Trajectory Generation

Dr.-Ing. John Nassour

Path vs. Trajectory

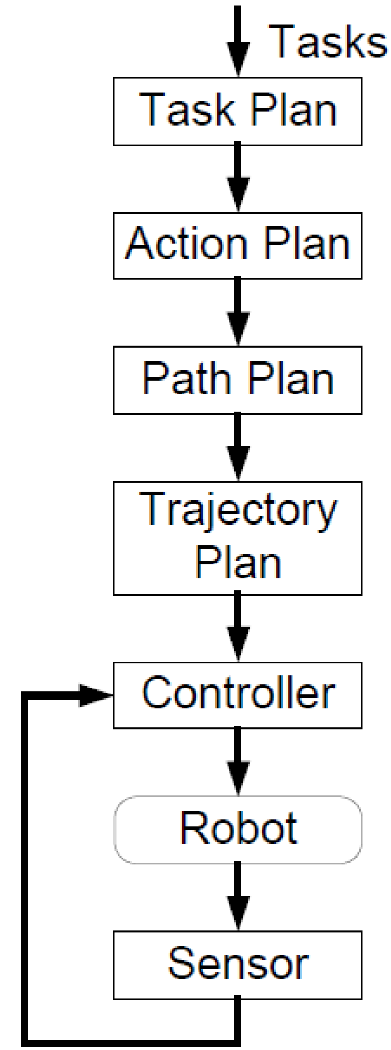
Path: is an ordered locus of points in the space (either joint or operational), witch the manipulator should follow.
It is a pure geometric description of motion.

Trajectory: a path on which timing law is specified, e.g., velocities and accelerations in each point.

Robot Motion Planning

Path planning:
- Geometric path
- Issues: optimal path.

Trajectory:
- Interpolate or approximate the desired path by a class of polynomial functions.
- Generate a sequence of time-based "control set points" for the control of manipulator from the initial configuration to its destination.



Trajectory Generation

The **aim of the trajectory generation** is to generate inputs to the motion control system which ensures that the planned trajectory is executed.

The user or the upper-level planner describes the desired trajectory by some parameters, usually:

- Initial and final point (point-to-point control).
- Finite sequence of points along the path (motion through sequence of points).

Trajectory planning/generation can be performed either in the joint space or operational space.

Minimal Requirements

Capability to move robot arm and its end effector from the initial posture to the final posture.

Motion laws have to be considered in order **not to**:

- violate saturation limits of joint drives.
- excite the modeled resonant modes of the mechanical structure.

Generation of **smooth** trajectories.

Joint Space vs. Operational Space

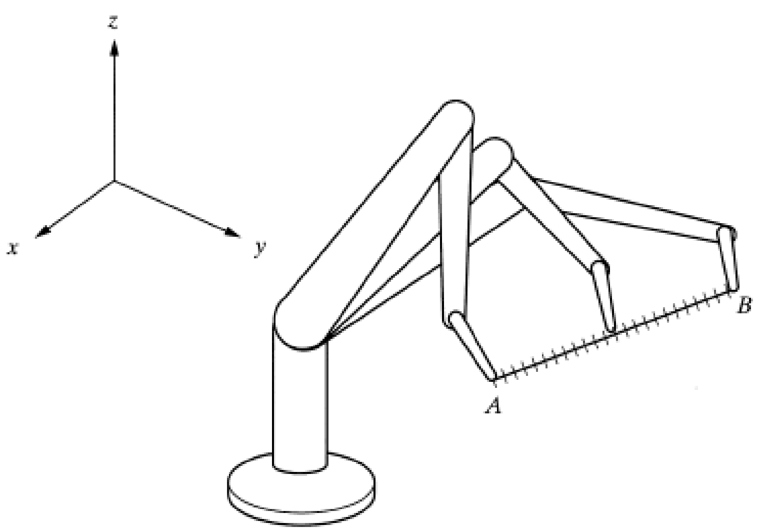
Joint-space description:

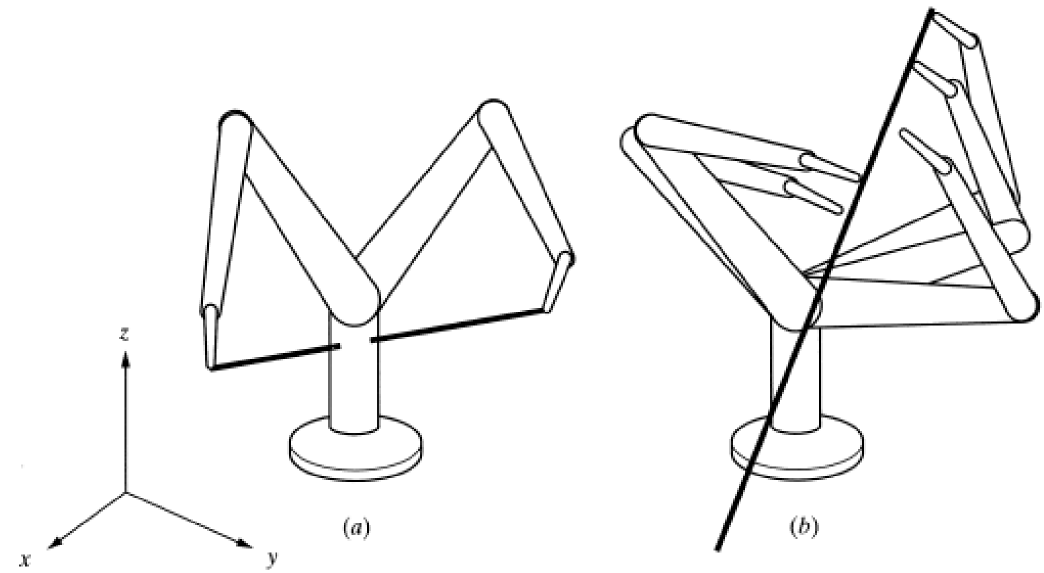
- The description of the motion to be made by the robot by its joint values.
- The motion between the two points is unpredictable.

Operational space description:

- The motion between the two points is known at all times and controllable.
- It is easy to visualize the trajectory, but it is difficult to ensure that singularity does not occur.

Joint Space vs. Operational Space Example





Sequential motions of a robot to follow a straight line.

Cartesian-space trajectory
(a) The trajectory specified in Cartesian coordinates may force the robot to run into itself, and (b) the trajectory may requires a sudden change in the joint angles.

Trajectory in the Operational Space

- Calculate path from the initial point to the final point.
- Assign a total time $\mathbf{T_{path}}$ to traverse the path.
- Discretize the points in time and space.
- Blend a continuous time function between these points.
- Solve inverse kinematics at each step.

Advantages:

- Collision free path can be obtained.

Disadvantages:

- Computationally expensive due to inverse kinematics.
- It is unknown how to set the total time $\mathbf{T_{path}}$.

Trajectory in the Joint Space

- Calculate inverse kinematics solution from initial point to the final point.
- Assign total time $\mathbf{T_{path}}$ using maximal velocities in joints.
- Discretize the individual joint trajectories in time.
- Blend a continuous function between these point.

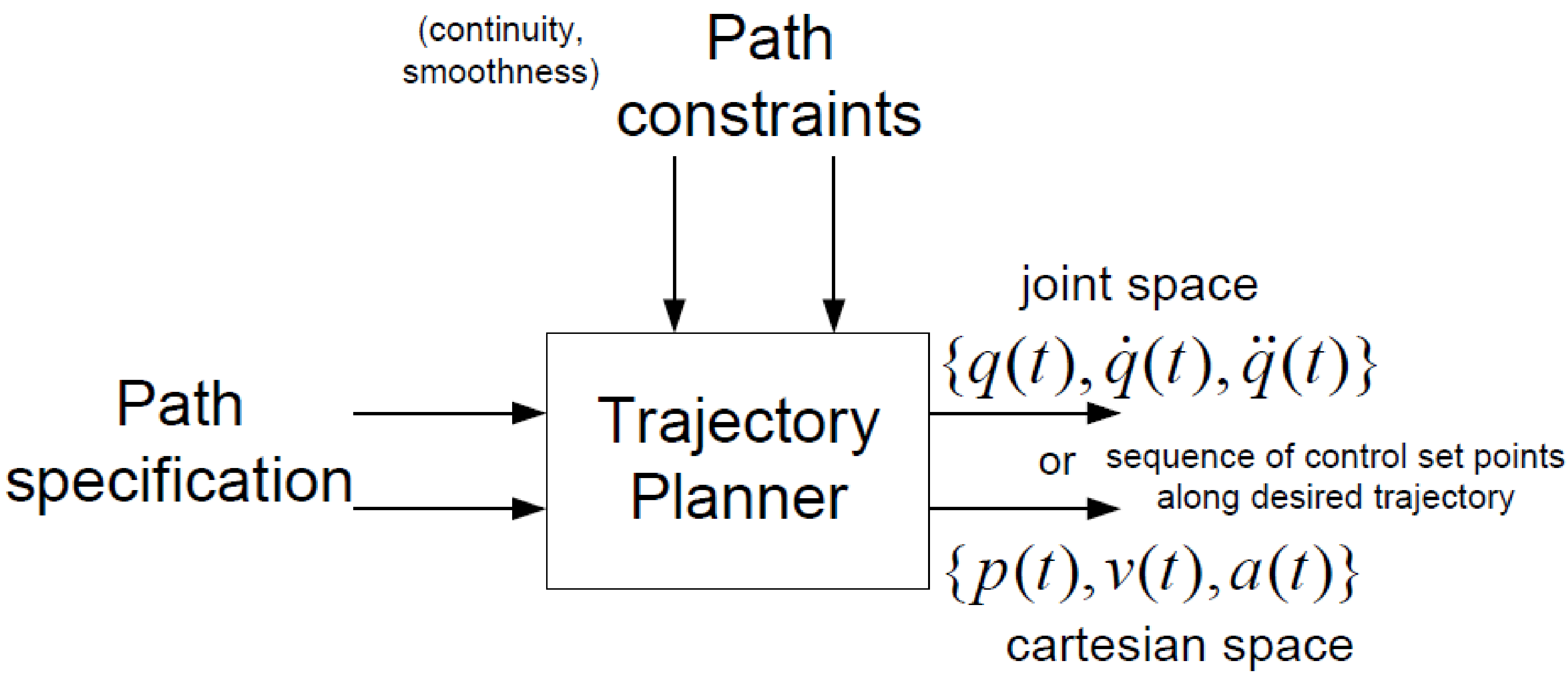
Advantages:

- Inverse kinematics is computed only once.
- Can easily take into account joint angle, velocity constraints.

Disadvantages:

- Cannot deal with operational space obstacles.

Trajectory Planning



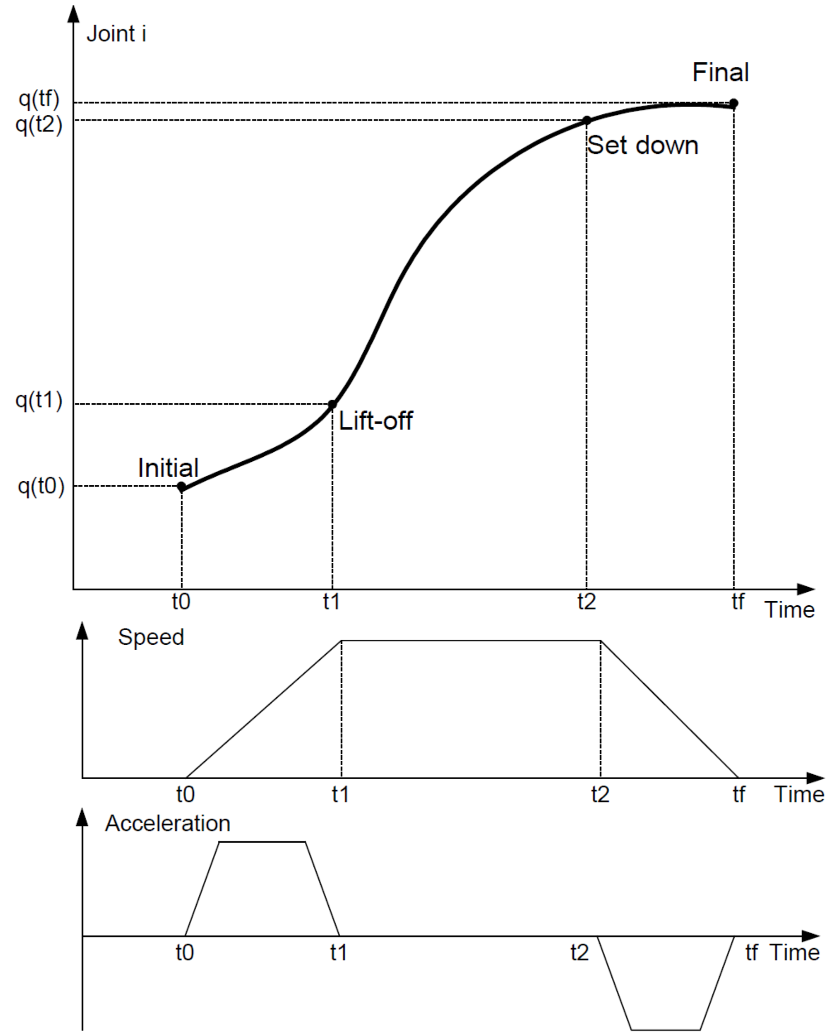
Best Planning Approach

- Combination of via points (global plan) and point to point (locally between two points).

