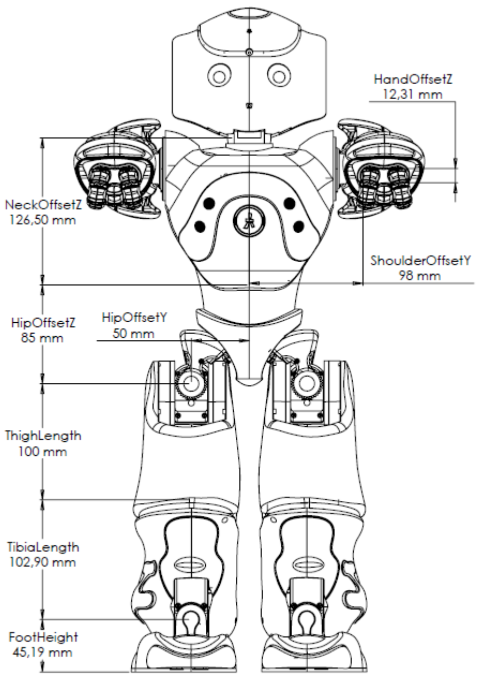
NAO Left Arm

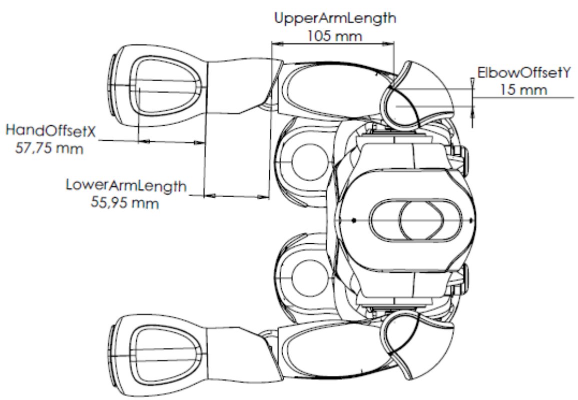

HandOffsetZ
12,31 mm
NeckOffsetZ
126,50 mm
ShoulderOffsetY
98 mm
HipOffsetZ
85 mm
HipOffsetY
50 mm
ThighLength
100 mm
TibiaLength
102,90 mm
FootHeight
45,19 mm

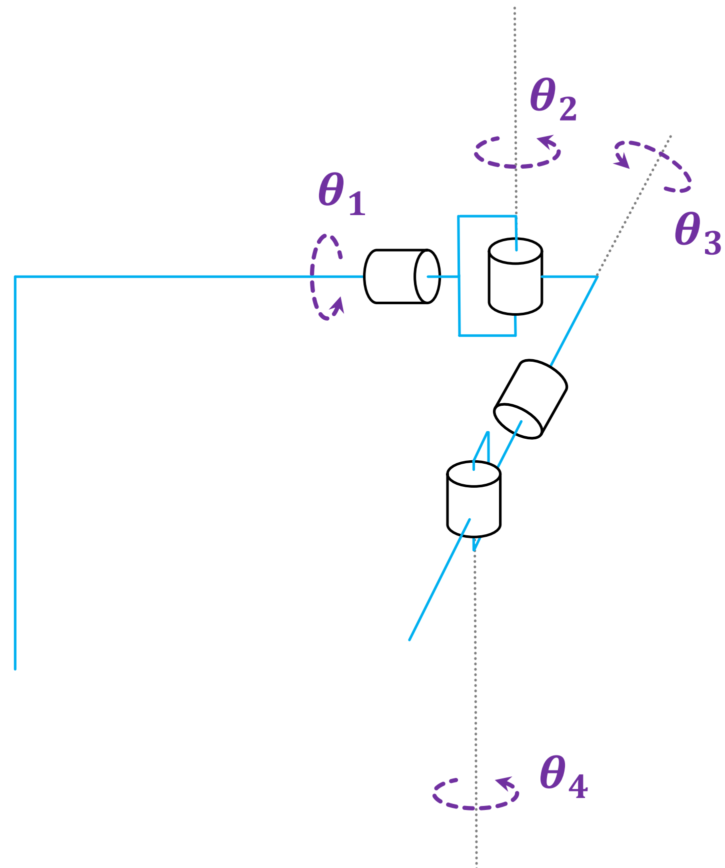

UpperArmLength
105 mm
ElbowOffsetY
15 mm
HandOffsetX
57,75 mm
LowerArmLength
55,95 mm


θ_2
θ_1

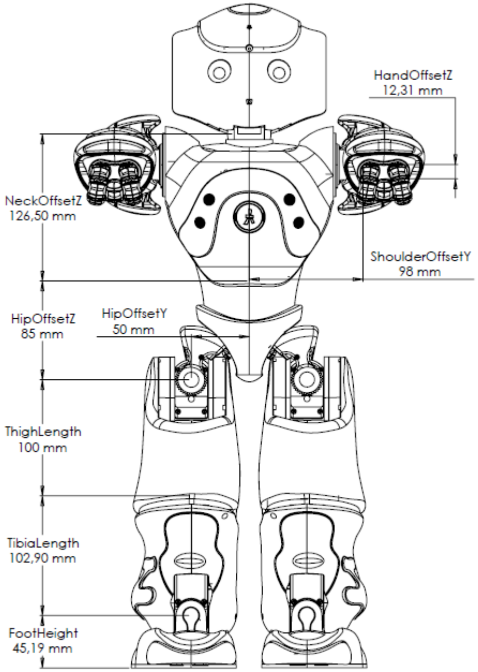
NAO Left Arm

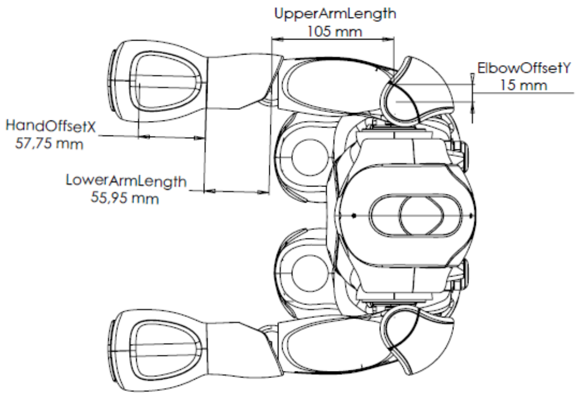

HandOffsetZ
12,31 mm
NeckOffsetZ
126,50 mm
ShoulderOffsetY
98 mm
HipOffsetZ
85 mm
HipOffsetY
50 mm
ThighLength
100 mm
TibiaLength
102,90 mm
FootHeight
45,19 mm

