Analyzing the Impact of Probabilistic Estimates on Communication Reliability at Intelligent Crossroads

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***Abstract*—Intelligent crossroads aim to substitute conventional traffic lights by coordinating the order in which vehicles cross an intersection. Since vehicles come and go at arbitrary points in time, this results in an open-ended setting that is difficult to analyze with deterministic methods. In particular, deterministic methods fail to provide meaningful estimates of the maximum number of vehicles at the intersection, which is paramount to assess communication reliability and, in the end, guarantee safety. In contrast, statistical and probabilistic techniques are more suitable for this purpose and constitute the focus of this paper. We especially investigate how different driving directions and vehicle lengths influence the quality of probabilistic estimates in approximating the maximum number of vehicles at the intersection. These estimates are then incorporated into the design and analysis of the crossroad VANET to derive guarantees on communication reliability. Our results show that such estimates can greatly reduce pessimism and overdesign compared to deterministic approaches. These and other benefits are illustrated by means of a detailed case study and simulations using OMNeT++.**

I. Introduction

Open-ended cyber-physical systems (CPS) are becoming increasingly common in various areas such as smart homes and buildings, intelligent traffic and transportation systems, smart grids, etc. Their defining feature is a constantly changing number of autonomous actors who can join and leave the system at any time. In general, this makes it difficult to estimate the maximum number of actors in such settings, i.e., the *maximum actor count*. However, on the other hand, this is a crucial factor for design and analysis, particularly under safety requirements, since it directly affects computation and communication overhead.

In this paper, we are concerned with an intelligent crossroad, which coordinates traffic flow with the aim of improving throughput and reducing congestion. The idea is to replace traditional traffic lights by a roadside unit (RSU) that assigns speed values to approaching vehicles so that these can traverse the intersection safely without unnecessary braking and accelerating. This requires the RSU to periodically calculate new speed values and communicate them to each vehicle. Clearly, it is paramount to guarantee a reliable communication between vehicles and the RSU, which requires a specialized Vehicular Ad Hoc Network (VANET).

In general, communication reliability strongly depends on the number of vehicles at the intersection. The more vehicles there are, the more interference these produce leading to a higher chance that data packets are lost, consequently, reducing the VANET's reliability.[1] Note that using mechanisms to avoid interference, e.g., by synchronizing nodes using TDMA (Time Division Multiple Access), yields a huge overhead due to the constantly changing operation conditions and is not suitable for this application.

In order to guarantee safety, we need to estimate the maximum number of vehicles at the intersection. Deterministic approaches assume the worst-case number of vehicles, i.e., that the intersection is completely filled with vehicles, which leads to a considerable amount of pessimism and overdesign. Probabilistic approaches, on the other hand, generally yield better results, since they better match the random nature of such settings. That is, many system properties are stochastic, e.g., traffic density, packet loss, interference, but also vehicle length among others. However, these probabilistic approaches do not provide absolute certainty, i.e., there will be always a (rather small, but non-zero) probability that the estimated maximum number of vehicles is exceeded and, hence, that any safety guarantees based on them will stop being valid at some point in time.

In order to avoid accidents, probabilistic estimates can be combined with fail-safe mechanisms. If, for example, the estimated maximum number of vehicles is exceeded, the intelligent intersection can switch back to behave as conventional traffic lights. Although this temporarily reduces service/usability (i.e., once safety conditions are restored, the intelligent crossroad will resume its intended operation), it maintains safety. In addition, the loss of usability can be reduced to a minimum by selecting the right probabilistic estimates, which is the ultimate goal of this work.

Contributions: In this paper, we propose an approach to probabilistically estimate the number of vehicles at an intersection. To this end, we make use of a traffic protocol (controlling the order in which vehicles cross the intersection) and analyze the space requirements of different actions such as left/right turns, driving through, etc., to derive an estimate of the vehicle count. This estimate is further extended by considering the following:

- Vehicles have different probabilities to turn left/right or drive through at the intersection.
- Vehicles have different lengths which can be described statistically.

[1] We consider only systemic interference caused by simultaneous transmissions. However, adding external interference is straightforward, assuming that its upper bound is known.

- The intersection is not completely filled with vehicles during hours of low traffic.

Our probabilistic estimate is then incorporated into the design and analysis of a specialized VANET, for which we derive guarantees of reliability. Since these guarantees are also of a probabilistic nature, there is always a residual risk that they will not be met, which should be contained by fail-safe mechanisms as mentioned above.

Lastly, the advantages of the proposed approach are discussed with a detailed case study and simulations using OMNeT++ [15].

Structure of this work: Section II discusses the state of the art, while Section III covers the underlying models and assumptions introducing a crossroad example used for later analysis. Section IV deals with the used traffic protocol and derives probabilistic estimates for the vehicle count. Next, Section V describes the used VANET and Section VI evaluates its communication reliability. Finally, Section VII concludes the paper.