- Via points provides an approximation of the path.

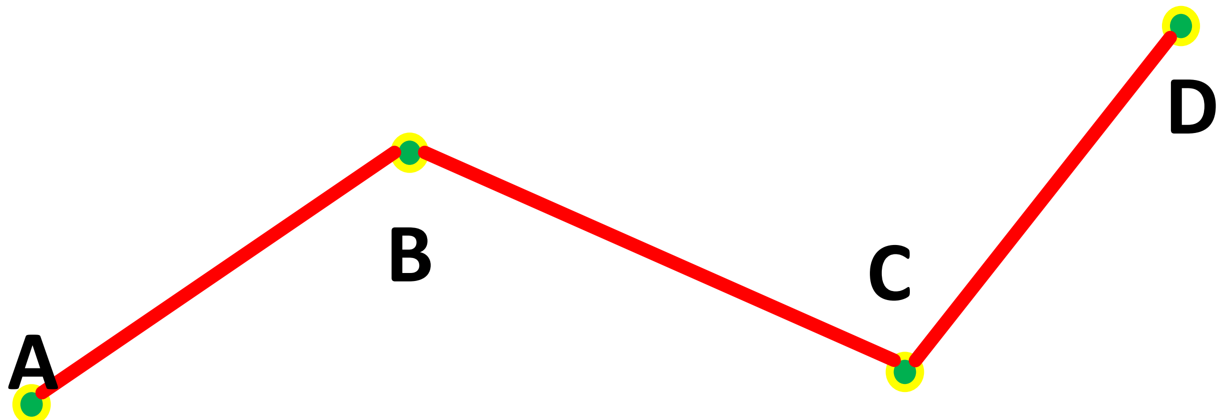
Trajectory Planning

- Path Profile
- Velocity Profile
- Acceleration Profile



Candidate Curves For Interpolation

- **Straight line (discontinuous velocity at path points).**
- Linear functions with parabolic blends.
- Cubic polynomials (splines).
- High order polynomials (quantic: polynomial of degree 5, ..).



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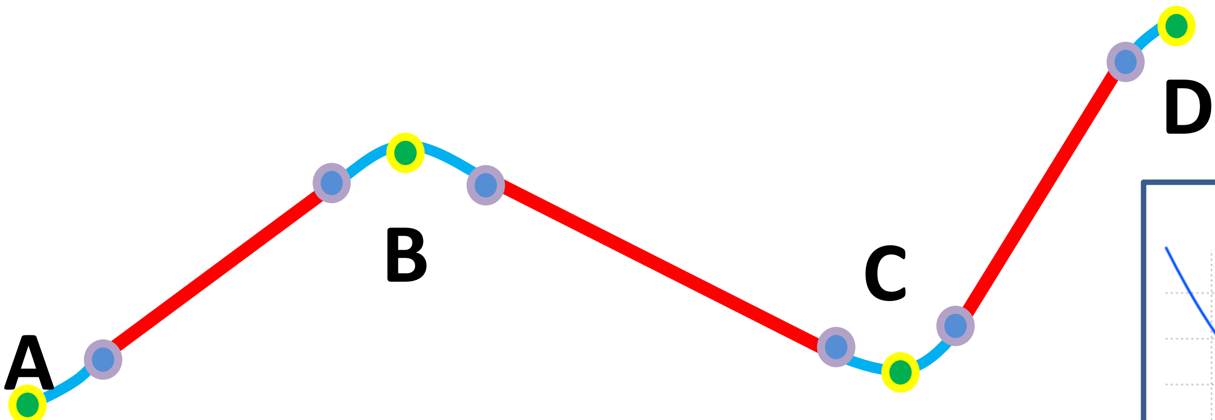

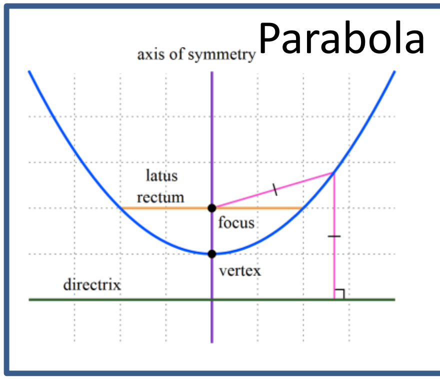

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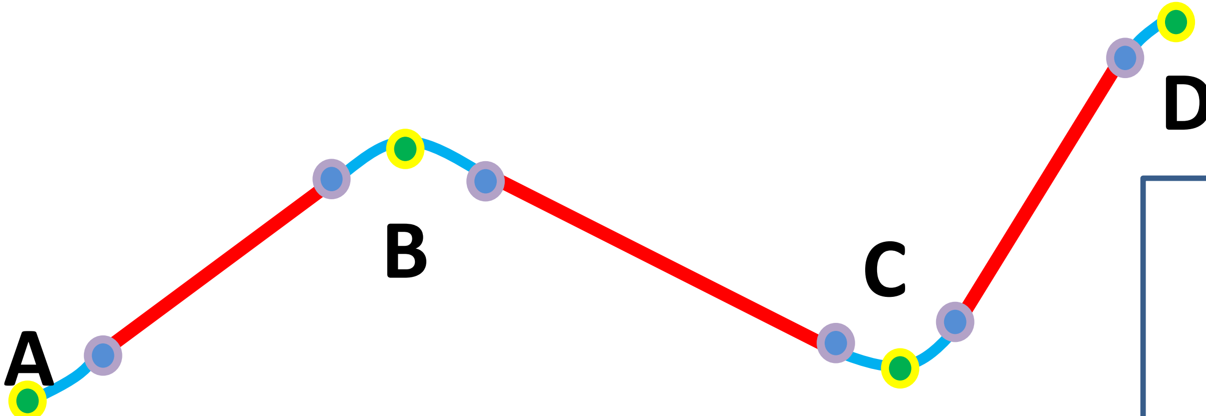
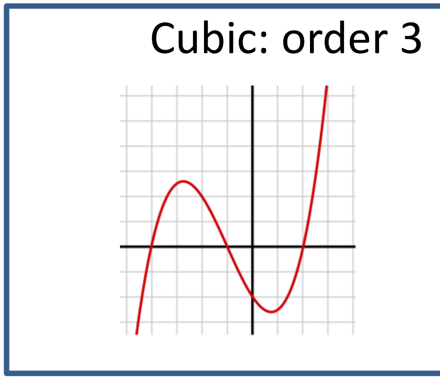
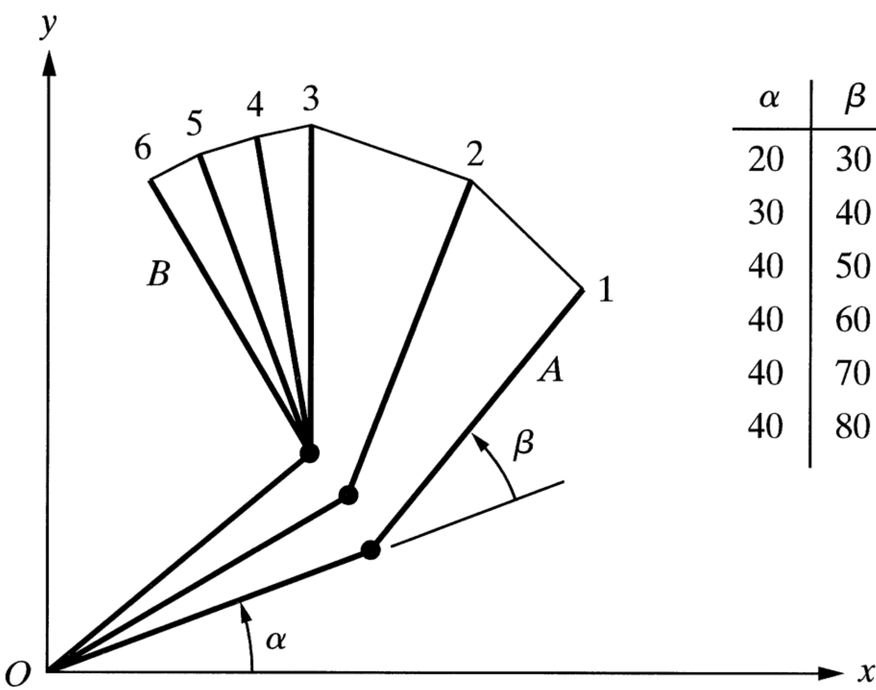




Basics for Trajectory generation

- Different approaches will be demonstrated on a simple example.
- Let us consider a simple 2 degree of freedom robot.
- We desire to move the robot from Point A to Point B.
- Let's assume that both joints of the robot can move at the maximum rate of 10 degree/sec.

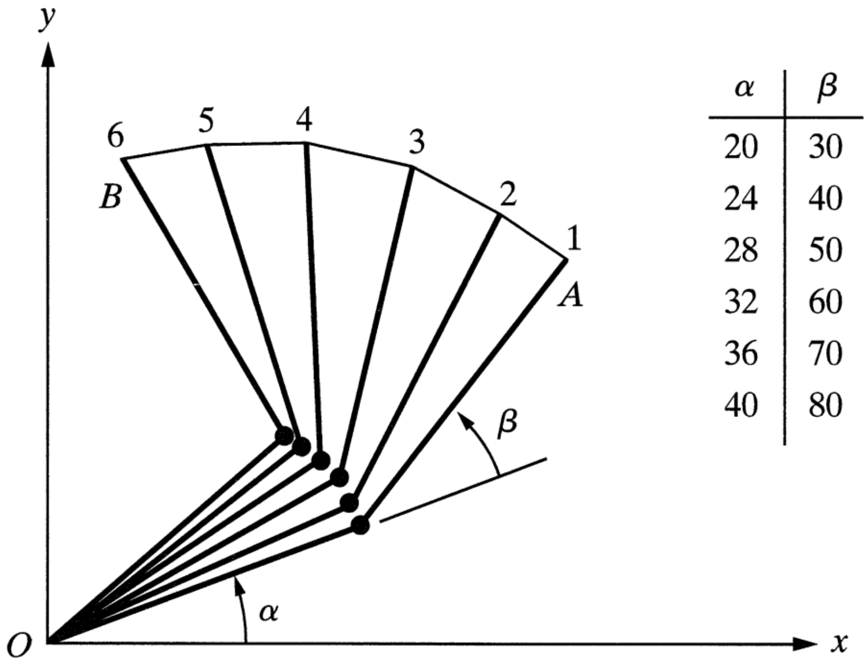
Non-Normalized Movement



α	β
20	30
30	40
40	50
40	60
40	70
40	80

- Move the robot from A to B, to run both joints at their maximum angular velocities.
- After 2 [sec], the lower link will have finished its motion, while the upper link continues for another 3 [sec].
- The path is irregular and the distances traveled by the robot's end are not uniform.