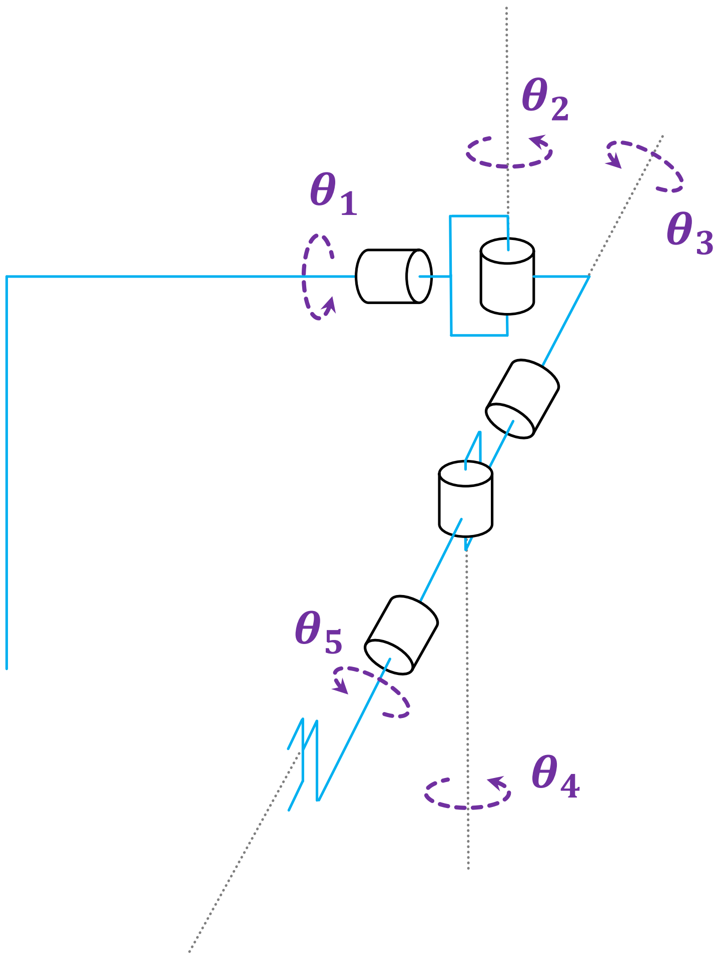

UpperArmLength
105 mm
ElbowOffsetY
15 mm
HandOffsetX
57,75 mm
LowerArmLength
55,95 mm


θ_2
θ_1
θ_3
θ_4

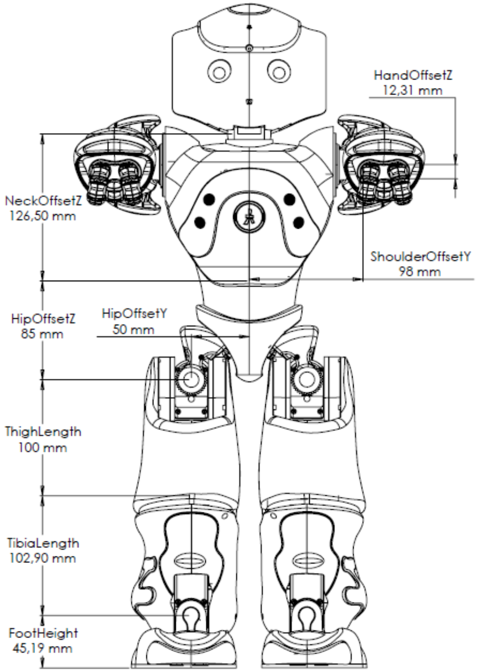
NAO Left Arm

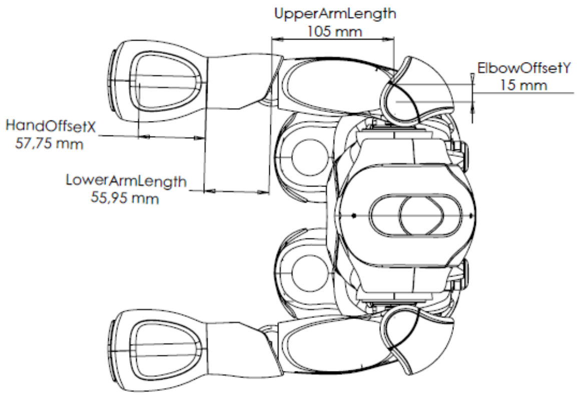

HandOffsetZ
12,31 mm
NeckOffsetZ
126,50 mm
ShoulderOffsetY
98 mm
HipOffsetZ
85 mm
HipOffsetY
50 mm
ThighLength
100 mm
TibiaLength
102,90 mm
FootHeight
45,19 mm


UpperArmLength
105 mm
ElbowOffsetY
15 mm
HandOffsetX
57,75 mm
LowerArmLength
55,95 mm


θ_2
θ_1
θ_3
θ_5
θ_4

NAO Left Arm


HandOffsetZ
12,31 mm
NeckOffsetZ
126,50 mm
ShoulderOffsetY
98 mm
HipOffsetZ
85 mm
HipOffsetY
50 mm
ThighLength
100 mm
TibiaLength
102,90 mm
FootHeight
45,19 mm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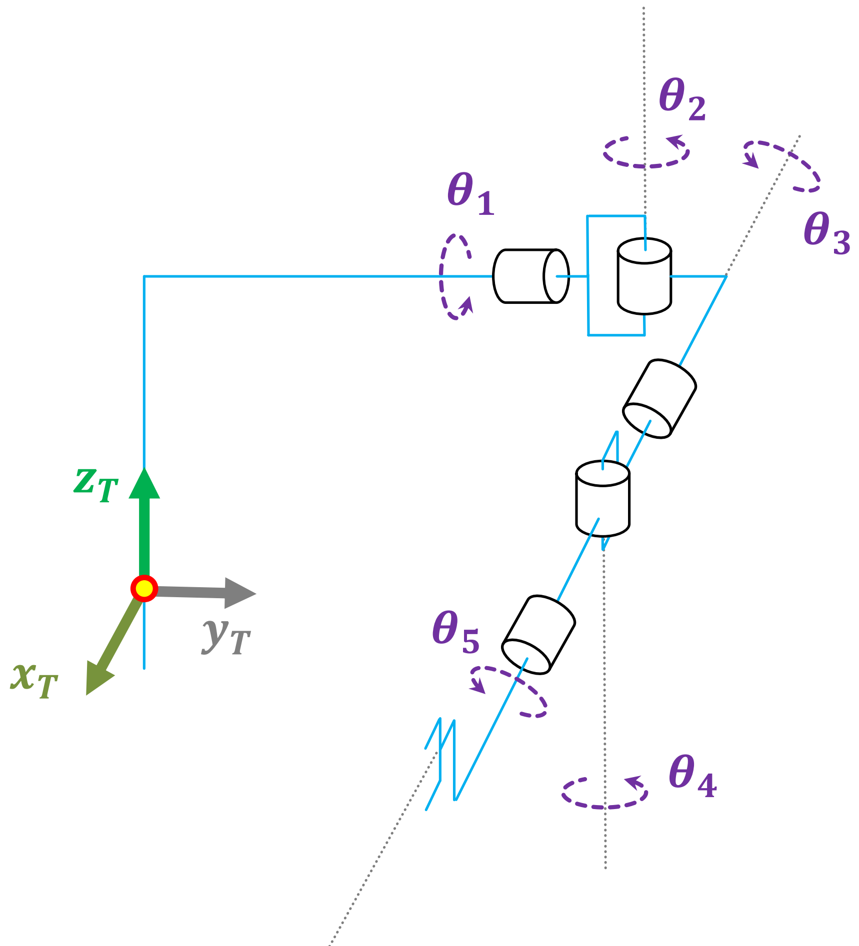
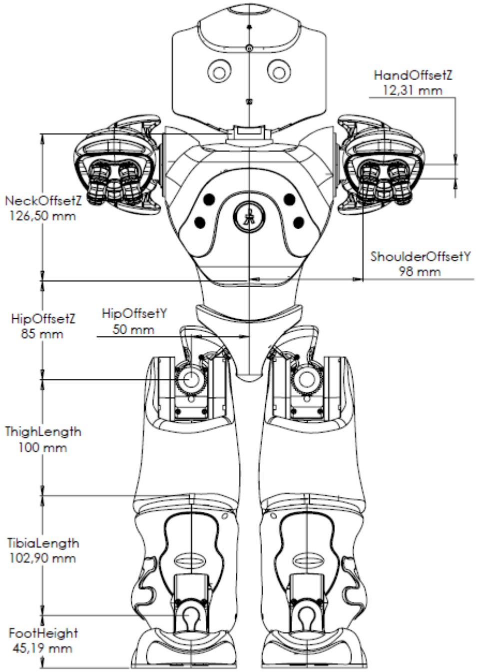

