# Using Probabilistic Estimates to Guarantee Reliability in Crossroad VANETs 

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#### Abstract

We consider an intelligent crossroad where conventional traffic lights are substituted by a roadside unit (RSU), which synchronizes vehicles at the intersection, minimizing waiting time and energy consumption (by avoiding unnecessary braking and accelerating). Clearly, in this case, a reliable communication needs to be guaranteed between vehicles and the RSU, for which we investigate the design and analysis of specialized Vehicular Ad Hoc Networks (VANETs). It turns out that reliability strongly depends on the number of vehicles at the crossroad, i.e., the more vehicles, the more interference and, hence, the lesser reliability. As a result, to guarantee a desired level of reliability, we first need to estimate the worstcase number of vehicles at the crossroad. However, straightforward, deterministic approaches - computing the maximum number of vehicles that physically fit into the crossroad's area - lead to a great amount of pessimism and overdesign. In this paper, we propose using probabilistic estimations for the number of vehicles instead, which greatly reduces the amount of pessimism while still guaranteeing safety. Our approach is based on vehicles' statistical information and allows computing the probability of having a certain number of vehicles at the crossroad in the worst case. We incorporate this probabilistic estimate into the VANET's design and analysis to derive guarantees on reliability. Finally, we illustrate the benefits of the proposed approach by means of a detailed case study and simulations using $\mathrm{OMNeT}++$.


## CCS CONCEPTS

- Computer systems organization $\rightarrow$ Embedded and cyberphysical systems; • Networks $\rightarrow$ Network protocol design; • Computing methodologies $\rightarrow$ Simulation evaluation;


## KEYWORDS

VANET; Intelligent Crossroad; OMNeT++; Reliability; Simulation

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## 1 INTRODUCTION

There are an increasing number of vehicles on roads and highways, which makes it necessary to control traffic flow in a more efficient manner. Current solutions such as conventional road infrastructure or traffic lights are unable to (or can only partially) adapt to changes in the traffic and, hence, largely fail to achieve this goal. As a result, there is a need for innovative solutions that help relieving congestion and preventing traffic jams in the future.

Motivated by autonomous and semi-autonomous driving, intelligent crossroads appear to be one such solution. Thereby, whenever vehicles arrive at a given distance from an intersection, a roadside unit (RSU) assigns them speeds so as to synchronize the points in time at which they arrive at and cross the intersection. This requires periodically collecting data from vehicles and computing new speed values to guarantee a collision-free crossing and, at the same time, avoid unnecessarily braking and accelerating vehicles at the intersection with the aim of minimizing waiting time and energy consumption.

Clearly, these computations need to be performed in real time, for which it is paramount to guarantee a reliable communication between vehicles and the RSU. To this end, a specialized Vehicular Ad Hoc Network (VANET) needs to be designed carefully. In this case, note that reliability strongly depends on the number of vehicles at the crossroad. In particular, the more vehicles there are at the crossroad, the more interference ${ }^{1}$ there will be and, consequently, the lesser the VANET's reliability. Note that synchronizing access to the communication channel to avoid interference between vehicles, e.g., using TDMA (Time Division Multiple Access), yields a huge overhead due to the constantly changing operation conditions and is not suitable in this context.

As a result, to be able to guarantee a desired level of reliability, we first need to compute the worst-case number of vehicles at the crossroad. Straightforward, deterministic approaches are based on estimating how many vehicles can physically fit in the surroundings of the crossroad. To this end, usually, the minimum possible length of a vehicle is taken into account, e.g., that of a motorbike. This leads to a pessimistic estimation and inefficient design, since most vehicles are, in fact, longer than motorbikes.

Contributions: In this paper, we propose an approach to probabilistically estimate the worst-case number of vehicles at the crossroad. Our approach takes vehicles' statistical data into account. In particular, we consider the number of units of all different vehicle models sold in Europe over the last years and derive a probability

[^1]distribution of the length of vehicles. Based on this, we can compute the probability of having vehicles with a given length at the intersection and, therefore, estimate the worst-case number of vehicles in the crossroad's surroundings.

We then incorporate these probabilistic estimates into the design and analysis of a specialized VANET, for which we assess interference in the network and derive guarantees on reliability. These guarantees are also of probabilistic nature and, hence, there is always a residual risk of not meeting them. As a result, fail-safe behavior needs to be implemented, e.g., the crossroad should switch back to conventional traffic lights in case something goes wrong.