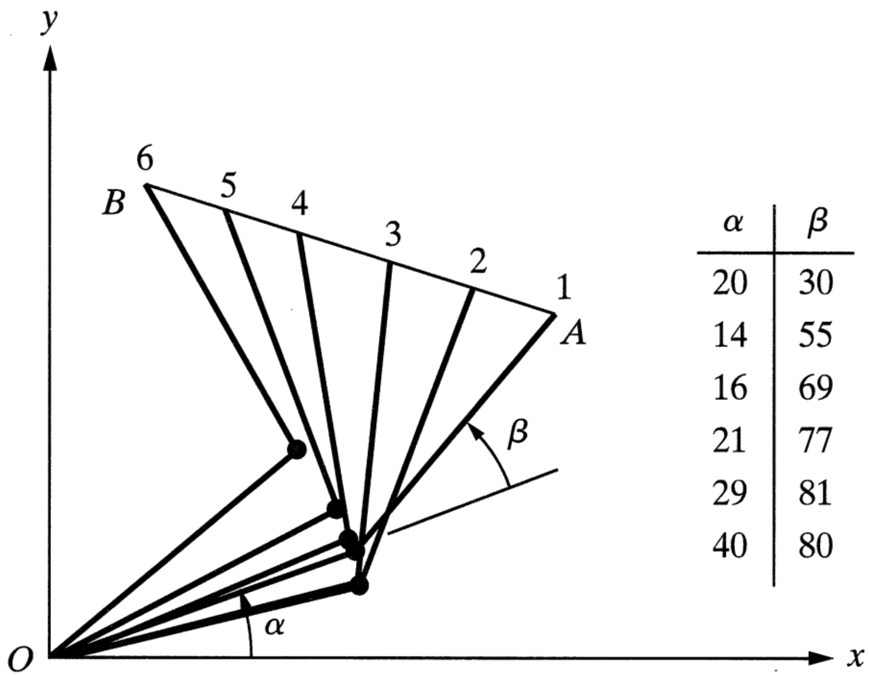
Normalized Movement



α	β
20	30
24	40
28	50
32	60
36	70
40	80

- Let’s assume that the motions of both joints are normalized by a common factor such that the joint with smaller motion will move proportionally slower and the both joints will start and stop their motion simultaneously.
- Both joints move at different speeds, but move continuously together.
- The resulting trajectory will be different.

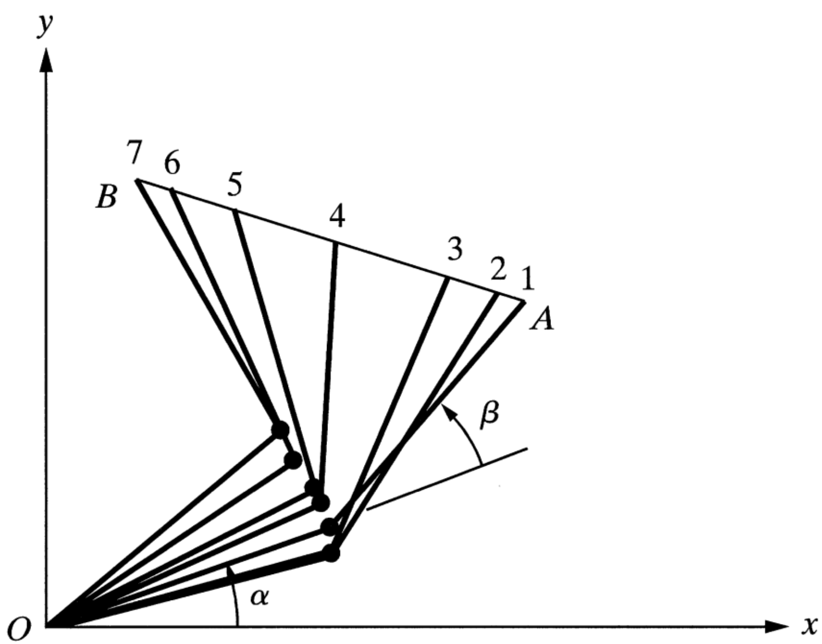
Straight Line Movement



α	β
20	30
14	55
16	69
21	77
29	81
40	80

- Let us assume that the robot hand follows a straight line between points A and B.
- The simplest way is to draw a line (interpolate) between A, B.
- Divide the line into five segments and solve for necessary angles α and β at each point.
- The joint angles do not change uniformly.

Straight Line Movement

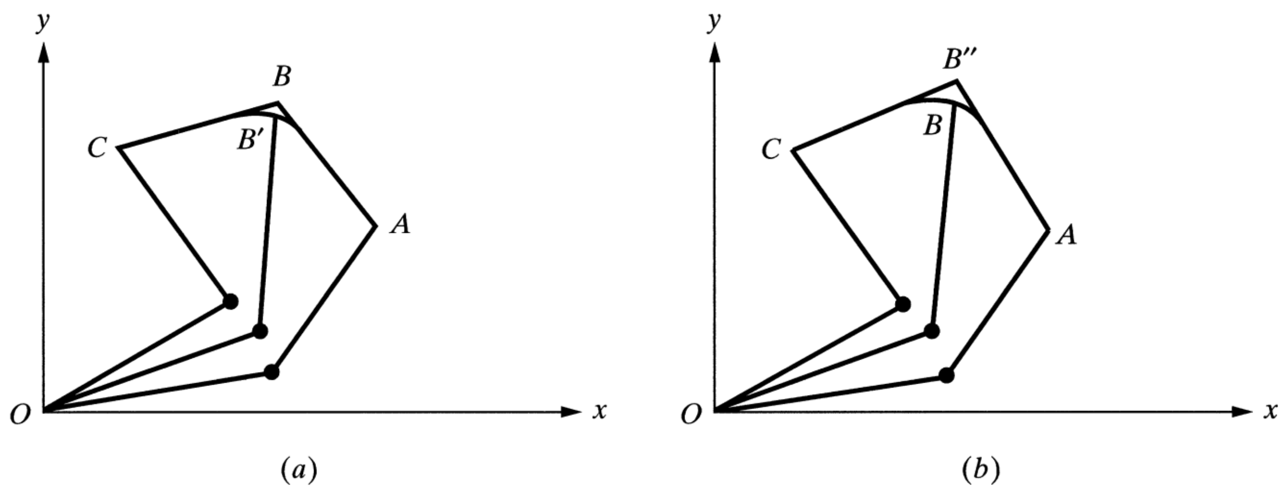


- Again interpolation between A, B by a straight line.
- The aim is to accelerate at the beginning and decelerate at the end.

- Divide the segments differently.
  - The arm move at earlier segments as we speed up at the beginning.
  - Go at a constant cruising rate.
  - Decelerate with later segments as approaching point B.

Continuous Transition Between Points

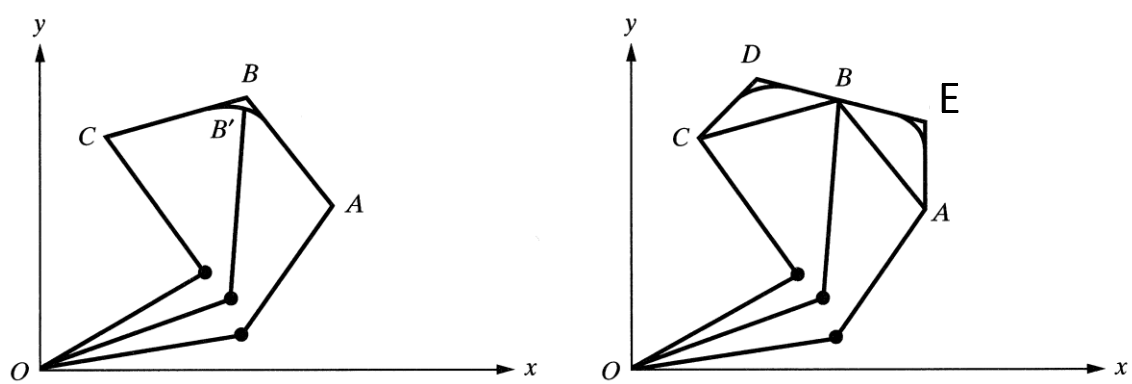
- Stop-and-go motion through the via-point list creates jerky motions with unnecessary stops.
- Solution: take multiple neighbouring trajectory into account and enforce constraints on the same tangent and acceleration on the trajectory point.
- How? Blend the two portions of the motion at point B.



Continuous Transition Between Points

Alternative scheme ensuring that the trajectory passes through control points.

- Two via points D and E are picked such that point B will fall on the straight-line section of the segment ensuring that the robot will pass through point B.



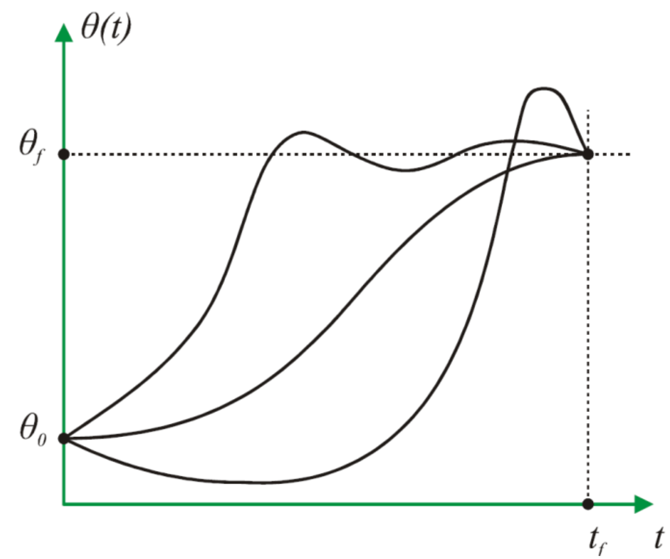
(a) (b)

Cubic Polynomials

Consider the problem of moving the tool frame from its initial position to a desired goal position. The initial position of the manipulator is known in the form of a set of joint angles. The set of joint angles for the terminal position is calculated by means of inverse kinematics.

What is required is **a function for each joint** whose value at t_0 is the initial position of the joint and whose value at t_f is the desired goal position of that joint.

As shown in the figure, there are many smooth functions, $\theta(t)$, that might be used to interpolate the joint value.



In making a single smooth function, at least four constraints of $\theta(t)$ are evident:

- Initial value $\theta(0) = \theta_0$,
- Final value$\theta(t_f) = \theta_\mathrm{f}$,
- Initial velocity $\dot{\theta}(0) = 0$,
- Final velocity $\dot{\theta}(t_f) = 0$.

Cubic Polynomials

These four constraints can be satisfied by a polynomial of third order:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Joint velocity and acceleration along this path are

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$

$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$

Applying the four constraints gives four equations for the unknown a_i:

$$\theta_0 = a_0$$
$$0 = a_1$$
$$\theta_f = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^3$$
$$0 = a_1 + 2a_2 t_f + 3a_3 {t_f}^2$$

The solution:

$$a_0 = \theta_0, \qquad a_1 = 0, \qquad a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0), \qquad a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0)$$

Cubic Polynomials

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$$a_0 = \theta_0, \qquad a_1 = 0, \qquad a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0), \qquad a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0)$$

This cubic polynomial can be used to connect any initial joint-angle position with any desired final position.

This solution is valid only for the case when the joint starts and finishes at zero velocity.

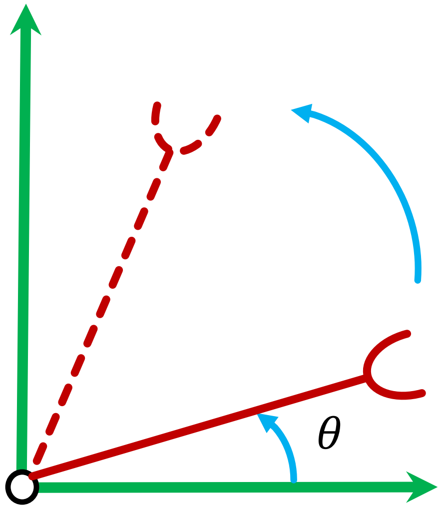
The single Cubic Polynomial equation that satisfies these conditions is:

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\boldsymbol{\theta(t) = \theta_0 + \frac{3}{{t_f}^2}(\theta_f - \theta_0)t^2 - \frac{2}{{t_f}^3}(\theta_f - \theta_0)t^3}$$

Example

A single-link manipulator with a revolt joint stopping at $\theta = 15$ degrees.
It is desired to move the joint in a smooth manner to $\theta = 75$ degrees in 3 seconds.
Find the coefficients of a cubic that accomplishes this motion and brings the manipulator to rest at the goal.