θ_2
θ_1
θ_3
z_T
y_T
x_T
θ_5
θ_4

NAO Left Arm



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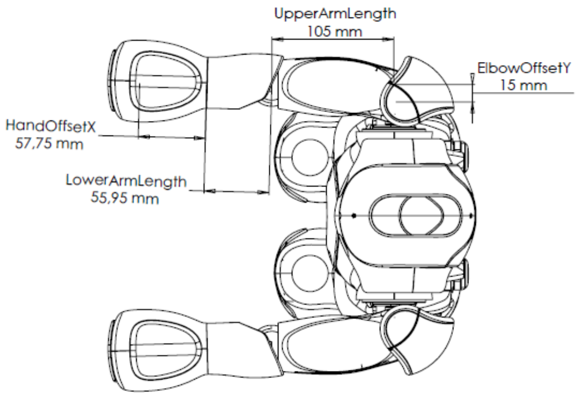

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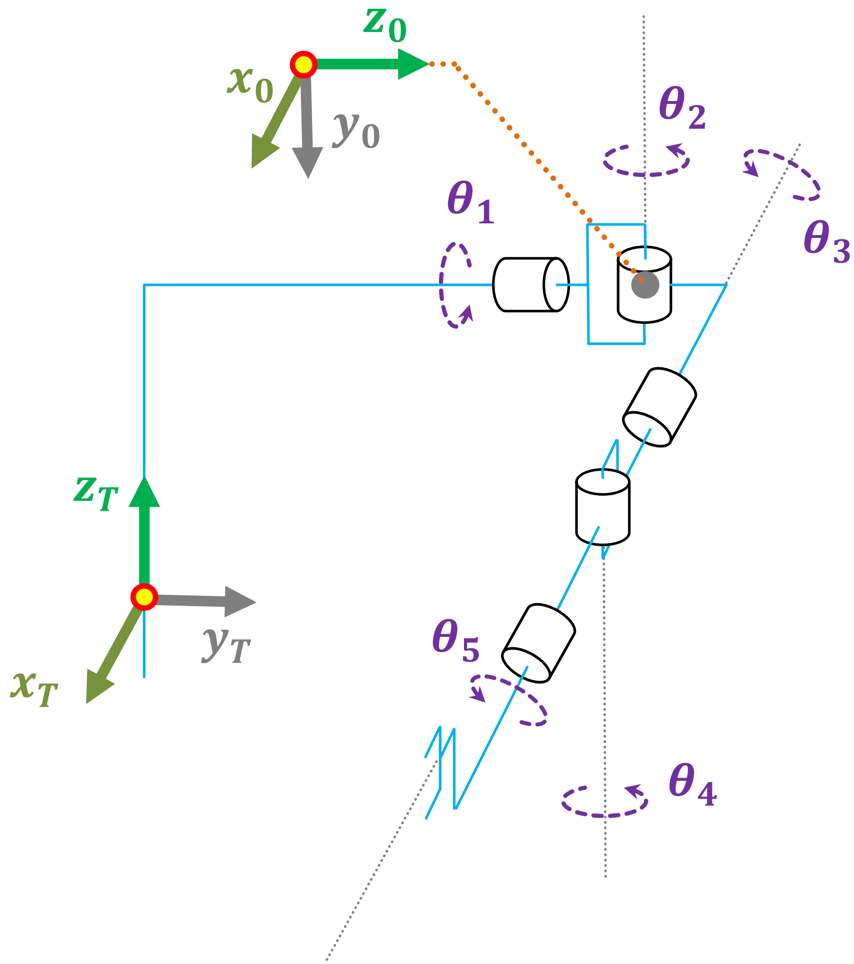
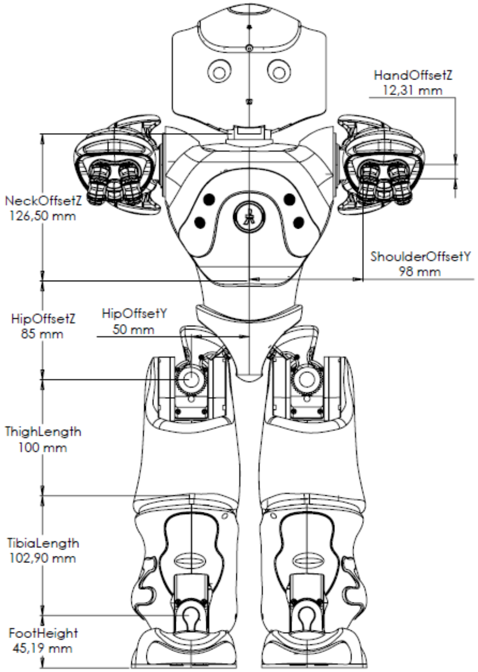


z_0
x_0
y_0
θ_1
θ_2
θ_3
z_T
y_T
x_T
θ_5
θ_4

NAO Left Arm



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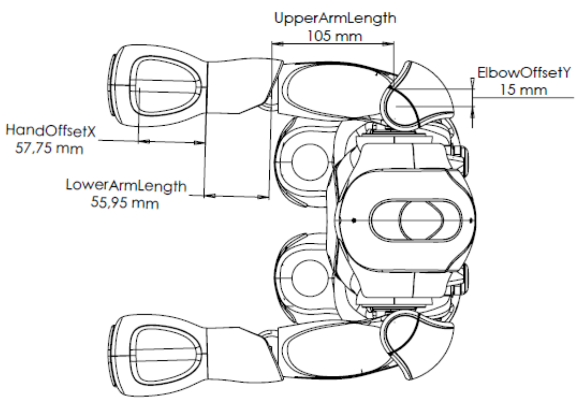