As discussed later in more detail, the proposed method has the advantage of being independent of the crossroad and its operation conditions, i.e., it does not require collecting statistical data from individual crossroads such as, for example, the number of vehicles per hour of the day, etc., which is considerably more cumbersome to obtain.

Finally, we illustrate the benefits of the proposed approach on a detailed case study and by means of simulations using OMNeT++ [15].

Structure of the paper: Section 2 discusses related work, while Section 3 presents our case study consisting of an intelligent crossroad. Apart from the crossroad's operation, we discuss how to obtain the worst-case number of vehicles both in a deterministic and a probabilistic fashion. Section 4 describes the used communication scheme and Section 5 provides an evaluation comparing the proposed probabilistic with the deterministic approach on the basis of OMNeT++. Finally, Section 6 concludes the paper.

## 2 RELATED WORK

A considerable number of research papers are concerned with the design of intelligent crossroads. In the following, we provide a brief overview of the most relevant works for this paper.

The need for automated road traffic to relieve congestion and prevent traffic jams has been acknowledged before [1]. Since then, many approaches were presented to solve typical problems or to improve overall effectiveness of roads, intersections, etc. In [12], for example, route guidance and driver information systems were proposed to manage and improve traffic flow. This also included several strategies for intersection control. However, the described method from [12] relies on traffic lights managing themselves with no connection to the vehicles and, hence, it cannot adapt to changes in the traffic.

In [2], the so called Ballroom Intersection Protocol (BRIP) was proposed, which achieves deadlock-free traffic management at a constant speed. In contrast to this paper, BRIP assumes vehicles to have identical lengths, which limits its performance when being applied to real-world crossroads (where vehicles with different lengths are typically encountered). In addition, this work rather focuses on the crossroad's operation and does not analyze communication requirements.

Another approach in [16] proposed a fuzzy-logic controller for traffic lights at intelligent intersections. The phases of red and green lights are adapted dynamically to the vehicle density on each lane
using fuzzy rules. The concept in [16] can be seen as an intermediate stage between conventional traffic lights and the concept by this paper. However, in contrast to our approach, no considerations concerning communication reliability and the impact of high vehicle counts are made.

The approach in [4] proposed a VANET-based control for intelligent traffic lights. Here, again, periods of red and green lights are adjusted according to the traffic flow at the intersection. This is based on information about vehicle numbers and flow densities obtained in real time. In contrast to the approach presented here, vehicle counts are not available for consideration at design time, and their impact on communication reliability is not analyzed.

A concept combining clustering of vehicles with VANETs is proposed in [7]. Here, vehicles are grouped/clustered depending on their turn direction. The used communication scheme is also based on clustering, which leads to a significant communication overhead in the case of high numbers of isolated members looking for clusters to join. Unlike the concept we propose, the approach in [7] does not take vehicle numbers and their impact on communication performance into account.

In [3], VANETs are used for improving communication between traffic lights in an urban environment. Here, multiple traffic lights interact to optimally balance vehicles between them. The vehicle numbers are observed considering influential factors like the time of day, yet this is done through extensive surveying beforehand, which we intend to avoid in our approach by using existing statistical data. Furthermore, unlike in the proposed concept, the impact of high vehicle numbers on communication reliability is not considered.

In [11], a VANET-based intelligent signaling system is proposed, which focuses on improving efficiency and safety over multiple traffic zones. Here, fixed vehicle lengths and flow densities are assumed, which results in decreased performance when applied to real-world scenarios. In contrast to our approach, the impact of varying vehicle lengths and of communication reliability is not analyzed.

## 3 INTELLIGENT CROSSROAD

As discussed above, at an intelligent crossroad, conventional traffic lights are replaced by an RSU, which then synchronizes the order in which vehicles cross the intersection. In this paper, for simplicity, we consider a two-lane crossroad where only straight trajectories are allowed at the intersection. Although the operation logic will be different, note that our proposed design and analysis of the crossroad VANET can be easily extended to multi-lane roads where vehicles are allowed to turn right/left.

The RSU tracks and controls the speeds of vehicles in its surroundings, which we have set to a radius $R=200 \mathrm{~m}$ from the intersection's center. This poses real-time requirements on the response time between vehicles and the RSU, which has to be below a certain boundary so as to guarantee safety.