II. Related Work

Intelligent crossroads are a relatively new application. As a result, only a few works (mentioned in the following) investigate design and analysis methods for this case. Generally, existing work can be divided into two types: decentralized protocols where vehicles communicate directly with each other, and centralized approaches where vehicles communicate with a dedicated component, e.g., a roadside unit.

Decentralized approaches are discussed in detail in [2]. Current works towards distributed intersection management are [12] and [7]. In the former, virtual platooning is used to greatly increase throughput and reduce waiting times. In the latter, vehicles are grouped into clusters based on their locations and driving directions. These clusters select a cluster head, which performs communications with other clusters and its members. The clusters then pass the intersection one after another. While this increases throughput compared to traditional, non-clustered traffic, it does not allow crossing in more than one direction at the same time. Furthermore, communication safety is not addressed apart from packets being sent twice to mitigate packet loss. Lastly, both works [7][12] do not consider the number of vehicles in the intersection range as a factor for communication reliability.

Towards centralized approaches, the general idea of scheduling cars via an intersection manager was first proposed in [3][4]. Here, vehicles are coordinated by a traffic management system based on reservations, leading to synchronized crossing patterns. In this case, to obtain the maximum number of vehicles, the crossroad is assumed to be completely filled with vehicles according to a reservation logic which fixes the inter-vehicle spacing. For example, a centralized approach is presented in [5], which implements a bi-level controller to coordinate automated vehicles at intersections. However, this approach does not discuss how to estimate the maximum number of vehicles in the system so as to account for reliability.

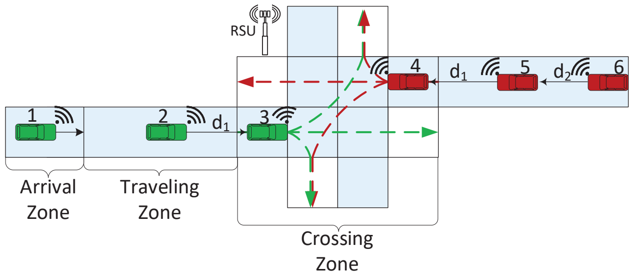


Fig. 1: An exemplary intelligent crossroad with 4 lanes in which new vehicles arrive periodically at the west/east lane.

To estimate the maximum number of vehicles at the crossroad, deterministic approaches have been used in the past [10][11]. However, these approaches do not consider the effects of crossroad protocols, vehicle lengths, travel directions etc. on the number of vehicles. On the contrary, they are based on restrictive simplifications that aim to guarantee safety at the cost of high pessimism.

To the best of our knowledge, probabilistic approaches for estimating the maximum number of vehicles at a crossroad have been first used in [9]. However, in this approach, no turning behavior or traffic density is considered and the crossroad was restricted to a single lane only, which clearly does not reflect real-world crossroads. In this paper, our goal is to remove these restrictions and extend the work in [9] to improve estimates of the vehicle count at the crossroad.

III. Background Knowledge and Definitions

As mentioned earlier, the idea behind an intelligent intersection is to replace traditional traffic lights with a RSU that coordinates vehicles to cross the intersection safely and efficiently. Fig. 1 shows such an exemplary intersection consisting of four lanes (north, east, south and west), which we will use in the following as a basis for our analysis. Note that, for the sake of exposition, we assume that vehicles arrive only in the west/east lane, while the north/south lane is for exiting only. However, our analysis can easily be extended for more complex intersection types with multiple lanes as shown later.

Whenever a vehicle comes within a range R of the intersection, it enters the *arrival zone* and must first register at the RSU — in our example, we set $R = 150\,\text{m}$. To this end, the vehicle sends an identifier containing its vehicle type, mass, length, etc. as well as speed and desired direction, i.e., whether it wants to turn left, right or drive through. Once a vehicle is registered, the RSU keeps tracking it and controlling its speed by periodically broadcasting control messages containing speed values for all vehicles that are currently within range. After registration, the vehicle then enters the *traveling zone* where it drives at the given speed and keeps a certain distance d to its front vehicle. This way, all approaching vehicles can be synchronized to pass the intersection efficiently and collision-free [1][8]. Lastly, at the intersection itself, the *crossing zone*, vehicles can either drive through or turn left or right — this is denoted by T, L and R respectively. Note that once a vehicle leaves the intersection, it is automatically de-registered by the RSU and not regarded anymore.

Each vehicle has to keep a minimum distance d_{min} to its front vehicle, which depends on the crossroad layout and protocol used, as explained in the following section in detail. To this end, we divide the intersection into sectors/cells to create a grid-like structure, which is a common procedure for centralized crossroads [2][12][14]. The length S of each sector is a multiple of d_{min} and stands for the distance each vehicle travels within a given unit of time we call *cycle*. This allows us to design crossroad protocols independent of speed, provided that all vehicles have the same speed and keep it constant — this is enforced in the *traveling zone*. For simplicity, we assume that S is equal to a standard vehicle's length (e.g., 5m). In addition, we also assume that the longest vehicle — a truck — is at most 10m long, i.e., it fits into $2S$.