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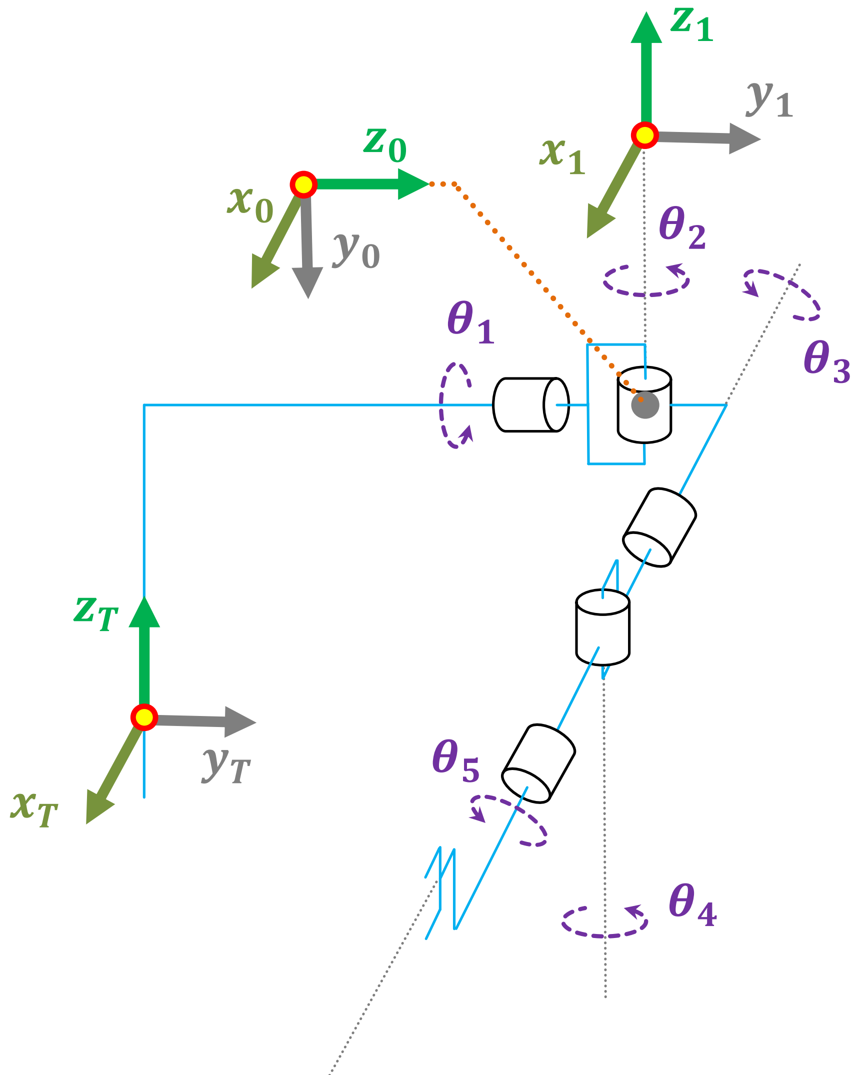
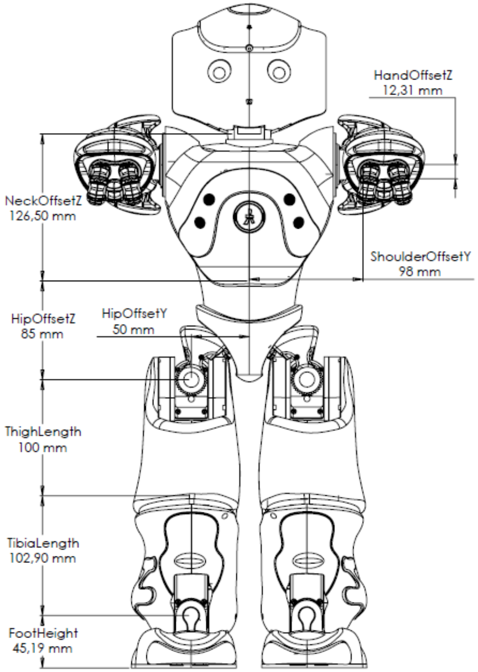