Operation logic: Since only straight trajectories are allowed, to minimize congestion at the intersection, it is meaningful to synchronize vehicles on opposite directions such that these cross simultaneously. That is, the vehicle going from sector 1 to 2 and the vehicle going from sector 3 to 4 in Fig. 1 are synchronized to cross
simultaneously with a constant speed $V$. To cross the intersection, these require a time equal to:

$$
t_{1 / 2}=t_{3 / 4}=\frac{2 S}{V},
$$

where $S$ is the size of one sector. The vehicle on the $1 / 2$ lane requires an additional time equal to $\frac{L_{1 / 2}}{V}$ being $L_{1 / 2}$ the vehicle's length to leave the intersection, during which this remains (at least partially) blocked. Similarly, the vehicle on the 3/4 lane will block the intersection by an additional time given by $\frac{L_{3 / 4}}{V}$ with $L_{3 / 4}$ being its length.

Now, considering that $\max \left(L_{1 / 2}, L_{3 / 4}\right)$ returns the greatest value between $L_{1 / 2}$ and $L_{3 / 4}$, the intersection will be blocked for a total time equal to:

$$
t_{\text {right } / \text { left }}=\frac{2 S+\max \left(L_{1 / 2}, L_{3 / 4}\right)}{V} .
$$

Similarly, the vehicle going from sector 2 to 3 and the vehicle going from sector 4 to 1 in Fig. 1 are synchronized to cross simultaneously with a constant speed $V$. Proceeding as before, we obtain:

$$
t_{\mathrm{up} / \mathrm{down}}=\frac{2 S+\max \left(L_{2 / 3}, L_{4 / 1}\right)}{V}
$$

where $L_{2 / 3}$ and $L_{4 / 1}$ are the lengths of the vehicles on the $2 / 3$ and on the $4 / 1$ lane respectively.

During $t_{\text {right/left }}$, the intersection is blocked by vehicles on lanes $1 / 2$ and $3 / 4$. Hence, vehicles on lanes $2 / 3$ and $4 / 1$ cannot cross. In the same way, during $t_{\text {up } / \text { down }}$, the intersection is blocked by vehicles on lanes $2 / 3$ and $4 / 1$ impeding that vehicles on $1 / 2$ and $3 / 4$ cross.

Assuming that vehicles on lanes $1 / 2$ and $3 / 4$ reach the intersection simultaneously at time $t_{0}$, vehicles on lanes $2 / 3$ and $4 / 1$ must also reach the intersection simultaneously at earliest at time $t_{0}+$ $t_{\text {right } / \text { left }}$. As a result, vehicles on lanes $1 / 2$ and $3 / 4$ can cross again at earliest at time $t_{0}+t_{\text {right } / \text { left }}+t_{\text {up } / \text { down }}$, i.e., they need to be separated by a distance equal to $4 S+\max \left(L_{1 / 2}, L_{3 / 4}\right)+\max \left(L_{2 / 3}, L_{4 / 1}\right)$ measured from front bumper to front bumper. This turns out to be the same inter-vehicle separation for vehicles on lanes $2 / 3$ and $4 / 1$. Note that the inter-vehicle separation cannot be less than this value according to the described operation logic.

### 3.1 Worst-case number of vehicles

In this section, we obtain the worst-case number of vehicles in the crossroad's surroundings, which we later need to design the used VANET. To this end, for the sake of comparison, we make use of a deterministic approach and of a probabilistic approach.

Deterministic approach: This is a straightforward approach, as discussed previously, which consists in computing the maximum number of vehicles that physically fit into the crossroad's surroundings, i.e., within a radius $R=200 \mathrm{~m}$ from the intersection's center. Clearly, the maximum number of vehicles results when considering that all vehicles have the minimum possible length denoted by $L_{\text {min }}$, leading to an inter-vehicle separation of $4 S+2 L_{\text {min }}$ on each of the four lanes.

On the other hand, vehicles on lanes $2 / 3$ and $4 / 1$ are shifted by $2 S+L_{\min }$ with respect to vehicles on lanes $1 / 2$ and $3 / 4$. To take this shifting into account, we observe that these vehicles physically


Figure 1: Two-lane intelligent crossroad allowing for synchronized straight trajectories in all directions. The intersection is divided into four sectors, i.e., $1,2,3$ and 4 , which are blocked by vehicles at different points in time. In the picture, for simplicity, vehicles' lengths are assumed to be the same, i.e., $L_{1 / 2}=L_{3 / 4}=L_{2 / 3}=L_{4 / 1}=L$.
fit into the empty slots between vehicles on lanes $1 / 2$ and $3 / 4$ as illustrated by red arrows in Fig. 1. In this case, the inter-vehicle separations on lanes $1 / 2$ and $3 / 4$ reduce to half the original value, i.e., to $2 S+L_{\text {min }}$, whereas lanes $2 / 3$ and $4 / 1$ are empty.