Unlike deterministic approaches, probabilistic ones cannot guarantee full safety, i.e., there is always a residual chance that estimates are not met. For this reason, a fail-safe mechanism must be implemented, e.g., the crossroad can switch to a conventional traffic light's behavior. Clearly, the system should be designed properly such that fail-safe mechanisms are not overused, but remain reserved for rare exceptions. This way, full safety can be achieved using probabilistic estimates, while at the same time reducing overdesign.

IV. Probabilistic Estimates for Vehicle Count

In this section, we derive probabilistic estimates for the vehicle count within an intersection. To this end, we first briefly introduce the traffic protocol from [8] and apply it to our crossroad example from Fig. 1. Next, we add probabilistic properties regarding turning behavior, vehicle length and whether vehicles are present or absent on a lane. Lastly, we provide an analytical framework to calculate probabilistic estimates of vehicle counts and discuss how to extend these to different, more complex intersection types.

A. Traffic protocol

In order to derive probabilistic estimates for the number of vehicles, we first need a suitable traffic protocol that fixes traffic throughput, required space, etc. at the intersection. To this end, we make use of the protocol from [8], which describes a centralized, RSU-based approach.

According to this protocol, vehicles cross the intersection in synchronized *pairs* on opposing lanes, i.e., in our example from Section III this means that two vehicles cross at a time. Since vehicles on opposing lanes cross simultaneously, there are multiple turn combinations that have to be executed in the crossing zone, each of which requires a specific number of cycles. Note that this number of cycles, called *cycle costs*, define the distance d to the following pairs of vehicles, see also Fig. 1. This ensures that the current pair of vehicles leaves the intersection before a new one arrives.

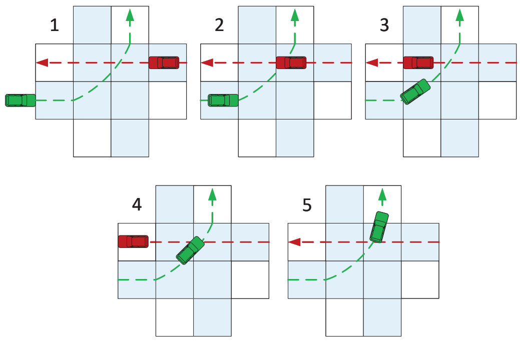


Fig. 2: Combination LT/TL in detail: left turns and through driving have a collision point, which requires the left turning vehicle to be delayed by one cycle.

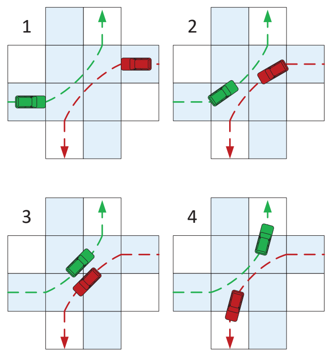


Fig. 3: During an LL combination, left turning vehicles cross simultaneously in front of each other.

The distance d allows us to draw conclusions about the total number of vehicles that fit in a radius R from the intersection. However, for an accurate estimate, other factors must be considered as well, for example, turning behavior as explained next.

B. Considering driving direction

At the center of the intersection, each vehicle can either turn left/right or drive through (L,R,T). Considering two lanes, this results in $3^2 = 9$ different combinations: LL, LT, LR, TL, TT, TR, RL, RT, RR. However, since vehicles are synchronized and their speeds and lengths are the same, some of these turning combinations are identical and actually only mirrored. That is, LT equals TL, LR equals RL, and RT equals TR, resulting in 6 distinct combinations in total. In the following, we will discuss these different combinations in more detail and analyze their cycle costs.

LT/TL is the only combination with overlapping trajectories between the opposing vehicles, see Fig. 2. Here, the left turn vehicle has to be delayed by an entire cycle before entering the crossroad center such that it does not collide with the vehicle driving through. Even though the vehicle driving through has exited the crossroad center after 3 cycles, the whole crossing procedure ends after 5 cycles when the left turning vehicle also exits — during that time, no new vehicles can enter the intersection.

When considering the combination LL, vehicles cross in front of each other to avoid potential collision points and therefore further delay. Here, it should be noted that the vehicles' widths have to be considered, i.e., depending on widths and on the sector size S, vehicles might come too close to each other as shown in Fig. 3. However, as long as the average width of both vehicles is below a critical distance, no modifications are necessary [8]. Note that, for simplicity, we disregard exceptionally wide vehicles.

Lastly, the remaining combinations focus on through driving and right turns, which do not influence each other and can therefore be considered independently of each other. A right turn allows exiting the crossroad after 2 cycles, whereby driving through requires 3 cycles. Again, each turn combination can be assigned a cycle cost, i.e., how many cycles have to pass before the last vehicle leaves the crossroad and a new pair can enter.