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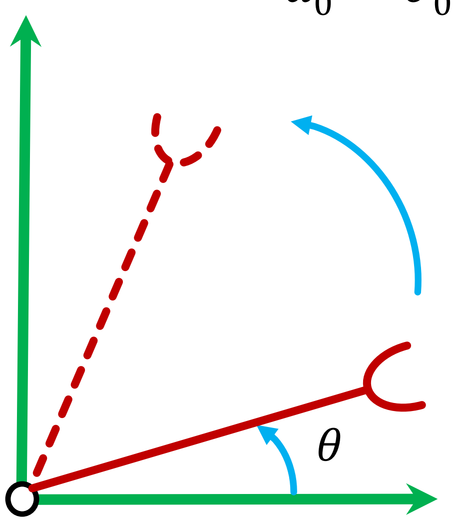
Solution:

$$a_0 = \theta_0, \qquad a_1 = 0, \qquad a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0), \qquad a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0)$$



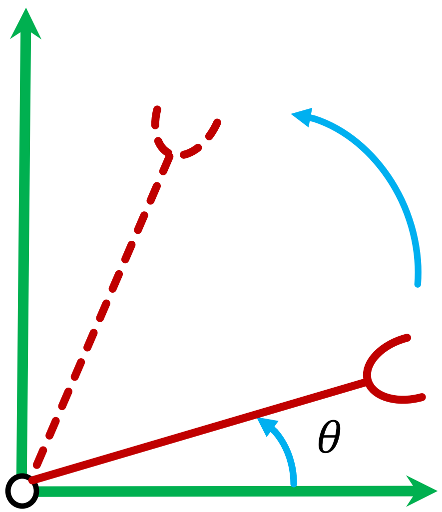
$$a_0 = 15,$$
$$a_1 = 0,$$
$$a_2 = \frac{3}{9}(75 - 15) = 20,$$
$$a_3 = -\frac{2}{27}(75 - 15) = -4.44$$

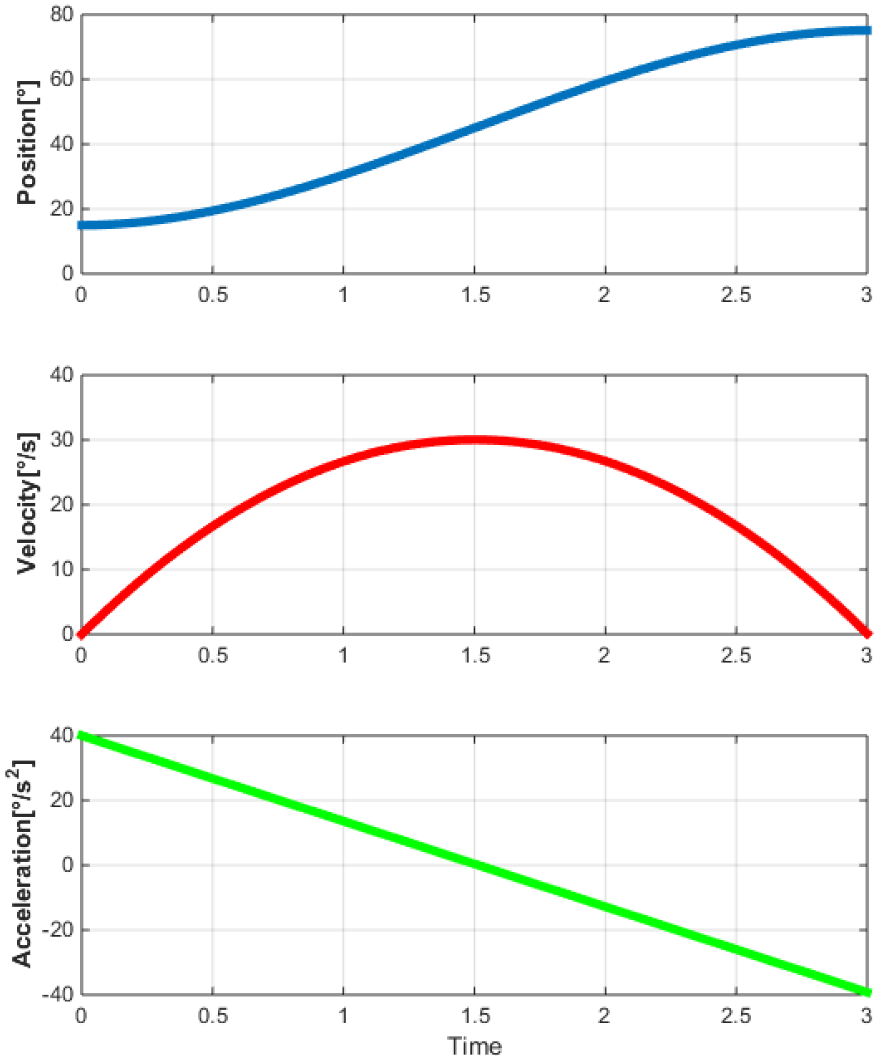
Example

Joint position:
$\theta(t) = 15 + 20\,t^2 - 4.44\,t^3$

Joint velocity along this path:
$\dot{\theta}(t) = 40\,t - 13.32\,t^2$

Joint acceleration along this path:
$\ddot{\theta}(t) = 40 - 26.4\,t$





Example

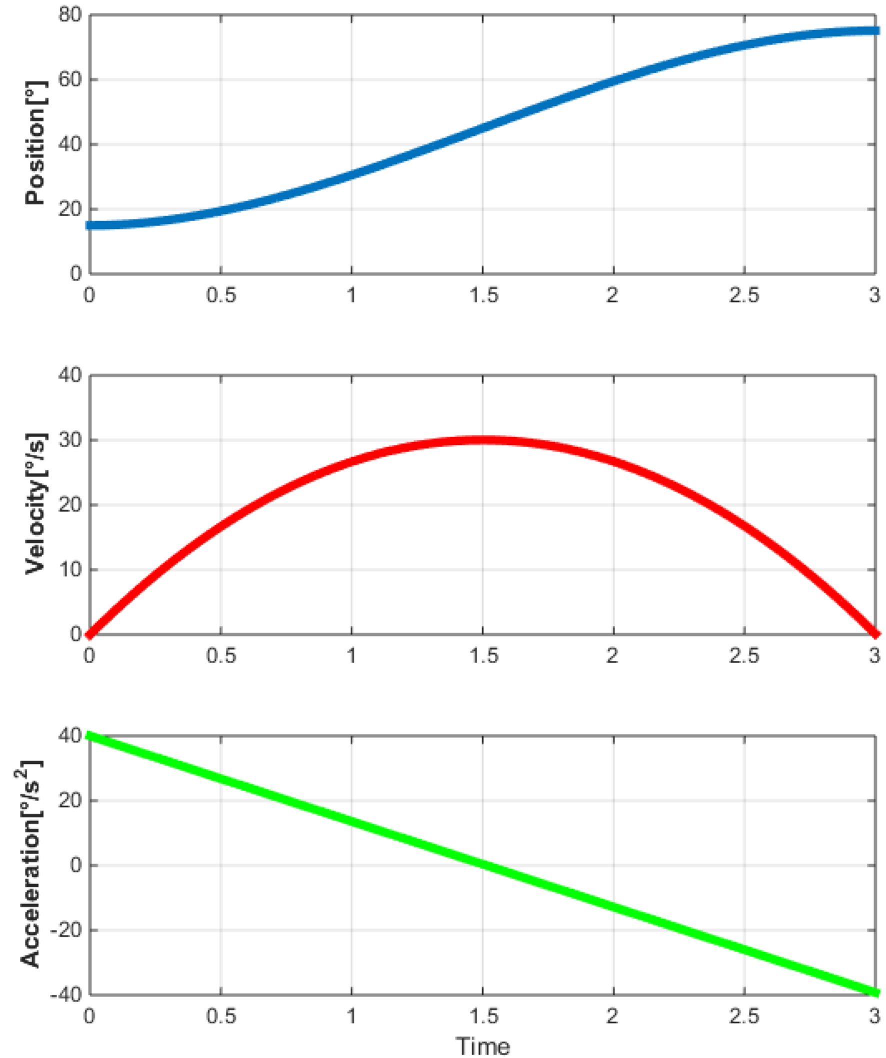
```
t = 0:0.01:3;

theta_t = 15 + 20.*t.*t - 4.44.*t.*t.*t;
theta_d_t = 40.*t - 13.32.*t.*t;
theta_2d_t = 40 - 26.4.*t;

subplot(3,1,1);
plot(t,theta_t); grid on;
ylabel('Position[°]');

subplot(3,1,2);
plot(t,theta_d_t); grid on;
ylabel('Velocity[°/s]');

subplot(3,1,3);
plot(t,theta_2d_t); grid on;
ylabel('Acceleration[°/s^2]');
```



Cubic Polynomials With via Points

If the desired velocities of the joints at the via points have non-zero values, then we can construct cubic polynomials as before with considering new constraints:

- Initial value $\theta(0) = \theta_0$,
- Final value$\theta(t_f) = \theta_\mathrm{f}$,
- Initial velocity $\dot{\theta}(0) = \dot{\theta}_0$,
- Final velocity $\dot{\theta}(t_f) = \dot{\theta}_f$.

Applying the four constraints gives four equations for the unknown a_i:

$$\theta_0 = a_0$$
$$\dot{\theta}_0 = a_1$$
$$\theta_f = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^3$$
$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 {t_f}^2$$

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$$\theta_f = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^3$$
$$\dot{\theta}_f = a_1 + 2a_2 t_f + 3a_3 {t_f}^2$$

The solution:

$$a_0 = \theta_0$$
$$a_1 = \dot{\theta}_0$$
$$a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0) - \frac{1}{t_f}\left(2\dot{\theta}_0 + \dot{\theta}_f\right)$$
$$a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0) + \frac{1}{{t_f}^2}\left(\dot{\theta}_0 + \dot{\theta}_f\right)$$

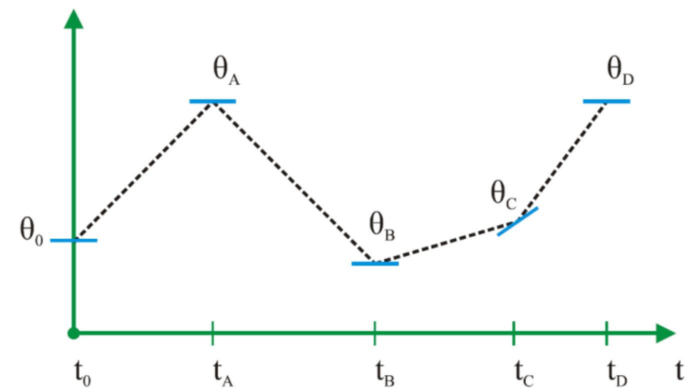
Now we are able to calculate the cubic polynomial that connects any initial and final positions with any initial and final velocities.

Velocities at via Points

There are several ways to work out the desired velocity at the via points:

The user specifies the desired velocity at each via point in terms of a Cartesian linear and angular velocity of the tool frame at that instant. Cartesian velocities at the via points are mapped to the desired joint velocity by using the inverse Jacobian of the manipulator at that point.

The system automatically chooses the velocities at the via points. Desired velocities at the points are indicated with the tangents. The via points are connected with straight line segments. If the slope of these lines changes the sign at the via point, choose zero velocity (point A and B), else, choose the average of the two slopes as the via velocity (point C).



Velocities at via Points

The system automatically chooses the velocities at the via points in such a way that acceleration is continuous at the via points.

To do this, a new approach is needed. We will replace the two velocity constraints at the connection of two cubics with the two constraints that velocity and acceleration be continuous.