Now considering a sector size $S=4 \mathrm{~m}$ and a vehicle's minimum possible length $L_{\text {min }}=1.8 m$, which corresponds to a motorbike, the worst-case number of vehicles on one such lanes is given by:

$$
\begin{equation*}
n_{\max }=\left\lceil\frac{R}{2 S+L_{\min }}\right\rceil=\left\lceil\frac{200 m}{9.8 m}\right\rceil=21 . \tag{1}
\end{equation*}
$$

As a result, the deterministic estimation of the worst-case number of vehicles at the crossroad is given by $2 \times n_{\max }$, i.e., two lanes with inter-vehicle separations of $2 S+L_{\text {min }}$. This deterministic vehicle count is safe, but overly pessimistic, which results in an inefficient design of the VANET. As a consequence, we propose using the more realistic, probabilistic estimation as discussed next in detail.

Probabilistic approach: Although $n_{\max }$ as per (1) is possible, its occurrence is extremely unlikely. In other words, it is extremely unlikely that only motorbikes are in the crossroad's surroundings.

To obtain a more realistic estimation, we propose taking into account the probability of occurrence of a given number of vehicles. To this end, we need to derive a probability distribution of the number of vehicles at the crossroad, which can be done by collecting statistical data from the crossroad either by observation or simulation. However, this turns out to be time-consuming and strongly depends on the particular crossroad being observed or simulated.


Figure 2: Length of vehicles sold in Europe from 2005 to 2015 (green dots) and the resulting weighted distribution (dashed line).

To overcome this problem, we propose using publicly available statistical data of vehicles' sales on the European market from 2005 to 2015 [6], combined with sales of exceptionally short cars [5], motorbikes [14] and trucks [6]. This leads to a weighted distribution of 258 different vehicle lengths in Europe as displayed by green dots in Fig. 2.

Computing all possible combinations of up to 21 vehicles within the crossroad - i.e., the worst-case number of vehicles as per (1) - amounts to $21^{258}$ possibilities. Clearly, it is not feasible to use exhaustive methods for this. As a result, to make this tractable, we propose clustering vehicles according to similarities. This way, we obtained $x=8$ weighted categories, whose resulting probability distribution is displayed by the dashed line in Fig. 2.

As discussed previously, the worst-case number of vehicles at the crossroad can be effectively computed as the number of vehicles on lanes $1 / 2$ and $3 / 4$ with inter-vehicle separations of $2 S+$ $\max \left(L_{1 / 2}, L_{3 / 4}\right)$. The longest of two vehicles $L_{1 / 2}$ or $L_{3 / 4}$ determines the separation to the next vehicle on both these lanes. Therefore, the probability distribution gets shifted from single vehicles to paired vehicles representing the probability that either $L_{1 / 2}$ or $L_{3 / 4}$ is longer, see Fig. 3.

From here onward, it is possible to describe the number of vehicles on a given lane with a multinomial distribution [10], which has the following probability mass function $f(\cdot)$ :

$$
\begin{equation*}
f\left(k_{1}, \ldots, k_{x}, p_{1}, \ldots, p_{x}\right)=\frac{n!}{k_{1}!\ldots k_{x}!}\left(p_{1}^{k 1} \times \ldots \times p_{x}^{k_{x}}\right) . \tag{2}
\end{equation*}
$$

Here, $x$ is the number of different categories of vehicles, $n=$ $\sum_{i=1}^{x} k_{i}$ is the number of vehicles on a lane and $p_{i}$ is the corresponding probability that the given vehicle is of type $i$ with $1 \leq i \leq x$. Eq. (2) gives the probability of having $n$ vehicles on a lane, whereby $k_{1}$ are of type $1, k_{2}$ are of type 2 , and so on. Note that the order of vehicles on the lane plays no role.