Now we can also assign a probability to the driving direction of vehicles, i.e., how likely it is on average that a vehicle follows a given trajectory at the intersection. This probability can be derived from statistical data, e.g., by observing traffic flow in the intersection over a sufficiently long period of time. For example, in [6], the eastbound traffic of the city of Redmond was observed for a year and probabilities were determined to be $p_L = 0.3$, $p_T = 0.6$ and $p_R = 0.1$ for turning left, turning right and driving through respectively. For example, the probability of combination TT is that vehicle 1 and 2 both drive through, i.e., $p_{TT} = p_T \cdot p_T = 0.36$. For mirrored combinations, both must be considered, e.g., $p_{TR} = p_T \cdot p_R + p_R \cdot p_T = 0.12$. Knowing the probability of all turns allows us to estimate the likeliness that certain cycle costs are encountered and, therefore, allows drawing conclusions on the maximum vehicle count.

C. Considering traffic density

So far, we have assumed that the intersection is completely filled with vehicles and that always a pair of vehicles cross at the same time. However, in case of low traffic, i.e., when the arrival rate of vehicles is lower than the exiting rate at the intersection, it may happen that one vehicle is absent in a pair and only one vehicle crosses the intersection at a time — we denote this as a *half-pair*. In this case, the crossing behavior is executed as efficiently as the driving direction allows. Note that we assume that there is at least one vehicle crossing, since, if both vehicles are absent, nothing happens and cycle cost is zero.

The probability p_E of having half-pairs is lower bounded by 0, i.e., the intersection is completely filled with cars, and upper bounded by $p_E = 1$, i.e., there are half-pairs only. Now, in order to determine p_E, we have to consider the arrival rate r_{arr} in relation to the exiting rate r_{exit}. In particular, if there are only half-pairs at the intersection, i.e., $p_E = 1$, then p_{arr} must be equal to $\frac{1}{2} p_{exit}$. Similarly, if there are no half-pairs

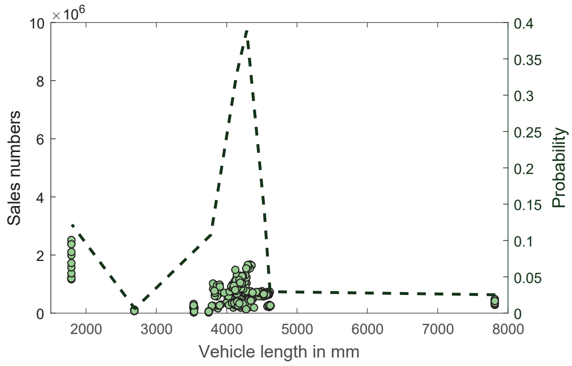


Fig. 4: Vehicle length distribution

at the intersection, i.e., $p_E = 0$, p_{arr} must be equal to p_{exit} in order that no traffic jam originates. (Otherwise, more vehicles enter than actually leave the intersection.) Now, modeling p_E as a linear function of r_{arr} results in the following:

$$p_E = 2 \cdot \left(1 - \frac{r_{arr}}{r_{exit}}\right) \tag{1}$$

Note that the exiting rate is fixed for a given crossroad, i.e., this describes the maximum number of vehicles that can leave the crossroad per cycle. The arrival rate, on the other hand, is variable and depends on several factors such as crossroad layout, time, location, etc. [6]. In our example, we set the arrival rate to 95% of the exiting rate, which results in $p_E = 2 \cdot (1 - 0.95) = 0.1$. However, later in Section VI, we will also investigate different values of p_E.

D. Considering vehicle lengths

Next, let us discuss the effects of vehicle lengths on our probabilistic estimates. For this purpose, let us first look at Fig. 4, which shows the various vehicle lengths and their market share on the basis of car sales statistics from [9]. As we can see here, there are three main groups of vehicles: motorcycles, cars, and trucks/buses, with cars making up the majority of vehicles. As a result, it is very likely to encounter a car at the intersection, whereas it is relatively unlikely to encounter long trucks and buses.

In the case of a deterministic approach, all vehicles are assumed to have the length of the longest vehicle, i.e., that of a truck or bus, despite of the low probability to encounter these. Clearly, this leads to a very high pessimism and overdesign. Probabilistic estimates, on the other hand, can reduce this pessimism substantially as discussed next.

Now, in order to derive the impact of vehicle lengths on our probabilistic estimates, let us first discuss the effects on the traffic protocol from [8]. Here, whenever a vehicle is longer than the sector length S — which was set to 5 m in our example — it is considered to be an overlength (OL) vehicle. Note that, for simplicity, overlength is discrete and can be expressed in multiples of S. Now, whenever there is an overlength vehicle present — which has the probability p_{OL} — the protocol adds extra cycles to its turns to prevent possible collisions. For every S that the OL vehicle is longer than one sector, the protocol adds one cycle to its cycle costs, regardless of the driving direction (e.g., L, T or R). For example, a truck is $2S$ long and the protocol adds one cycle to its cycle costs. Note that if there are two OL vehicles in one pair of vehicles, extra cycles are added only once based on the longer vehicle.