Example

Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous velocity and acceleration at the via point. The given values are:

- **the initial angle θ_0,**
- **the via point θ_v,**
- **the goal point θ_g.**

$$\boldsymbol{\theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3}$$
$$\boldsymbol{\theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3}$$

Angular constraints for first cubic:
Initial position $\boldsymbol{\theta}_0 = \boldsymbol{a_{10}}$
Terminal position $\boldsymbol{\theta_v = a_{10} + a_{11}t_{f1} + a_{12}{t_{f1}}^2 + a_{13}{t_{f1}}^3}$

Angular constraints for second cubic:
Initial position $\boldsymbol{\theta}_v = \boldsymbol{a_{20}}$
Terminal position $\boldsymbol{\theta_g = a_{20} + a_{21}t_{f2} + a_{22}{t_{f2}}^2 + a_{23}{t_{f2}}^3}$

Example

Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous velocity and acceleration at the via point. The given values are:

- **the initial angle θ_0,**
- **the via point θ_v,**
- **the goal point θ_g.**

$$\theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$
$$\theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

Angular velocity constraint for first cubic: Start from rest:

$$0 = a_{11}$$

Angular velocity constraint for second cubic: End at rest:

$$0 = a_{21} + 2\,a_{22}t_{f2} + 3\,a_{23}{t_{f2}}^2$$

Both cubics must have the same angular velocity and acceleration at the via point:

$$a_{11} + 2\,a_{12}t_{f1} + 3\,a_{13}{t_{f1}}^2 = a_{21}$$
$$2\,a_{12} + 6\,a_{13}t_{f1} = 2\,a_{22}$$

Example

$\boldsymbol{\theta}_0 = \boldsymbol{a_{10}}$

$\boldsymbol{\theta_v = a_{10} + a_{11}t_{f1} + a_{12}t_{f1}^{\,2} + a_{13}t_{f1}^{\,3}}$

$\boldsymbol{\theta}_{\mathrm{v}} = \boldsymbol{a}_{20}$

$\boldsymbol{\theta_g = a_{20} + a_{21}t_{f2} + a_{22}t_{f2}^{\,2} + a_{23}t_{f2}^{\,3}}$

$\boldsymbol{0 = a_{11}}$

$\boldsymbol{0 = a_{21} + 2\,a_{22}t_{f2} + 3\,a_{23}t_{f2}^{\,2}}$

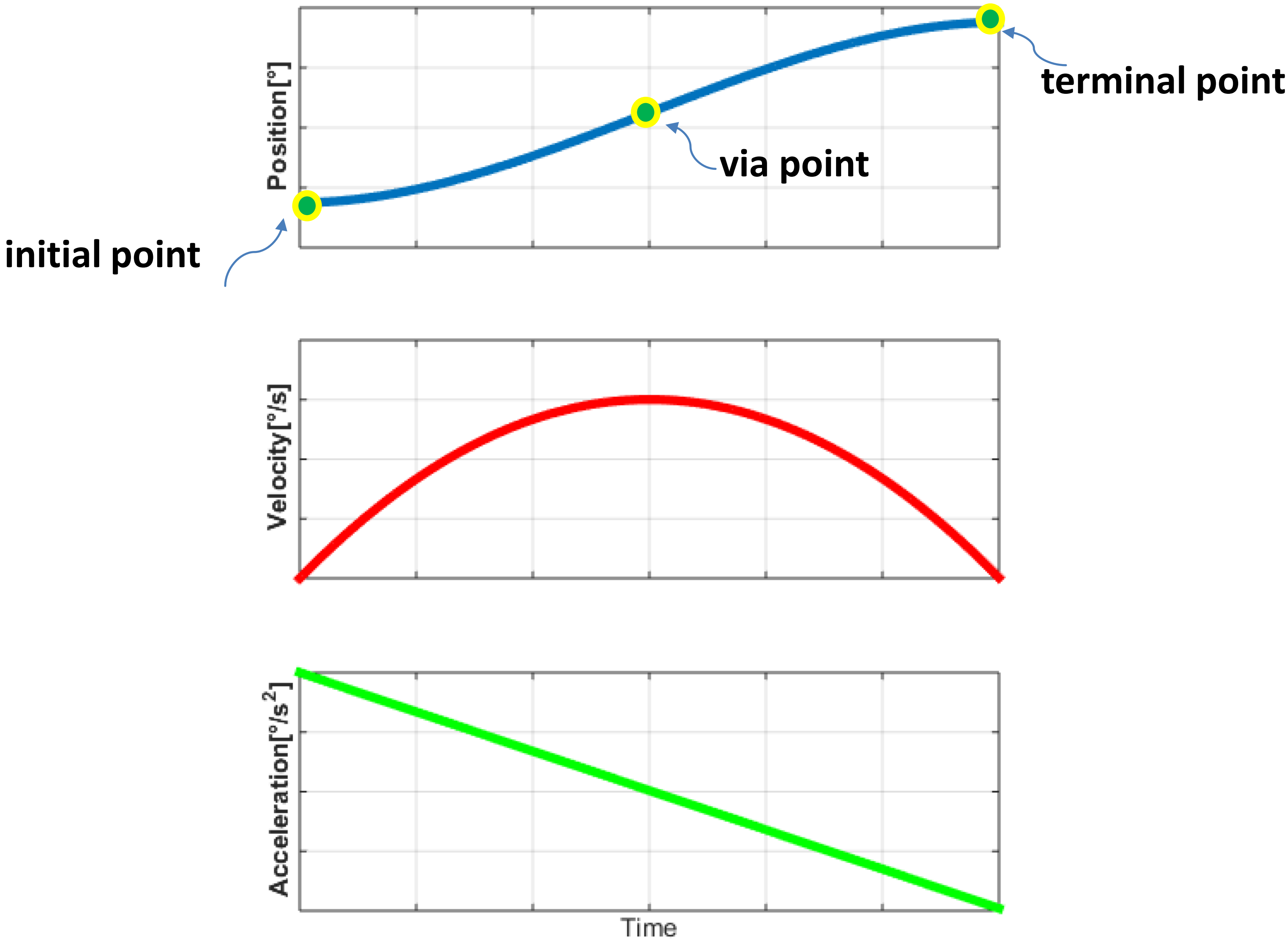
$\boldsymbol{a_{11} + 2\,a_{12}t_{f1} + 3\,a_{13}t_{f1}^{\,2} = a_{21}}$

$\boldsymbol{2\,a_{12} + 6\,a_{13}t_{f1} = 2\,a_{22}}$

If we consider $t_f = t_{f1} = t_{f2}$, solution:

$$\boldsymbol{a_{10}} = \boldsymbol{\theta}_0$$
$$\boldsymbol{a_{11} = 0}$$
$$\boldsymbol{a_{12} = \frac{12\theta_v - 3\theta_g - \theta_0}{4t_f^{\,2}}}$$
$$\boldsymbol{a_{13} = \frac{-8\,\theta_v + 3\theta_g + 5\,\theta_0}{4t_f^{\,3}}}$$

$$\boldsymbol{a_{20}} = \boldsymbol{\theta}_{\mathrm{v}}$$
$$\boldsymbol{a_{21} = \frac{3\,\theta_g - 3\,\theta_0}{4t_f}}$$
$$\boldsymbol{a_{22} = \frac{-12\theta_v + 6\theta_g + 6\theta_0}{4t_f^{\,2}}}$$
$$\boldsymbol{a_{23} = \frac{8\theta_v - 5\theta_g - 3\theta_0}{4t_f^{\,3}}}$$



terminal point
via point
initial point
Position[°]
Velocity[°/s]
Acceleration[°/s²]
Time

Quintic Polynomials

If we wish to be able to **specify the position, velocity, and acceleration** at the beginning and end of a path segment, a **quintic polynomial** is required:

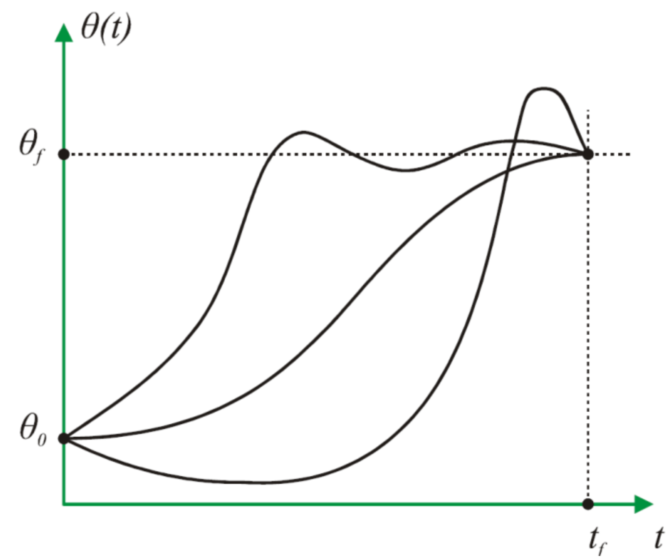
$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

Where the constraints are given as:

$$\theta_0 = a_0$$
$$\theta_f = a_0 + a_1 t_f + a_2 t_f^{\,2} + a_3 t_f^{\,3} + a_4 t_f^{\,4} + a_5 t_f^{\,5}$$

$$\dot{\theta}_0 = a_1$$
$$\dot{\theta}_f = a_1 + 2\,a_2 t_f + 3\,a_3 t_f^{\,2} + 4\,a_4 t_f^{\,3} + 5\,a_5 t_f^{\,4}$$

$$\ddot{\theta}_0 = 2\,a_2$$
$$\ddot{\theta}_f = 2\,a_2 + 6\,a_3 t_f + 12\,a_4 t_f^{\,2} + 20\,a_5 t_f^{\,3}$$



These constraints specify a linear set of six equations for the six unknowns

Quintic Polynomials

The solution of this system of six linear equations is:

$$a_0 = \Theta_0, \quad a_1 = \dot{\Theta}_0, \quad a_2 = \frac{\ddot{\Theta}_0}{2},$$

$$a_3 = \frac{20\Theta_f - 20\Theta_0 - (8\dot{\Theta}_f + 12\dot{\Theta}_0)t_f - (3\ddot{\Theta}_0 - \ddot{\Theta}_f)t_f^{\,2}}{2t_f^{\,3}},$$

$$a_4 = \frac{30\Theta_0 - 30\Theta_f + (14\dot{\Theta}_f + 16\dot{\Theta}_0)t_f + (3\ddot{\Theta}_0 - 2\ddot{\Theta}_f)t_f^{\,2}}{2t_f^{\,4}},$$

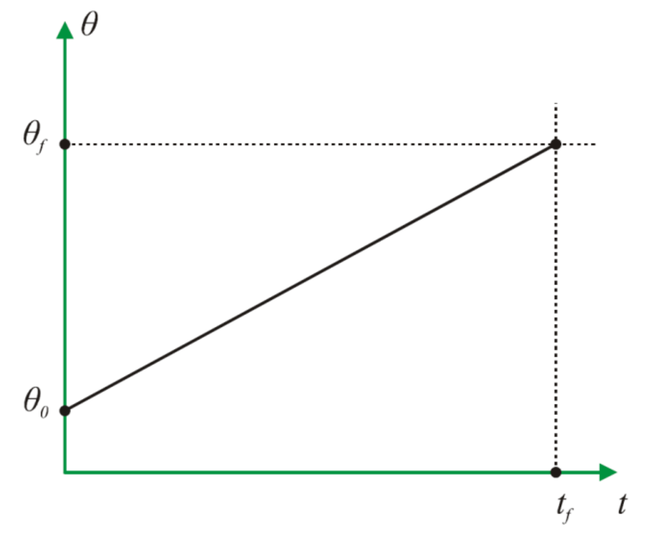
$$a_5 = \frac{12\Theta_f - 12\Theta_0 - (6\dot{\Theta}_f + 6\dot{\Theta}_0)t_f - (\ddot{\Theta}_0 - \ddot{\Theta}_f)t_f^{\,2}}{2t_f^{\,5}}.$$

Linear interpolation

Another choice of joint-path shape is linear.
That is, we simply interpolate linearly to move from the present joint position to the final position.
Remember that, although the motion of each joint in this scheme is linear, the end-effector in general does not move in a straight line in Cartesian space.

However, straightforward linear interpolation would cause the velocity to be **discontinuous at the beginning and at the end of the motion**.

To create a smooth path with continuous position and velocity, we start with the linear function but add a **parabolic blend region** at each path point.