z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
z_T
y_T
x_T
θ_5
θ_4

NAO Left Arm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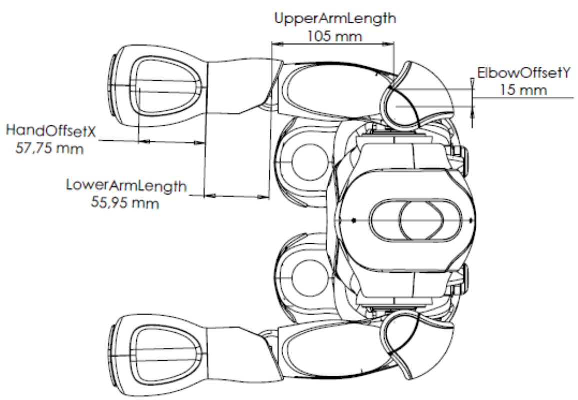
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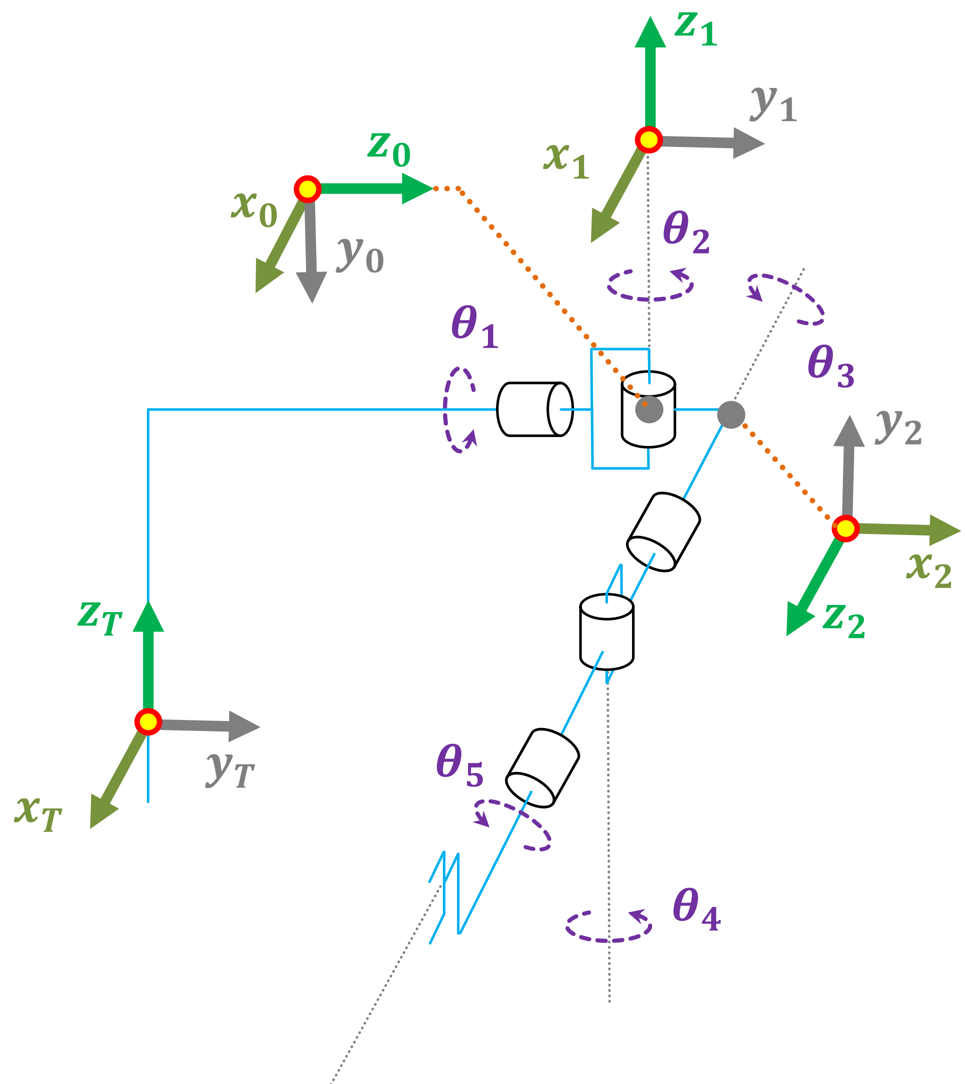
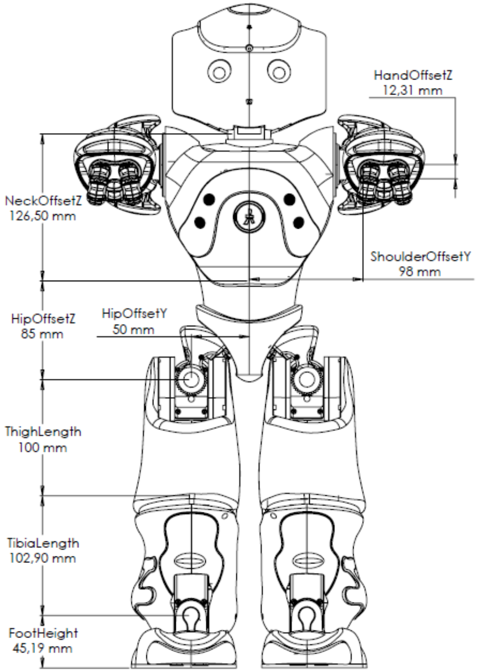

z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
y_2
x_2
z_2
z_T
y_T
x_T
θ_5
θ_4

NAO Left Arm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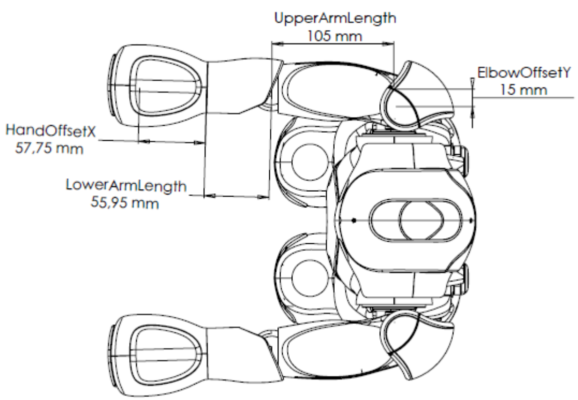
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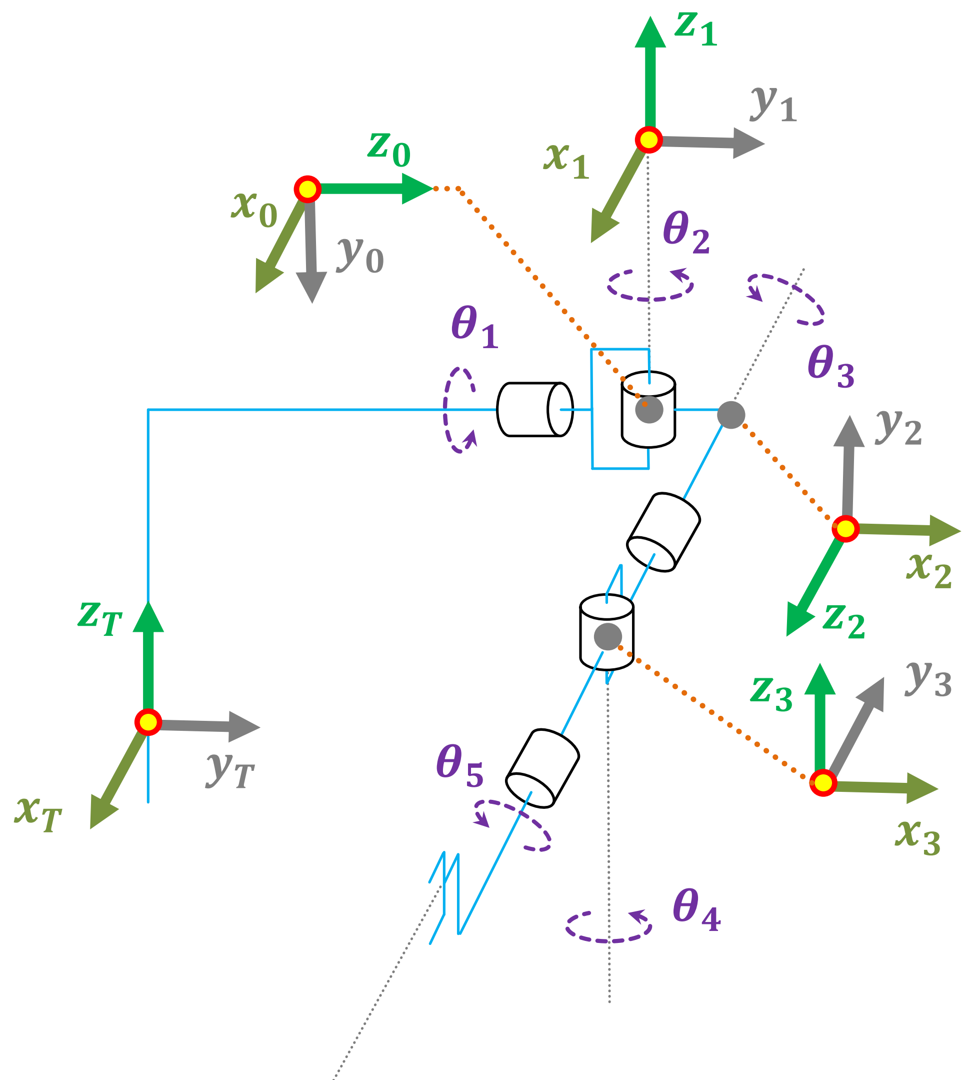
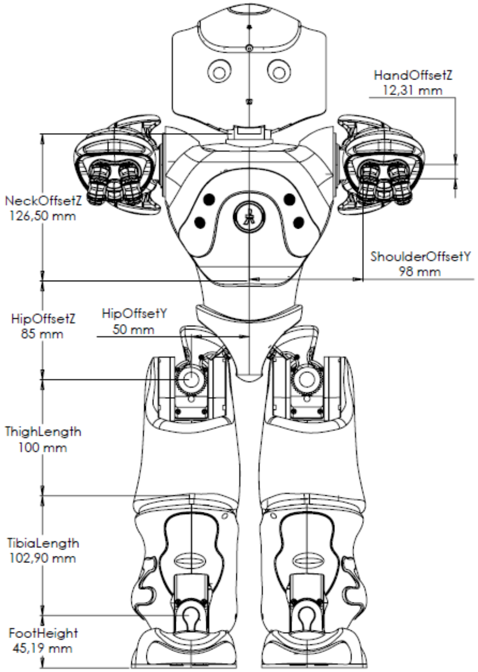