Now, we can assign each turning combination or combination of driving directions a cycle cost g. These range from $g = 2$ for RR to $g = 6$ for TL/LT with OL penalty and are shown in Table I.

No OL	**West**			
East	**R**	**T**	**L**	**E**
R	2	3	4	2
T	3	3	5	3
L	4	5	4	4
E	2	3	4	-

OL	**West**			
East	**R**	**T**	**L**	**E**
R	3	4	5	3
T	4	4	6	4
L	5	6	5	5
E	3	4	5	-

Table I: Cycle cost g for each combination of driving directions with and without overlength (OL) penalty. E stands for an *empty* slot on one of the lanes, i.e., only one vehicle crossing at a time. We refer to this situation as a *half-pair*.

With exemplary values of $p_E = 0.1$, $p_L = 0.3$, $p_T = 0.6$, $p_R = 0.1$, $R = 150m$, we can determine the different cases and their occurrence probabilities as shown in Table II.

Cycle Cost (g)	**Probability**
2S	0.0185
3S	0.4800
4S	0.1733
5S	0.3200
6S	0.0082

Vehicles (n)	**Probability**
1	0.1
2	0.9

	Weighted Average
Cycle Cost	3.8193
Vehicles	1.9

Table II: Costs in cycles and probability of occurence for for a single pair of vehicles

E. Estimating vehicle count

To estimate the vehicle count within an intersection, we have to consider all different combinations of vehicles within the intersection's range R. Each pair has a certain cycle cost g which defines the number of cycles required for turning, driving through, etc. and, therefore, defines the distance to the following pair of vehicles. These distances can then be used to calculate how many pairs physically fit into the intersection's range R. Note that, since lanes are synchronized, the distances on each lane are the same — see again Fig. 1.

First, let us derive bounds for our probabilistic estimates, beginning with the the upper bound n_{max}. This determines the maximum vehicle count at the intersection and can be derived using a deterministic approach. To this end, we assume that all vehicles have standard length S, there are no empty slots (i.e., half-pairs) and only RR turns are performed, which have the lowest cycle costs of $g = 2$. This results in:

$$n_{max} = 2 \cdot \left\lceil \frac{R}{\min\limits_{\forall i}(g_i) \cdot S} \right\rceil . = 2 \cdot \left\lceil \frac{R}{2S} \right\rceil . \quad (2)$$

To determine the lower bound on the vehicle count n_{min}, we consider that all vehicles have the maximum length of $2S$ (e.g., trucks), there are only half-pairs, and vehicles follow trajectories with the greatest cycle costs, i.e., TL/LT with $g = 5$. This results in:

$$n_{min} = \left\lceil \frac{R}{\max\limits_{\forall i}(g_i) \cdot S} \right\rceil = \left\lceil \frac{R}{5S} \right\rceil . \quad (3)$$

When generating different combinations of pairs of vehicles, we must ensure that these fit into the crossroad's range R according to the used traffic protocol. Following [9], a *valid combination* of vehicles must meet the following condition:

$$R - 2S < k_2 \cdot 2S + k_3 \cdot 3S + k_4 \cdot 4S + k_5 \cdot 5S + k_6 \cdot 6S < R,$$

where k_2, k_3, k_4, k_5 and k_6 represent the amount of combinations leading to a total inter-vehicle distance of $2S, 3S, 4S, 5S$ and $6S$ respectively. The above equation can be generalized to the following:

$$D = \sum_{j=\min\limits_{\forall i}(g_i)}^{\max\limits_{\forall i}(g_i)} k_j \cdot j \cdot S. \quad (4)$$

If $R - 2S < D < R$ is fulfilled, the given vehicle set is a valid combination, i.e., it fits into the intersection's range R and is greater than $R - 2S$ meaning that no further vehicle fits into it.

Now, in order to estimate the likeliness for a certain n to occur, we have to consider the probabilities leading to that case. More specifically, we have to consider the probability of a vehicle having overlength p_{OL}, the probability that pairs of vehicles perform a certain combination of turns/follow certain driving directions p_C, and the probability of having a half-pair p_E. Combining these, we can calculate the probability p_n of having a given n by:

$$p_n = p_{OL,n} \cdot p_{C,n} \cdot p_{E,n}. \quad (5)$$

The probability p_{OL} can be derived from Fig. 4, i.e., by applying the distribution from [9]. In the case of the combined turning/driving probabilities p_C, these can be calculated by multiplying individual such probabilities, e.g., $p_{TT} = p_T \cdot p_T$ or $p_{LT} = p_L \cdot p_T + p_T \cdot p_L$ — see again Section IV-B. Lastly, the probability of having half-pair of vehicles p_E can be obtained by using (1) together with a traffic observation or prediction leading to an arrival vehicle rate r_{arr} [6].