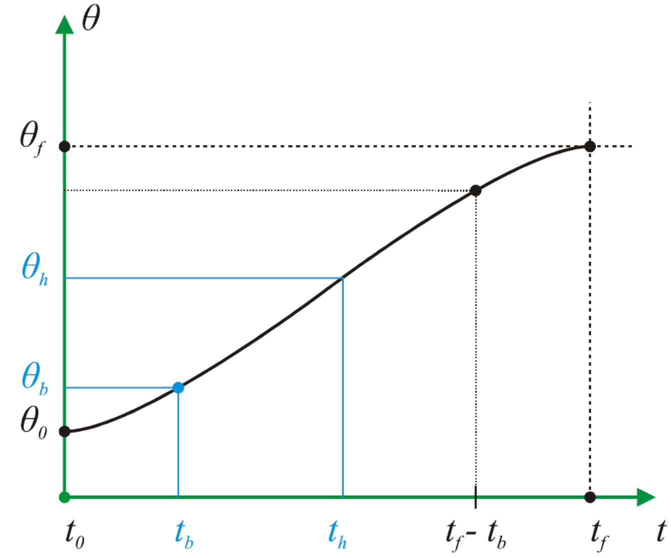


Linear Int. with Parabolic Blends

During the blend part of the trajectory, **constant acceleration** is used to **change velocity smoothly**.

The linear function and the two parabolic functions are “splined” together so that the entire path is continuous in position and velocity.

We will assume that the parabolic blends have the same duration; therefore, the same constant acceleration is used during the blends.

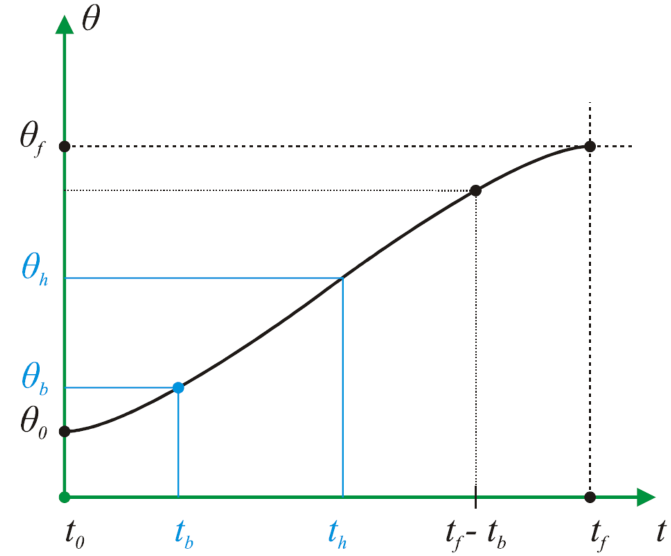


Linear Int. with Parabolic Blends

The velocity at the end of the blend region must equal the velocity of the linear section, and so we have:

$$\ddot{\boldsymbol{\theta}}\boldsymbol{t_b} = \frac{\boldsymbol{\theta_h} - \boldsymbol{\theta_b}}{\boldsymbol{t_h} - \boldsymbol{t_b}}$$

where $\boldsymbol{\theta_b}$ is the joint angle at the end of the blend region, and $\ddot{\boldsymbol{\theta}}$ is the joint acceleration acting during the blend region.



The value of $\boldsymbol{\theta_b}$ is given by $\boldsymbol{\theta_b} = \boldsymbol{\theta_0} + \frac{1}{2}\ddot{\boldsymbol{\theta}}\boldsymbol{t_b}^2$

Combining the two equations and taking into account the symmetry of the path and its duration $\boldsymbol{t_f} = \boldsymbol{2t_h}$, we get:

$$\ddot{\boldsymbol{\theta}}\boldsymbol{t_b}(\boldsymbol{t_h} - \boldsymbol{t_b}) = \boldsymbol{\theta_h} - \boldsymbol{\theta_0} - \frac{1}{2}\ddot{\boldsymbol{\theta}}\boldsymbol{t_b}^2$$

Linear Int. with Parabolic Blends

$$\ddot{\theta} t_b (t_h - t_b) = \theta_h - \theta_0 - \frac{1}{2}\ddot{\theta} {t_b}^2$$

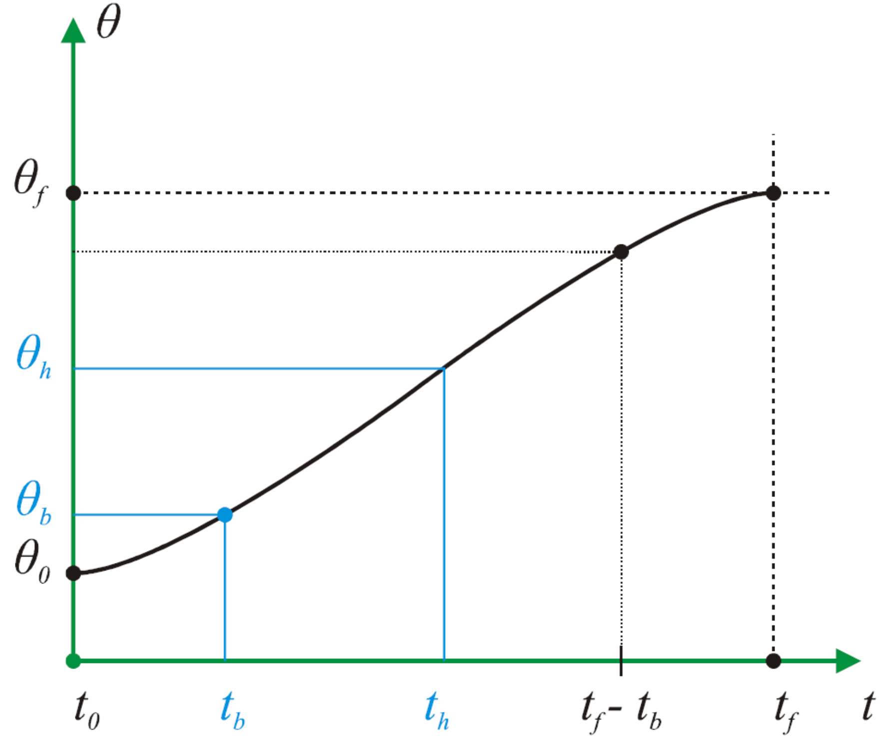
$$\Rightarrow \; -\ddot{\theta} {t_b}^2 + \ddot{\theta} t_b t_h = \theta_h - \theta_0 - \frac{1}{2}\ddot{\theta} {t_b}^2$$

$$\Rightarrow \; \frac{1}{2}\ddot{\theta} {t_b}^2 - \ddot{\theta} t_b t_h + (\theta_h - \theta_0) = 0$$

$$\Rightarrow \; \frac{1}{2}\ddot{\theta} {t_b}^2 - \ddot{\theta} t_b \frac{t_f}{2} + \frac{1}{2}(\theta_f - \theta_0) = 0$$

$$\Rightarrow \; \ddot{\theta} {t_b}^2 - \ddot{\theta} t_b t_f + (\theta_f - \theta_0) = 0$$

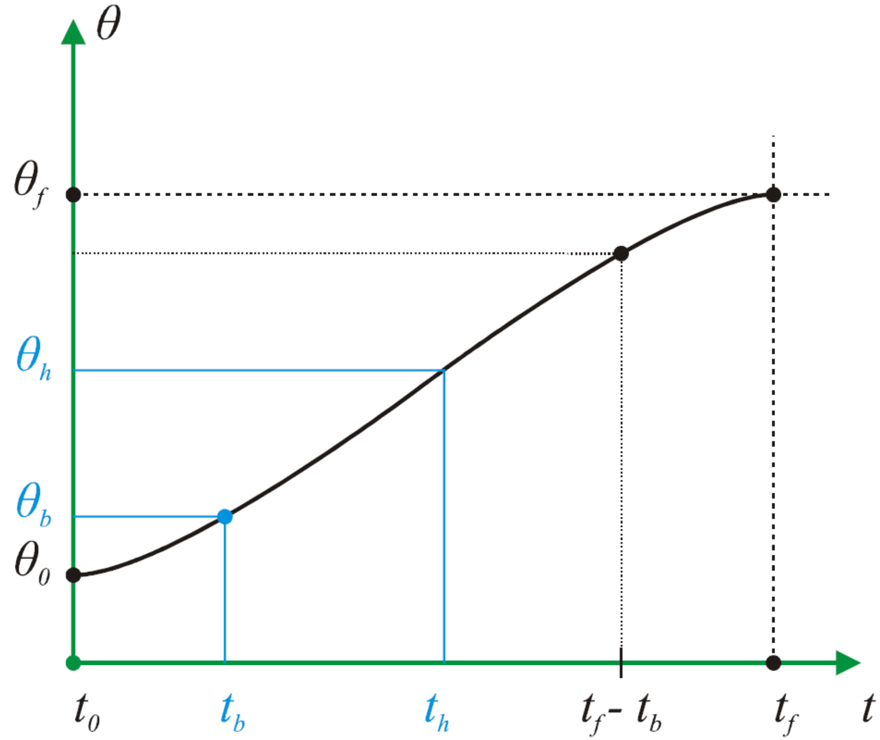
where $\mathbf{t_f}$ is the desired duration of the motion.



Linear Int. with Parabolic Blends

$$\ddot{\theta} t_b^2 - \ddot{\theta} t_b t_f + (\theta_f - \theta_0) = 0$$

Given θ_f, θ_0 and t_f we can follow any of the path given by the choices of $\ddot{\theta}$ and t_b that satisfy the previous equation.



The solution of the equation for the blend duration is:

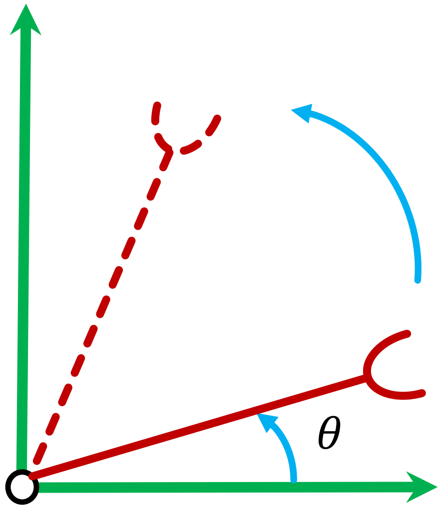
$$t_b = \frac{t_f}{2} - \frac{\sqrt{\ddot{\theta}^2 t_f^2 - 4\ddot{\theta}(\theta_f - \theta_0)}}{2\ddot{\theta}}$$