z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
y_2
x_2
z_2
z_T
y_T
x_T
θ_5
z_3
y_3
x_3
θ_4

NAO Left Arm



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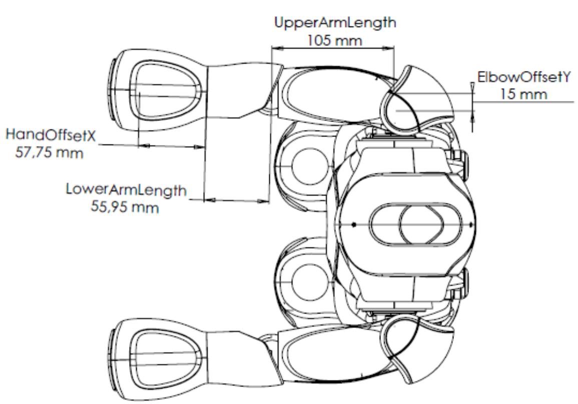

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LowerArmLength
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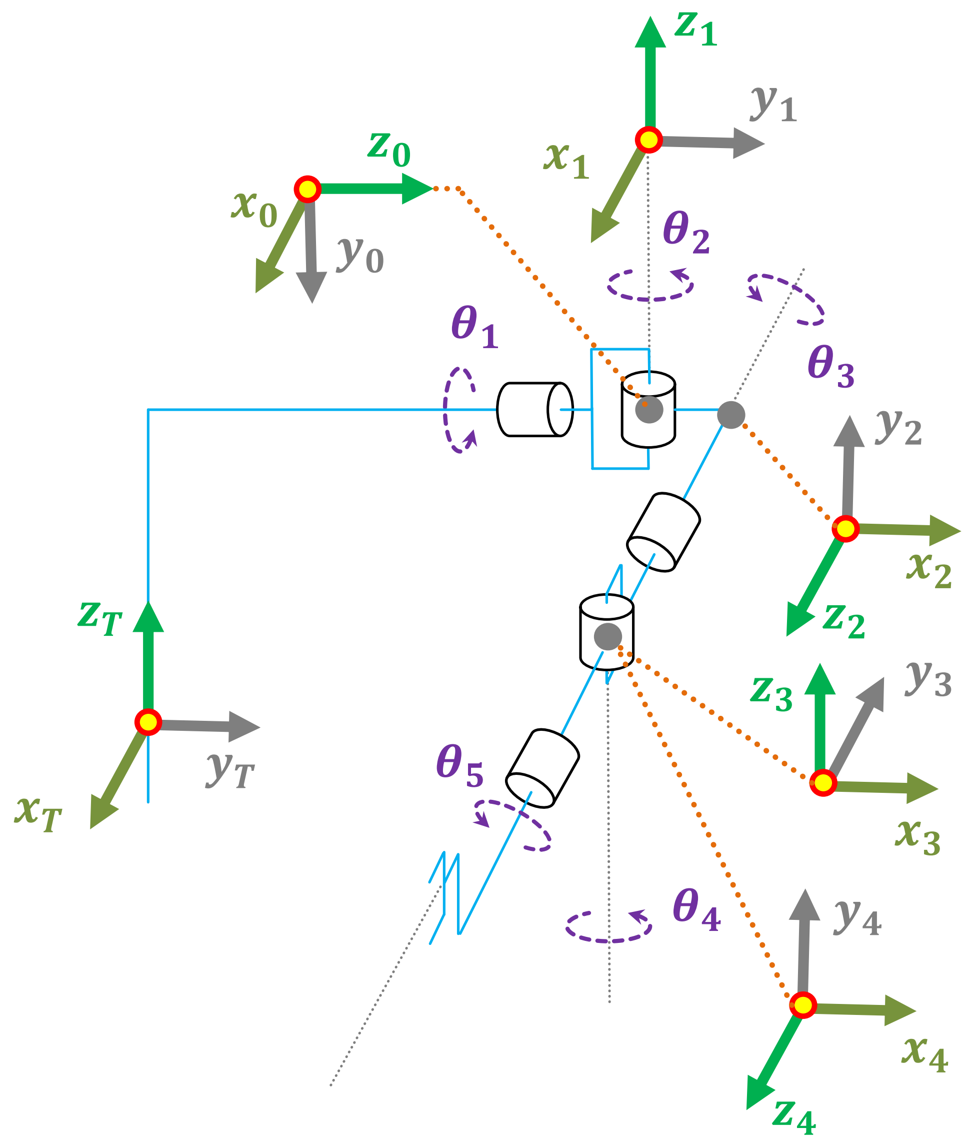
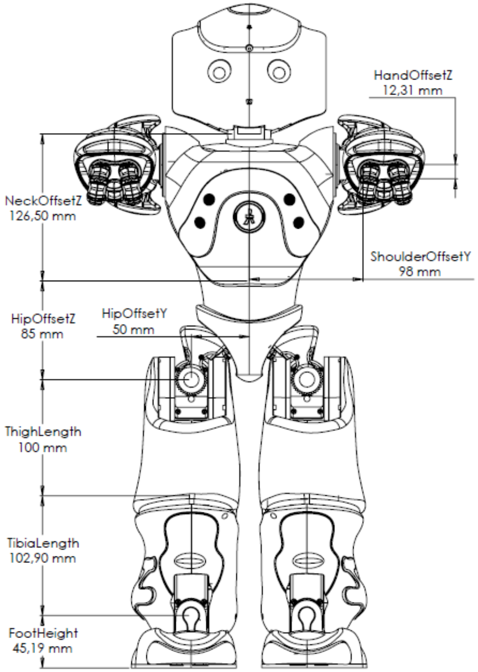