To illustrate the above formulas and derive values for later experiments in Section VI, let us now calculate the different probabilistic estimates for our crossroad example of Fig. 1. Assuming an intersection range of $R = 150m$ and a sector size of $S = 5m$, the maximum number of vehicles ranges between 6 and 30, with 30 being equivalent to the deterministic worst case. Analyzing all k_2, k_3, k_4, k_5 and k_6 that fulfill these combinations, we can calculate the estimates for n with $n \in [6, 30]$ and their corresponding probabilities p_n, as shown in table III. Note that these results clearly show the pessimism

Vehicle Count	Probability	Vehicle Count	Probability
<6	0	18	3.7074E-05
6	4.6859E-14	19	9.9612E-08
7	1.2831E-10	20	1.2186E-07
8	1.0539E-07	21	3.8954E-10
9	3.3684E-05	22	3.9974E-10
10	0.00406149	23	5.4333E-13
11	0.14303233	24	4.6334E-13
12	0.77750801	25	5.6528E-17
13	0.02003493	26	1.9801E-17
14	0.05526484	27	3.3938E-22
15	5.1611E-07	28	7.1491E-23
16	4.9100E-06	29	1.1927E-29
17	2.1892E-05	30	1.7865E-30

Table III: Resulting maximum number of vehicles with their corresponding probabilities for $p_E = 0.1$, $p_L = 0.3$, $p_T = 0.6$, $p_R = 0.1$, $R = 150m$ and $S = 5m$.

of deterministic approaches. That is, the probability of having $n = 30$ is 1.78×10^{-30}, i.e., it will effectively never occur. The probabilistic estimate, on the other hand, can be chosen according to a given safety level, i.e., a probability that the estimate fails. For example, when selecting $n = 19$, the chance that there is a higher n at the crossroad is less than 1×10^{-7}.

In summary, this section described how to derive probabilistic estimates on the maximum vehicle count at an intersection, whereby our analysis was performed on a simplified crossroad layout as shown in Fig. 1. Nevertheless, the proposed methods can also be applied to other, more complex intersection types. In this case, as described before in Section IV-A, the intersection has to be divided into different turn combinations with different cycle costs depending on the chosen traffic protocol. These combinations are then weighted with their occurrence probability (Section IV-B), and, if needed, extended by estimates regarding traffic density and vehicle lengths (Section IV-C and IV-D).

F. Fail-safe behavior

Using probabilistic estimates brings about residual risk that these do not hold. For example, when selecting $n = 19$ as per Table III, the chance of having a higher vehicle count is $\approx 1 \times 10^{-7}$. Even though this value is very small, it does not guarantee full safety. To overcome this problem, additional fail-safe mechanisms are required. This could, for example, be an emergency braking system installed in each vehicle or the RSU switching to classic operation as traffic lights if the number of vehicles estimated is exceeded or communication fails. However, since these mechanisms interrupt normal operation, the system should be designed such these are not overused.

V. Impact on Crossroad Communication

The communication scheme used for the intelligent crossroad is based on the crossroad VANET from [9]. Here, vehicles and the road-side unit (RSU) periodically exchange data in cycles which have the following structure as shown in Fig. 5.

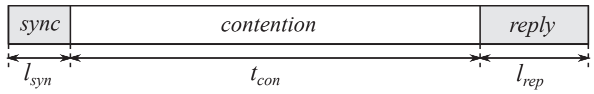


Fig. 5: Structure of a communication cycle

Each cycle starts with a sync field in which the RSU broadcasts a data packet containing information about the intersection (e.g., type, traffic load, etc.) that is received by all vehicles within range R. These then reply with a request message in the contention phase, sending their current speed, position, etc. The RSU collects these messages, calculates new speed values for each vehicle according to the traffic protocol described in Section III and communicates these during the reply phase. Note that after a cycle is complete, a new cycle starts immediately. The cycle interval or length is determined by the *physical resolution*, i.e., the maximum distance that a vehicle may travel before it requires an update from the RSU. For example, if we set the resolution to 1 m and assume a speed of 50 km/h, the cycle length is set to $\frac{1\,m}{50\,km/h} = 72\,\text{ms}$.

During contention phase, vehicles transmit data using the probabilistic medium access control (MAC) protocol from [13]. Here, each vehicle transmits its request message multiple times k, whereby the time between transmissions is is randomly selected from an interval $[t_{min}, t_{max}]$. Given the transmission time of a request message l_{req} and the number of vehicles n, it is possible to determine the worst-case reliability of the system, i.e., the probability that at least one out of k transmissions reaches the RSU:

$$p = 1 - \left(\frac{2(n-1)l_{req}}{t_{max} - t_{min}} \right)^k . \quad (6)$$

To include our probabilistic estimates of vehicle count n and their occurrence probabilities p_n, Eq. (6) is extended to:

$$\bar{p} = \sum_{n=n_{min}}^{n_{max}} p_n \left(1 - \left(\frac{2(n-1)l_{req}}{t_{max} - t_{min}} \right)^k \right) . \quad (7)$$