A real solution exists if:

$$\ddot{\theta} \geq \frac{4\,(\theta_f - \theta_0)}{t_f^2}$$

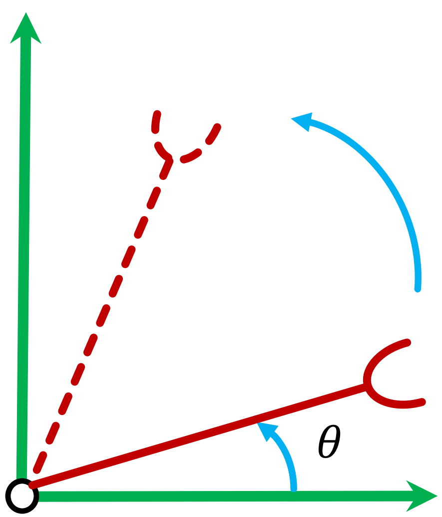
Example

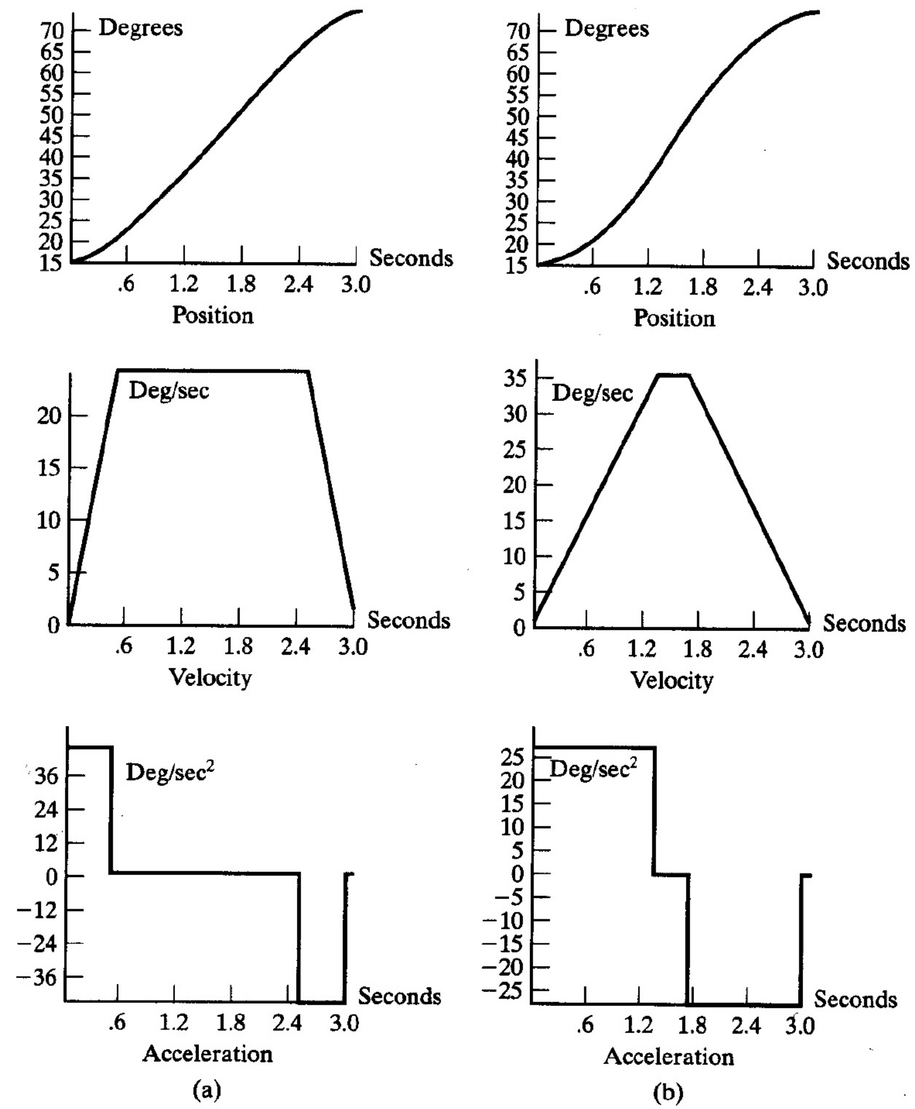
A single-link manipulator with a revolt joint stopping at $\theta = 15$ degrees.
It is desired to move the joint in a smooth manner to $\theta = 75$ degrees in 3 seconds.
Show two examples, one with high acceleration and one with low acceleration of a linear path with parabolic blends.



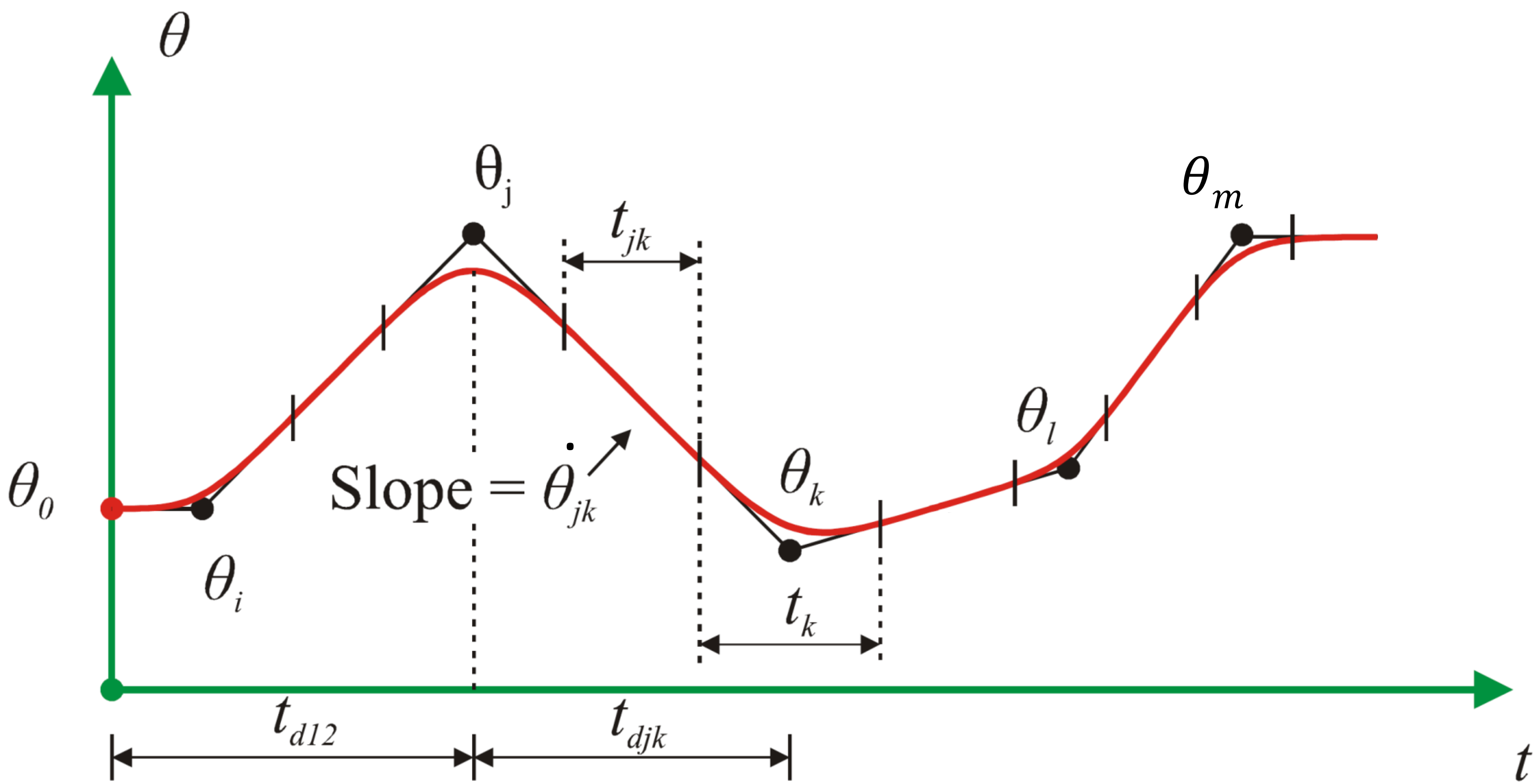
Example

Position, velocity, and acceleration profiles for linear interpolation with parabolic blends. The set of curves on the left is based on a higher acceleration during the blends than is that on the right.





Linear Int. with Parabolic Blends for Several Segments



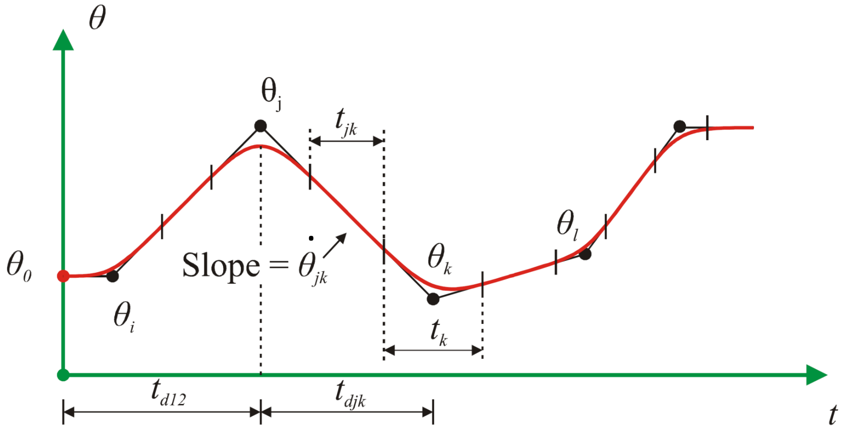
Linear Int. with Parabolic Blends for Several Segments

Given:

- Positions $\theta_i, \theta_j, \theta_k, \theta_l, \theta_m$
- Desired time durations t_{dij}, t_{djk}, t_{dkl}, t_{dlm}
- The magnitudes of the accelerations $|\ddot{\boldsymbol{\theta}}_{\mathrm{i}}|, |\ddot{\boldsymbol{\theta}}_{\mathrm{j}}|, |\ddot{\boldsymbol{\theta}}_{\mathrm{k}}|, |\ddot{\boldsymbol{\theta}}_{\mathrm{l}}|$

Compute:

- Blends times t_i, t_j, t_k, t_l, t_m
- Straight segment times t_{ij}, t_{jk}, t_{kl}, t_{lm}
- Slopes (velocities) $\dot{\theta}_{ij}, \dot{\theta}_{jk}, \dot{\theta}_{kl}, \dot{\theta}_{lm}$
- Signed accelerations



Linear Int. with Parabolic Blends for Several Segments

For inside segments:

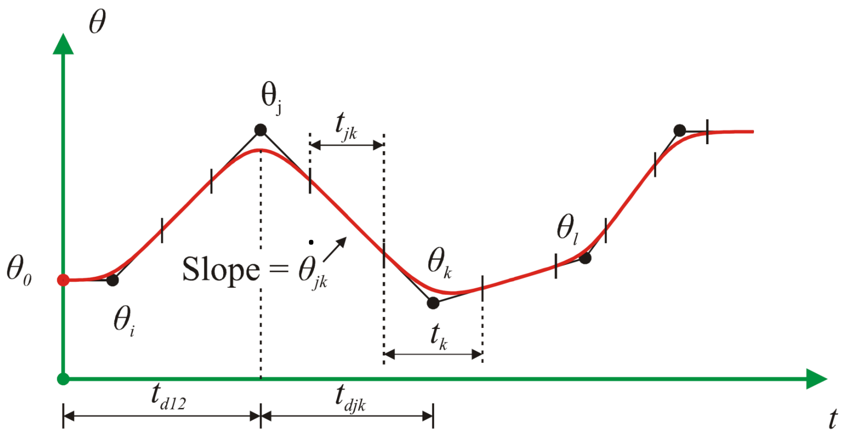
The duration of the blend region at path point *k* is t_k.
The duration of the **linear portion** between points *j* and *k* is t_{jk}.

$$\dot{\Theta}_{jk} = \frac{\Theta_k - \Theta_j}{t_{djk}},$$

$$\ddot{\Theta}_k = SGN(\dot{\Theta}_{kl} - \dot{\Theta}_{jk})\left|\ddot{\Theta}_k\right|,$$

$$t_k = \frac{\dot{\Theta}_{kl} - \dot{\Theta}_{jk}}{\ddot{\Theta}_k},$$

$$t_{jk} = t_{djk} - \frac{1}{2}\left(t_j + t_k\right).$$



Linear Int. with Parabolic Blends for Several Segments

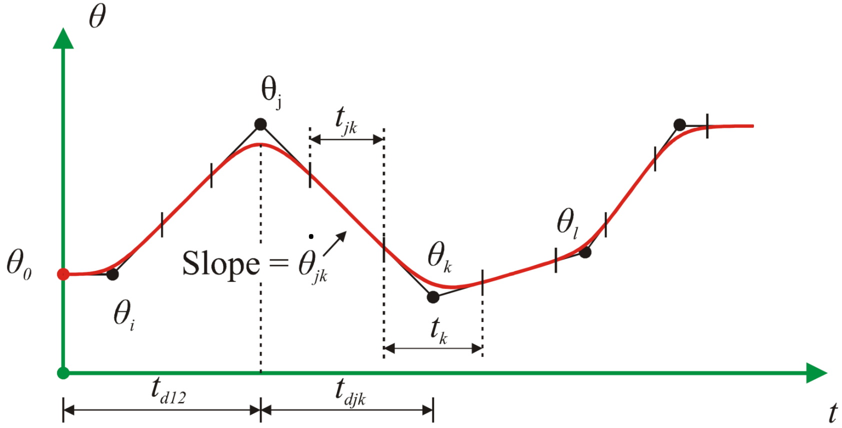
For the first segments:

$$\ddot{\Theta}_1 = SGN(\Theta_2 - \Theta_1)\left|\ddot{\Theta}_1\right|$$

$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(\Theta_2 - \Theta_1)}{\ddot{\Theta}_1}}$$