So far, we found an upper bound for $n$ with $n_{\max }=21$ as per (1), i.e., only motorbikes on the crossroad. The minimum bound $n_{\text {min }}$, on the other hand, can be found by assuming that the crossroad


Figure 3: Change in probability when considering (paired) vehicles combinations (red line), compared to the original, single vehicle distribution from Fig. 2 (blue line). The higher impact of longer vehicles leads to a probability shift towards the longer vehicles.
only contains the longest vehicles, i.e., trucks with $L_{\max }=7.82 \mathrm{~m}$ :

$$
\begin{equation*}
n_{\min }=\left\lceil\frac{R}{2 S+L_{\max }}\right\rceil=\left\lceil\frac{200 m}{15.82 m}\right\rceil=13 \tag{3}
\end{equation*}
$$

Next, we calculate all possible combinations of $k_{1}$ to $k_{x}$ for all possible $n \in[13,21]$, with $1 \leq x \leq 8$. Now, not every combination of $n$ vehicles is valid. If the combined lengths and inter-vehicle separations exceed $R$, the combination is not valid.

Similarly, if the combined length is below $R$, but there is room for additional vehicles, the combination is not valid either, since we are interested in the worst-case number of vehicles, i.e., the maximum possible number of vehicles that fit into $R$. As a result, a valid combination for a vehicle count $n$ is a set $\left(k_{1}, \ldots, k_{x}\right)$, where the following condition holds for any $1 \leq i \leq x$ :

$$
R-\left(2 S+L_{i}\right)<2 n \cdot S+k_{1} \cdot L_{1}+\ldots+k_{x} \cdot L_{x}<R
$$

Computing all these cases for all $n \in[13,21]$ leads to the probabilities of worst-case vehicle counts on a single lane and at the crossroad shown in Table 1 - nine different worst-case vehicle counts in total. Here, the worst-case number of vehicles at the crossroad equals twice the lane's worst case. This is due to the previously described mirroring of the opposing lanes. Therefore, these probabilities correspond to the vehicle counts of $26,28, \ldots, 42$. Accumulating these probabilities shows that the maximum number of vehicles at the crossroad is $\leq 32$ in $99.9942 \%$ of all cases, or $\leq 34$ in $\left(1-1.68 \times 10^{-10}\right) \%$ of all cases.

| On one lane $(n)$ | At the crossroad $(2 n)$ | Probability |
| :---: | :---: | ---: |
| 21 | 42 | $\approx 0$ |
| 20 | 40 | $1.1880 \mathrm{E}-33$ |
| 19 | 38 | $5.6016 \mathrm{E}-22$ |
| 18 | 36 | $1.6768 \mathrm{E}-12$ |
| 17 | 34 | $5.7923 \mathrm{E}-05$ |
| 16 | 32 | 0.59656894 |
| 15 | 30 | 0.40271516 |
| 14 | 28 | $6.5791 \mathrm{E}-04$ |
| 13 | 26 | $6.6878 \mathrm{E}-08$ |

Table 1: Resulting worst-case numbers of vehicles with their corresponding probabilities.

### 3.2 Fail-safe behavior

Note that, since we are now using probabilistic estimates of the worst-case number of vehicles, there exists a residual risk that these do not hold. As a result, the intelligent crossroad should be implemented in such a way that it allows for fail-safe behavior. In particular, the RSU switches to manual operation, i.e., it starts behaving as a conventional traffic light, if safety is compromised.

However, in this case, we need to consider that sudden mode switches might lead to hazardous situations. To ease this transition, mode switches will be preceded by a warning signal, e.g., a yellow traffic light. The conditions leading to a mode switch need to be defined clearly in advance. For example, these conditions can be (i) a vehicle deviates from the crossroad schedule for more than a previously specified time or (ii) the maximum number of vehicles expected or allowed at the crossroad - is exceeded.

A vehicle may deviate from schedule, if the driver - in case of a manual vehicle - disregards some signs or the vehicle loses communication to the RSU. Clearly, this leads to a mode switch so as to ensure the safety of all other participants. In this paper, we assume that vehicles are autonomous and only deviate from the given schedule in case of communication loss. Therefore, we focus on analyzing packet loss on the network. With respect to manual vehicles, a deviation can be detected by external sensors (e.g., radars) at the crossroad. This enables the RSU to detect vehicles independent of whether communication was successful or not, and to measure their speeds.