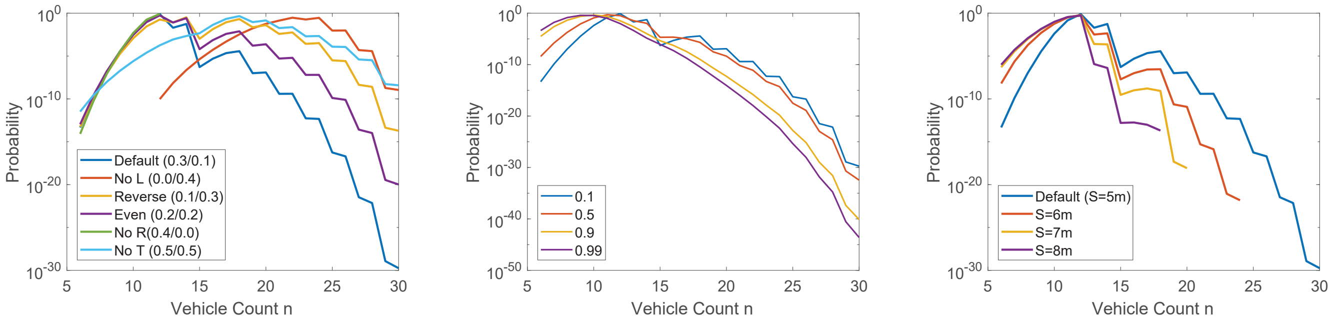
Here, $\bar{p}$ is the weighted reliability, i.e., the sum of all reliabilities of all different n multiplied by their occurrence probabilities p_n as per Eq. (5). Further, n_{min} and n_{max} are the lower and upper bound of the vehicle estimate as per Eq. (3) and (2). Note that for the evaluation presented in the next section, we selected $l_{req} = 78\,\mu\text{s}$ and $k = 3$ as per [9] and assumed $t_{min} = \frac{t_{max}}{2}$ and $t_{max} = \frac{t_{con} - l_{req}}{k}$ as per [13].

VI. Evaluation

In this section, we evaluate the impact of the crossroad's characteristics (driving direction, probability of half-pairs and overlength penalty) on the probabilistic estimates.

A. Driving direction

As discussed before in Section IV-B, different combinations of driving directions have different cycle costs and, therefore, affect the estimated maximum number of vehicles within the intersection. More specifically, right turns lead to a higher vehicle count, since they have lower cycle costs, while left



(a) Impact of driving direction (brackets denote (p_L/p_R))

(b) Impact of probability of half-pairs p_E

(c) Impact of sector size S

Fig. 6: Impact of turning/driving direction probabilities, half-pairs and sector size (i.e., overlength penalty) on our probabilistic estimate of n

turns and through driving decrease the vehicle count due to the low efficiency of LL and LT combinations.

Our example from Fig. 1 uses $p_L = 0.3$, $p_T = 0.6$ and $p_R = 0.1$ from [6]. Now, if we keep $p_T = 0.6$ and shift the remaining 0.4 between p_L and p_R, the previously discussed behavior can be observed in Fig. 6a. That is, the more right turns there are, the higher the chance of having high vehicle counts. For $p_L = 0.0$ and $p_R = 0.4$ (no left turns), it can even be observed that vehicle counts below 12 do not occur. Similarly, for $p_L = 0.4$ and $p_R = 0.0$ (no right turns), it is not possible to have more than 12 vehicles at the crossroad. For all other combinations, n is within $[6, 30]$ as shown in Table III.

B. Half-pairs

The impact of changing p_E, i.e., the probability of having half-pairs (i.e., only one vehicle) crossing the intersection, is depicted in Fig. 6b. Again, $p_E = 0$ means that there are no half-pairs, whereas $p_E = 1$ means that there are only half-pairs. As expected, the higher p_E, the more likely it is to have a lower vehicle count. On the other hand, very high vehicle counts, e.g., $n = 30$ are still possible, however, very unlikely to occur.

C. Overlength penalty

Changes to the sector size S impact the overlength probability p_{OL}, since having larger sector sizes (e.g., $S = 7m$) means that only vehicles longer than S are regarded as overlength ones. Effectively, larger sector sizes decrease p_{OL}, which leads to less extra cycles due to overlength penalty. However, on the other hand, all cycle costs are given in multiples of S and therefore increase for larger S. As a result, larger sector sizes effectively reduce the number of vehicles that can be processed at a time and are therefore not meaningful.