z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
y_2
x_2
z_2
z_T
z_3
y_3
x_3
θ_5
y_T
x_T
θ_4
y_4
x_4
z_4

NAO Left Arm



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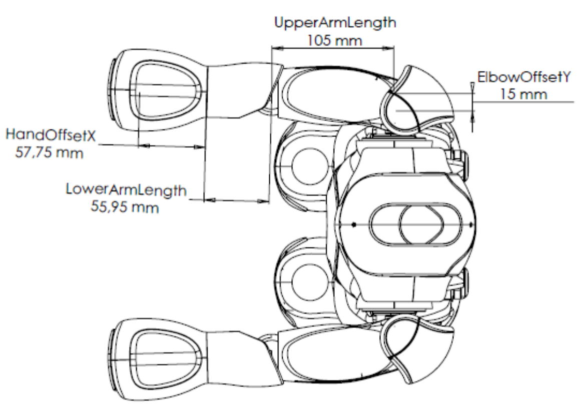

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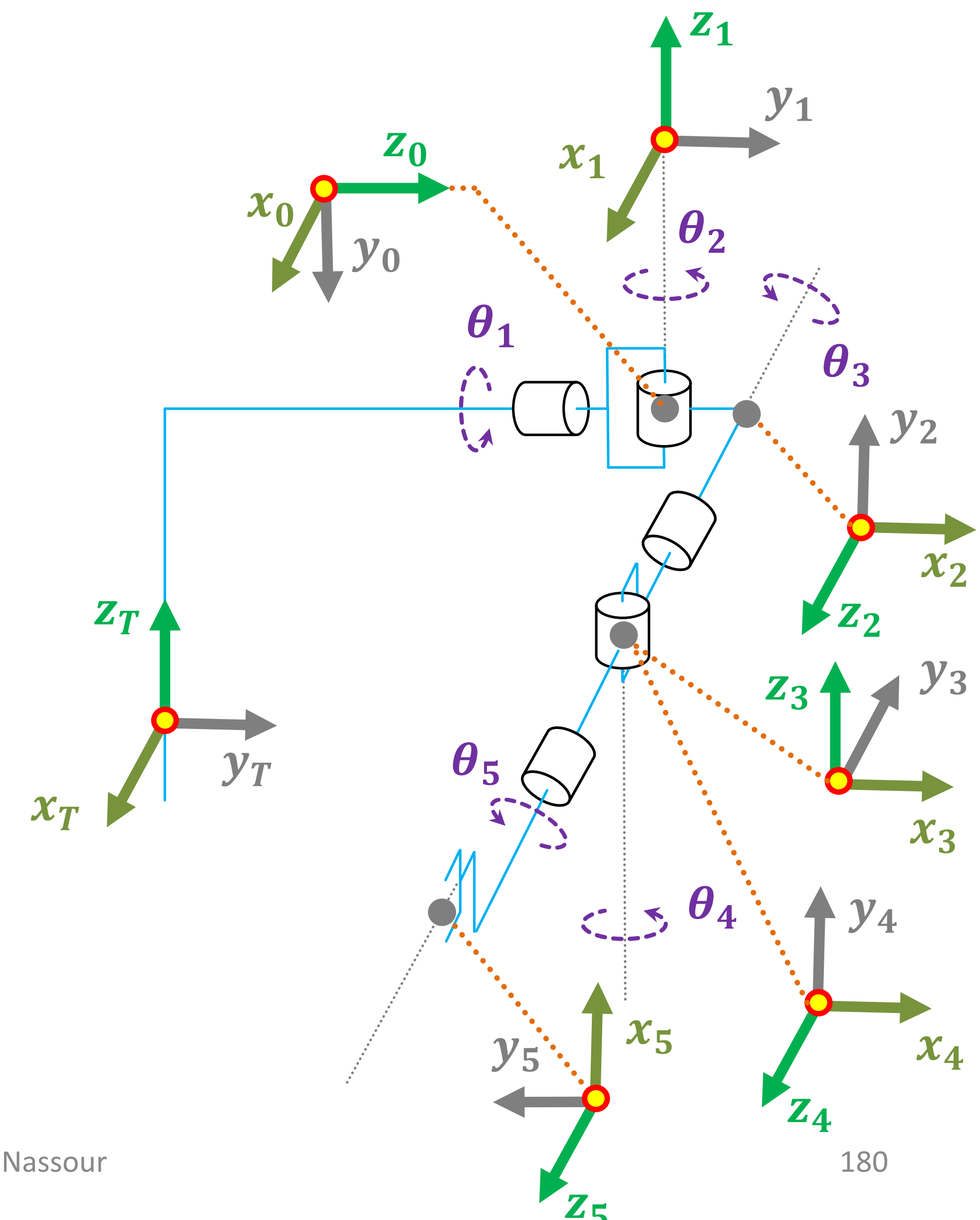
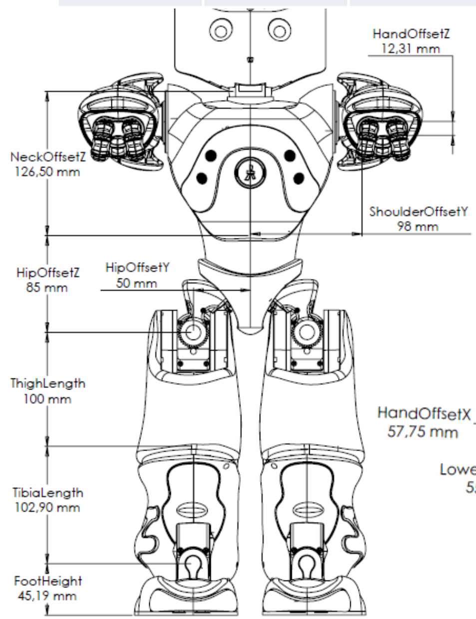