## 4 CROSSROAD VANET

In this section, we present a communication scheme for an intelligent crossroad, which we later use to evaluate the presented theory. All vehicles within a certain distance from the intersection periodically transmit their current coordinates and speed to the RSU. The RSU then processes this data and replies with updated speed values, which vehicles have to adopt. The periodicity of the transmission is determined by the speed of vehicles and the required physical resolution, i.e., the distance a car can travel before it must receive an update. For example, if the speed is $50 \mathrm{~km} / \mathrm{h}$ and an update must be received every 1 m traveled, the transmission interval is equal to $\frac{1 \mathrm{~m}}{50 \mathrm{~km} / \mathrm{h}}=72 \mathrm{~ms}$.

To lower complexity and decrease overhead, we assume that vehicles and the RSU are within range of each other and, hence, can communicate directly in a single-hop fashion. This avoids delays


Figure 4: Every transmission cycle consists of three parts: sync, contention phase and reply. A sync message informs vehicles about the length of the contention phase, where these can transmit a request message to the RSU. At the end of the cycle, the RSU replies to all vehicles with a single reply message.
over multiple hops and can be easily achieved with off-the-shelf hardware, as shown in [17], where tests showed possible ranges of $\geq$ 300 m using 5.9 GHz vehicle-to-vehicle modems. We further assume that there is no interference with neighboring systems, which is typically achieved by using multiple (different) radio channels.

Similar to other VANET scenarios, a peculiarity of an intelligent crossroad is that communication between vehicles and RSU is only established for a short time. Using classic synchronous communication methods for data exchange, such as TDMA, hence results in considerable overhead, since time slots must be re-assigned continuously. To reduce this complexity, we instead use a hybrid approach as depicted in Fig. 4. Here, the RSU periodically transmits a sync beacon, which indicates the start of a new communication cycle and informs newly arrived vehicles about the intersection. This is followed by a contention phase, in which vehicles transmit a request packet to the RSU using a random transmission pattern, as explained later. At the end of the cycle, i.e., after all request messages have been received, the RSU calculates a new speed value for each vehicle and transmits these together in one reply messages.

This hybrid, RSU-initiated topology has the advantage that there are only collisions, where these cannot be prevented, i.e., among request messages in the contention phase. The RSU messages, i.e., sync and reply, on the other hand, are not disrupted, which increases the overall reliability of the network. In the next section, we will analyze the collisions within the contention phase in more details.

### 4.1 Medium access control

As mentioned before, synchronous protocols would incur in high overhead due to the high mobility within the network. To solve this, we instead use a hybrid frame structure and implement an asynchronous protocol in the contention phase to effectively convey data. To this end, any asynchronous medium access control (MAC) protocol could be used, for example CSMA. However, we decided to use the probabilistic approach presented in [13], where vehicles transmit requests as a sequence of redundant packets with random back-off times in-between transmissions. This scheme has the advantage that it allows calculating worst-case transmission reliabilities, which facilitates a connection to the presented theory of this paper. In addition, it does not rely on sending acknowledgments - these are also not needed, since feedback is obtained from the reply message - which further reduces overhead and generated traffic and, therefore, improves the overall transmission reliability. Using the MAC protocol from [13], vehicles now transmit their request message $k$ times within the contention phase of length $t_{\text {con }}$.

Each of these transmission requires certain amount of time depending on the number of bits to be transmitted and the transmission rate. We refer to this time as packet length and denote the length of the request message by $l_{\text {req }}$. The time in-between two consecutive request messages is randomly selected within a lower boundary $t_{\text {min }}$ and an upper boundary $t_{\max }$ :

$$
\begin{gathered}
t_{\max }=\frac{t_{c o n}-l_{\text {req }}}{k}, \\
t_{\min }=\frac{t_{\text {max }}}{2} .
\end{gathered}
$$

To evenly spread messages across the whole contention phase $t_{c o n}$ and, therefore, achieve a lower collision rate, $t_{\max }$ is set to the highest possible value. That is, the $k$-th packet of a vehicle must start transmitting at latest at $t_{c o n}-l_{\text {req }}$ time to finish transmission before the end of the contention phase. Similarly, $t_{\min }$ is chosen as small as possible to achieve a greater interval $t_{\max }-t_{\min }$ and, hence, increase the number of possible choices for back-off times, which is again beneficial for reliability. However, $t_{\min }$ is lower bounded to $\frac{t_{\max }}{2}$, since smaller values would allow more than one request message within $t_{\text {max }}-t_{\text {min }}$, for example, if a vehicle randomly selects $t_{\text {min }}$ as a back-off multiple times. As shown in [13], this reduces the possible reliability and is therefore not desirable.