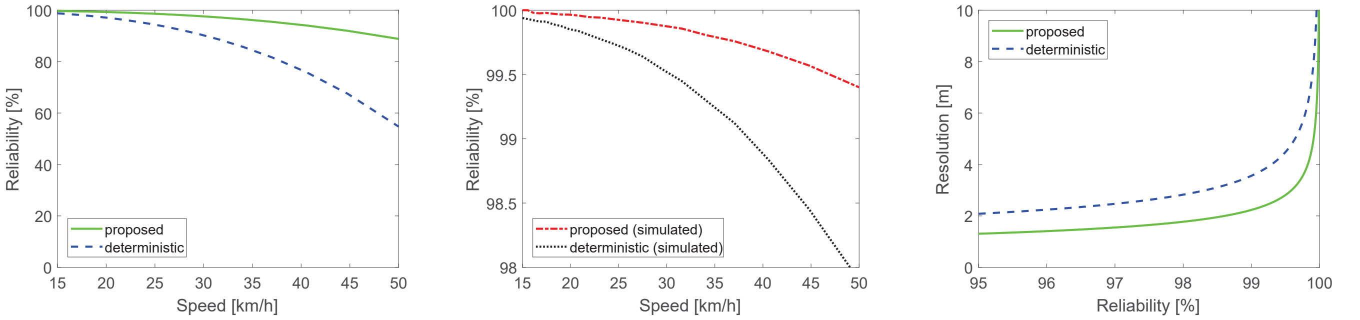
To illustrate the effects on our probabilistic estimates of n, let us now have a look at Fig. 6c. It can be seen that a larger S reduces the maximum n as expected. This complies with Eq. (2), which provides a deterministic upper bound of n. Inversely, a smaller S results in possibly larger n. Considering that larger S values reduce the throughput at the intersection, it is meaningful to set S to the length of the vehicle that occurs the most often at the intersection, i.e., $S = 5\,\text{m}$.

D. Communication reliability

Next, we will examine the impact of our probabilistic estimates on the communication reliability of the VANET introduced in Section V. To this end, we performed extensive simulations based on the OMNeT++ simulation framework [15], which allowed us to record statistical data of a very large number of transmissions — for each of the presented curves, at least $100,000$ communication cycles were simulated. We use the channel models and parameters from [16] and assume that there is no external interference present — however, this can be easily added as described in [13]. Note that we selected $n = 30$ for the deterministic approach and $n = 19$ for our proposed estimate for the following experiments, which is reasonable safe value, i.e., the chance of having a higher n is very small with $\approx 1 \times 10^{-7}$.

Fig. 7a shows the calculated (as per Eq. (7)) and Fig. 7b the simulated (average) transmission reliability in relation to the vehicle speed. Recall that the vehicle speed defines the cycle length and, therefore, the period of time t_{con} in which vehicles can send their request message. The higher the speed, the less time each node has to transmit its request message, resulting in a higher channel load and, therefore, less reliability. Also note that the simulated reliabilities in Fig. 7b are always higher than the calculated worst-case values in Fig. 7a.

Next, in Fig. 7c, we analyze the physical resolution (the distance a vehicle travels at a given speed) in relation to the achievable reliability p. The larger the physical resolution, the more time a vehicle has available to communicate with the RSU and, hence, reliability increases. It can be observed from Fig. 7c that the resolution increases very slowly at first for lower p. This makes it possible to strongly increase reliability at the cost of a only slightly larger resolution. For example, by increasing the resolution from $1\,\text{m}$ to $2\,\text{m}$,



(a) Calculated communication reliability for varying vehicle speed

(b) Simulated communication reliability for varying vehicle speed

(c) Physical resolution for varying communication reliability

Fig. 7: Comparing the proposed probabilistic with deterministic estimates of the number of vehicles taking communication reliability and physical resolution into account

we can increase reliability from 89 % to ≈ 99 % with the proposed technique. This is a meaningful step to take, if the application tolerates it. Note that the steep increase for very high p close to 100 % is due to the fact that the used MAC protocol cannot ensure perfect reliability (e.g., $p = 100\,\%$) — it requires increasingly more time (i.e., a higher resolution) as it approaches 100 % [13].

In summary, it can be seen that our probabilistic estimates for the maximum number of vehicles can greatly reduce pessimism compared to deterministic approaches. In particular, the VANET protocol benefits from it and can achieve a much higher communication reliability. Note that the safety/confidence for the chosen estimate is very high, i.e., it very unlikely to encounter a larger n at in the intersection ($\approx 1 \times 10^{-7}$ for $n = 19$). Our results show that in this example it is more likely that the communication fails rather than n is exceeded.

VII. Concluding Remarks

In this paper, we have investigated the quality of probabilistic estimates and their capability to overcome deterministic pessimism in open-ended settings such as an intelligent crossroad, which we used as a case study. To this end, we implemented a traffic protocol (controlling the order in which vehicles cross the intersection) and analyzed the space requirements of different actions such as turn left/right, driving through, etc. to derive an estimate of the maximum number of vehicles. This estimate was then extended to consider the probabilities of different vehicle lengths, their driving directions and traffic levels to further reduce pessimism.

To illustrate the benefits of the proposed approach, we simulated an exemplary VANET using the OMNeT++ framework. Our results show that probabilistic estimates can greatly reduce pessimism compared to deterministic approaches, while still maintaining a high level of safety.

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