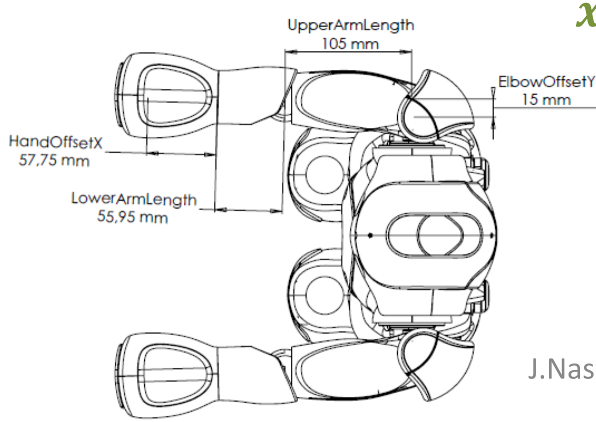
z_1
y_1
x_1
z_0
x_0
y_0
θ_2
θ_1
θ_3
y_2
x_2
z_2
z_T
y_T
x_T
z_3
y_3
x_3
θ_5
θ_4
y_4
x_4
z_4
x_5
y_5
z_5

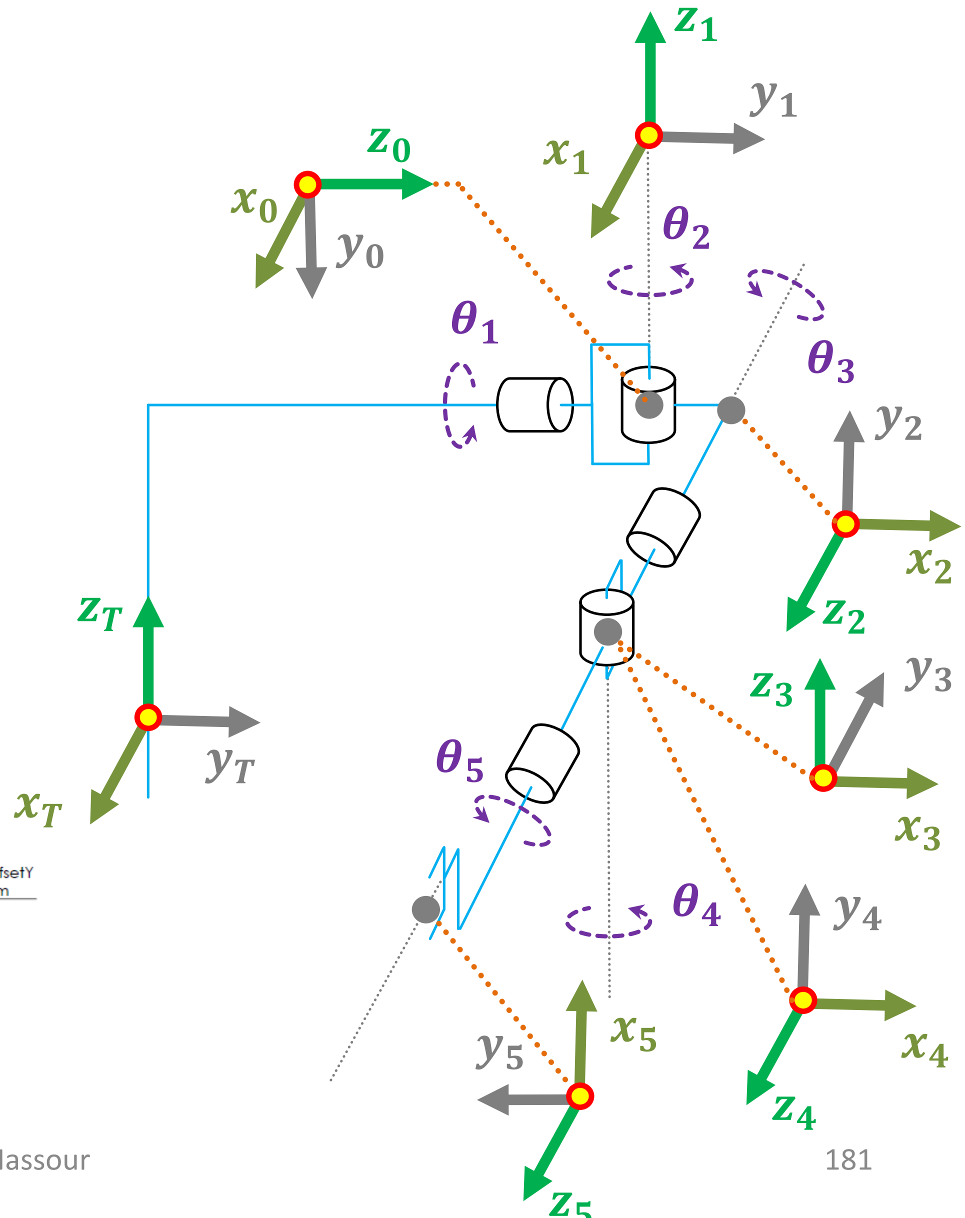
NAO Left Arm

a_i is distance from z_{i-1} to z_i measured along x_i.
α_i is angle from z_{i-1} to z_i measured about x_i.
d_i is distance from x_{i-1} to x_i measured along z_{i-1}.
θ_i is angle from x_{i-1} to x_i measured about z_{i-1}.

i	a_i	α_i	d_i	θ_i
0	T_0^{BASE} ((			
1				
2				
3				
4				
5				







NAO Left Arm

$\boldsymbol{a_i}$ is distance from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured along $\boldsymbol{x_i}$.
$\boldsymbol{\alpha_i}$ is angle from $\boldsymbol{z_{i-1}}$ to $\boldsymbol{z_i}$ measured about $\boldsymbol{x_i}$.
$\boldsymbol{d_i}$ is distance from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured along $\boldsymbol{z_{i-1}}$.
$\boldsymbol{\theta_i}$ is angle from $\boldsymbol{x_{i-1}}$ to $\boldsymbol{x_i}$ measured about $\boldsymbol{z_{i-1}}$.

i	a_i	α_i	d_i	θ_i
0	$T^{BASE}_{0}(0, ShoulderOffsetY, ShoulderOffsetZ)$			
1	0	90 °	0	θ_1 *
2	a_2	90 °	0	$\left(\frac{\pi}{2}\right) + \theta_2$ *
3	0	-90 °	d_3	θ_3 *
4	0	+90 °	0	θ_4 *
5	a_5	0 °	d_5	$\left(\frac{\pi}{2}\right) + \theta_5$ *

$$T^{BASE}_{5} = ?$$