The probability that one packet of a given vehicle is interfered results from the ratio between $2(2 n-1) l_{\text {req }}$, i.e., the fraction of the interval $\left[t_{\text {min }}, t_{\text {max }}\right]$ that is potentially being used by other vehicles, and $t_{\max }-t_{\min }$, i.e., the total length of this interval. The term $2(2 n-1) l_{\text {req }}$ results from the fact that potentially $2 n-1$ other vehicles are sending packets of length $l_{\text {req }}$ and that any overlapping between two packets is considered to yield packet loss, which explains the factor 2 . Since this probability is independent of the individual transmission, a binomial distribution can be used allowing to calculate the resulting worst-case reliability of the network:

$$
\begin{equation*}
p=1-\left(\frac{2(2 n-1) l_{\text {req }}}{t_{\text {max }}-t_{\text {min }}}\right)^{k} \tag{4}
\end{equation*}
$$

where again $2 n$ is the number of vehicles at the crossroad, $t_{\text {min }}$ and $t_{\text {max }}$ are back-off interval boundaries, $l_{\text {req }}$ is the length of a request message and $k$ is the number of (redundant) transmissions.

Eq. (4) assumes a deterministic value of $n$. However, we now have different possible values of $n$ with different probabilities as per Table 1. As a result, when integrating probabilistic estimates of the worst-case number of vehicles, (4) must be extended to:

$$
\begin{equation*}
\bar{p}=\sum_{n=13}^{21} p_{n}\left(1-\left(\frac{2(2 n-1) l_{r e q}}{t_{\max }-t_{\min }}\right)^{k}\right) \tag{5}
\end{equation*}
$$

where $n$ corresponds to the worst case vehicle count listed in Table 1 with $13 \leq n \leq 21$ and $p_{n}$ is the corresponding probability of $n$.

### 4.2 Physical layer

Using the physical layer (PHY) and protocol packet data unit (PPDU) header of the IEEE 802.11p standard [8], each transmission consists of a preamble field, a signal field and variable length data field see Fig. 5. The preamble consists of a series of pre-defined symbols that allow the demodulator to lock onto the carrier frequency and


Figure 5: Protocol packet data unit (PPDU) header from IEEE 802.11p.
decode the data. The signal field defines the data field length and the transmission rate, while the payload of the message is contained in the data field. In order to securely detect any corrupted transmission, we include a 2 bytes cyclic redundancy check (CRC) into the data field.

The IEEE 802.11p PHY can support data rates between 3 and 27 Mbps , when using a channel bandwidth of 10 MHz [8]. However, since the data rate is unknown prior to reception of the signal field, the preamble and signal field are transmitted at the lowest rate of 3 Mbps for improved robustness. This results in a duration of $32 \mu \mathrm{~s}$ and $8 \mu$ s respectively. For the data field, we select 6 Mbps , which is a good compromise of robustness and transmission speed. The total length of any packet can hence be calculated:

$$
l_{\text {packet }}=32 \mu s+8 \mu s+\frac{\text { payload }+2 \text { bytes }}{6 \mathrm{Mbps}}
$$

Regarding the payload, each request message contains the location and speed of a vehicle as well as an identifier, which is needed associate the speed values within the reply message to the corresponding vehicles. Assuming 8 bytes for the GPS position, 1 byte for speed and 17 bytes for the $\mathrm{ID}^{2}$, the length of a request message is $78 \mu \mathrm{~s}$. Similarly, the sync message holds 10 bytes of information about the intersection, e.g., number of lanes, etc., and, hence, has a length of $l_{\text {syn }}=56 \mu \mathrm{~s}$. The reply message, on the other hand, is of variable length, depending on how many request messages were (successfully) received. For each received message, it replies with 17 bytes ID and 1 byte speed.

## 5 EVALUATION

In this section, we present the results of a simulation based on the OMNeT++ network simulation framework [15] and an extension for mobile and wireless networks named MiXiM [9]. This allows us to effectively simulate our intelligent crossroad with different physical parameters and to record statistical values for very large numbers of transmissions.

In particular, each simulation was performed with different parameters, for which at least 50,000 cycles have been simulated each time. Channel parameters were taken from [17], i.e., transmission power, receiver sensitivity, etc. For simplicity, we neglect packet loss due to channel errors, fading, etc. Note that this assumption does not invalidate the presented results, since the proposed MAC is highly robust against such effects [13].

In the following, we compare the proposed approach by this paper and the more straightforward deterministic approach. The proposed approach incorporates probabilistic estimates of the worst-case number of vehicles into the VANET's design, computing reliability

[^2]

Figure 6: Relation between the speed of a vehicle and the resulting worst-case transmission reliability.


Figure 7: Relation between the speed of a vehicle and its transmission reliability.
as per (5). The deterministic approach considers a fixed worst-case number of vehicles equal to 42 based on (1) and uses (4) to compute reliability.

Determining $k$ : The underlying MAC layer is based on transmitting $k$ request messages within the contention phase, for which formulas are provided to calculate $t_{\text {min }}, t_{\text {max }}$, and reliability ( $p$ or $\bar{p}$ ). On the other hand, $k$ must be determined experimentally due to its non-linearity and dependency on the other parameters. For this reason, we conducted an experiment, as depicted in Fig. 6, where the relation between the speed of a vehicle and its transmission reliability is shown for different $k$. As we can see, a faster vehicle leads to a lower possible reliability. This is due to fact that the contention phase $t_{c o n}$ becomes shorter, which we previously defined as the time a vehicle needs for traveling 1 m at a given speed. Further, a higher $k$ leads to higher possible reliability for lower speeds, which then quickly decreases for higher speeds. To achieve good performance over a high range of speeds, we select $k=3$ for the following experiments.

Reliability vs speed: Next, we analyze how a vehicle's speed affects its transmission reliability as shown in Fig. 7. Here, we can see


Figure 8: Relation between the payload size of a request message and the transmission reliability.


Figure 9: Relation between the transmission reliability and the required physical resolution, i.e, the distance a vehicle travels at $50 \mathrm{~km} / \mathrm{h}$ to achieve this reliability.
that, with (5), i.e., a more precise estimate for the vehicle number $n$, the network can benefit with a considerable higher reliability. This effect is observable for both simulated and non-simulated proposed and deterministic approaches. Note that the simulated curves show the average reliability, which is typically higher than the computed worst-case reliability as per (4) and (5).

Reliability vs payload: So far, we considered a payload size of 28 bytes containing only the minimum required information, e.g., speed, ID, GPS, and CRC. In case additional information needs to be conveyed, for example, vehicle priority, etc., the request message becomes longer, which affects the transmission reliability, as shown in Fig. 8. As expected, an increasing payload size reduces the possible reliability, since a longer message is more vulnerable to being interfered by another packet. This effect is slightly non-linear due to channel saturation effects. Again, the proposed curves show a considerable improvement over the deterministic one.

Reliability vs physical resolution: When a vehicle travels faster through the intersection, it has less time to transmit its data within a given distance, e.g, 1 m . In Fig. 9, we show the dependency between
this distance, also called (physical) resolution, and the achievable reliability as per (4) and (5) for a speed of $50 \mathrm{~km} / \mathrm{h}$. As we can see, the physical resolution increases only very slowly for rising reliability until starting to strongly increase from a reliability of $99 \%$ onwards and getting unbounded. That is, reliabilities close to $100 \%$ are hard to achieve and, therefore, come at high costs when using asynchronous protocols.

## 6 CONCLUDING REMARKS

In this paper, we have incorporated probabilistic estimates into the design and analysis of a crossroad VANET, based on which we were able to derive more accurate guarantees on reliability. We showed that it is possible to reduce pessimism and overdesign inherently caused by deterministic estimations in this case, while still retaining a high level of safety.

We illustrated benefits by the proposed approach based on a detailed case study and by simulations using OMNeT++. These showed that using probabilistic estimates allows us to roughly achieve $20 \%$ higher speed, or $35 \%$ larger payloads in the VANET without affecting the amount of reliability.

In future work we plan to extend this approach by modeling the entire system probabilistically (not only the worst-case number of vehicles, but communication and computation processes as well). This way, we intend to achieve an entirely probabilistic design and analysis approach of a VANET.

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[^1]:    ${ }^{1}$ We consider internal/systemic interference due to simultaneous access to the medium. However, the proposed technique can be easily extended to external interference as well, assuming that we can find an upper bound for it. On the other hand, in this application, it is meaningful to grant vehicles exclusive use of the communication channel, thus, reducing external interference to a negligible minimum.

[^2]:    ${ }^{2}$ We use the vehicle identification number (VIN) as this does not only contain a unique production number, but also useful information about the vehicle itself, such as weight, length, etc.