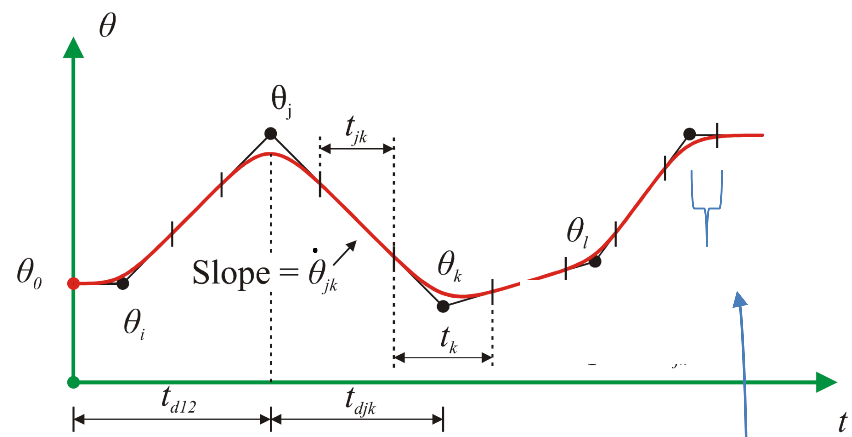
$$\dot{\Theta}_{12} = \frac{\Theta_2 - \Theta_1}{t_{d12} - \frac{1}{2}t_1}$$

$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$



Linear Int. with Parabolic Blends for Several Segments

For the last segments:



$$\ddot{\Theta}_n = SGN(\Theta_{n-1} - \Theta_n)\left|\ddot{\Theta}_n\right|,$$

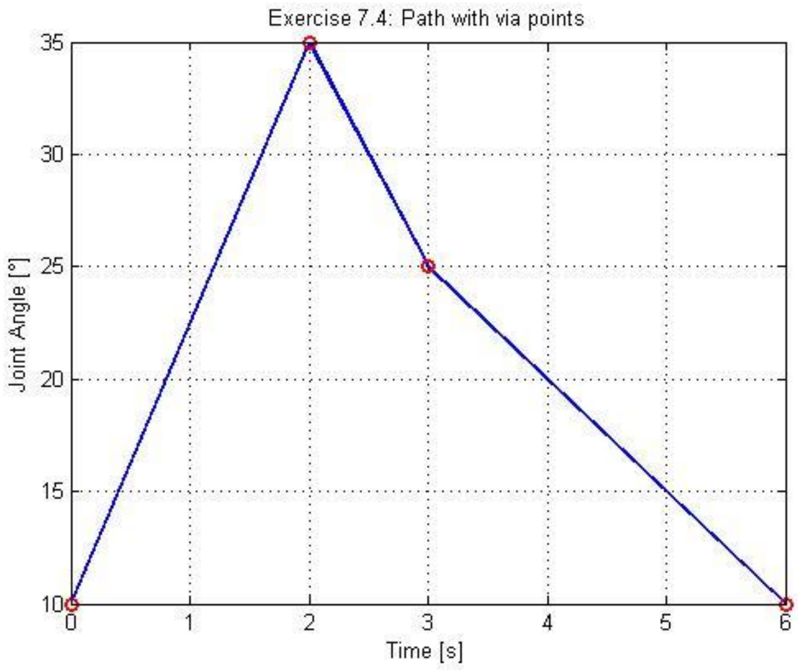
$$t_n = t_{d(n-1)n} - \sqrt{t_{d(n-1)n}^2 + \frac{2(\Theta_n - \Theta_{n-1})}{\ddot{\Theta}_n}}$$

$$\dot{\Theta}_{(n-1)n} = \frac{\Theta_n - \Theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n},$$

$$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2}t_{n-1}$$

Example

The trajectory of a particular joint is specified as follows: Path points in degrees: 10, 35, 25, 10. The duration of these three segments should be 2, 1, and 3 seconds, respectively. The magnitude of the default acceleration to use at all blend points is 50 degrees/second^2. **Calculate all segment velocities, blend times, and linear times and sketch the resulting trajectory.**



Example

For the first segment we apply:

$$\ddot{\Theta}_1 = SGN(\Theta_2 - \Theta_1)\left|\ddot{\Theta}_1\right|,$$

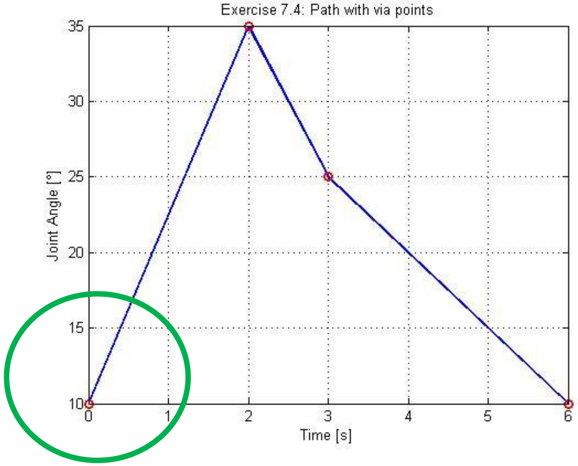
$$t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(\Theta_2 - \Theta_1)}{\ddot{\Theta}_1}},$$

$$\dot{\Theta}_{12} = \frac{\Theta_2 - \Theta_1}{t_{d12} - \frac{1}{2}t_1}.$$

$$\ddot{\Theta}_1 = 50\frac{\text{deg}}{s^2},$$

$$t_1 = 2s - \sqrt{4s^2 - \frac{2(35^\circ - 10^\circ)}{50\frac{^\circ}{s^2}}} = 2 - \sqrt{3} = 0.268s$$

$$\dot{\Theta}_{12} = \frac{35^\circ - 10^\circ}{2 - \frac{1}{2}0.268s} = 13.398\frac{\text{deg}}{s}.$$



Example

For the second segment we use:

$$\dot{\Theta}_{jk} = \frac{\Theta_k - \Theta_j}{t_{djk}},$$

$$\ddot{\Theta}_k = SGN\left(\dot{\Theta}_{kl} - \dot{\Theta}_{jk}\right)\left|\ddot{\Theta}_k\right|,$$

$$t_k = \frac{\dot{\Theta}_{kl} - \dot{\Theta}_{jk}}{\ddot{\Theta}_k},$$

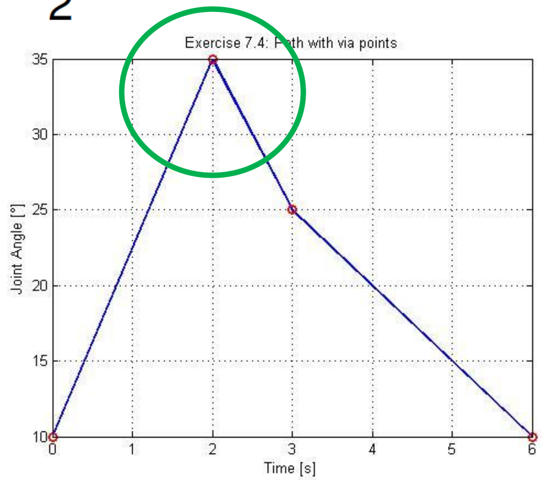
$$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2 .$$

$$\dot{\Theta}_{23} = \frac{\Theta_3 - \Theta_2}{t_{d23}} = \frac{25° - 35°}{1s} = -10\frac{deg}{s},$$

$$\ddot{\Theta}_2 = SGN\left(\dot{\Theta}_{23} - \dot{\Theta}_{12}\right)\left|\ddot{\Theta}_2\right| = -50\frac{deg}{s^2},$$

$$t_2 = \frac{-10\frac{deg}{s} - 13.398\frac{deg}{s}}{-50\frac{deg}{s^2}} = 0.468s,$$

$$t_{12} = 2 - 0.268s - \frac{1}{2}0.486s = 1.489s.$$



Example

For the last segment we apply:

$$\ddot{\Theta}_n = SGN\left(\Theta_{n-1} - \Theta_n\right)\left|\ddot{\Theta}_n\right|,$$

$$t_n = t_{d(n-1)n} - \sqrt{t_{d(n-1)n}^2 + \frac{2\left(\Theta_n - \Theta_{n-1}\right)}{\ddot{\Theta}_n}},$$

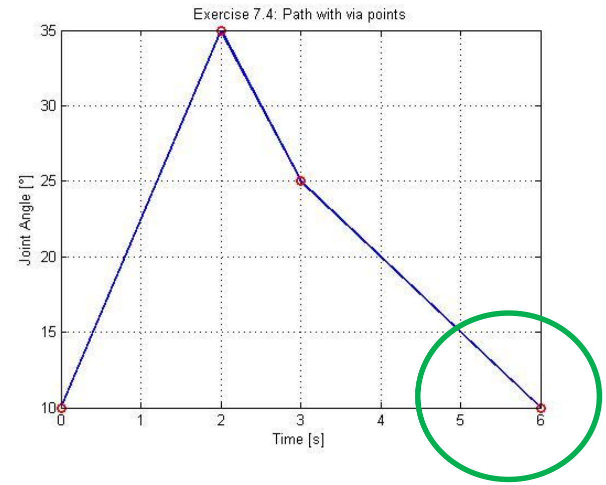
$$\dot{\Theta}_{d(n-1)n} = \frac{\Theta_n - \Theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n}.$$

$$\ddot{\Theta}_4 = SGN\left(\Theta_3 - \Theta_4\right)\left|\ddot{\Theta}_4\right| = \ddot{\Theta}_4 = 50\frac{\text{deg}}{s^2},$$

$$t_4 = t_{d34} - \sqrt{t_{d34}^2 + \frac{2\left(\Theta_4 - \Theta_3\right)}{\ddot{\Theta}_4}}$$

$$t_4 = 3s - \sqrt{9s^2 + \frac{2\left(10° - 25°\right)}{50\frac{°}{s^2}}} = 0.102s,$$

$$\dot{\Theta}_{34} = \frac{\Theta_4 - \Theta_3}{t_{d34} - \frac{1}{2}t_4} = \frac{10° - 25°}{3s - \frac{1}{2}0.102s} = -5.086\frac{\text{deg}}{s}.$$



Example

Next we use:

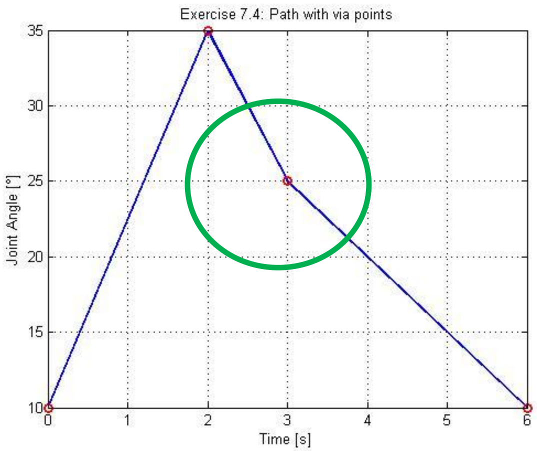
$$\ddot{\Theta}_k = SGN\left(\dot{\Theta}_{kl} - \dot{\Theta}_{jk}\right)\left|\ddot{\Theta}_k\right|,$$

$$t_k = \frac{\dot{\Theta}_{kl} - \dot{\Theta}_{jk}}{\ddot{\Theta}_k}.$$

$$\ddot{\Theta}_3 = SGN\left(\dot{\Theta}_{34} - \dot{\Theta}_{23}\right)\left|\ddot{\Theta}_3\right|$$

$$= SGN\left(-5.086\frac{deg}{s} + 10\frac{deg}{s}\right)50\frac{deg}{s^2} = 50\frac{deg}{s^2},$$

$$t_3 = \frac{\dot{\Theta}_{34} - \dot{\Theta}_{23}}{\ddot{\Theta}_3} = \frac{-5.086\frac{deg}{s} + 10\frac{deg}{s}}{50\frac{deg}{s^2}} = 0.098s.$$



Example

Finally we have

$$t_{jk} = t_{djk} - \frac{1}{2}\left(t_j + t_k\right)$$

and

$$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2} t_{n-1}.$$

to calculate the linear parts of the trajectory between points 2, 3 and 3, 4, respectively.

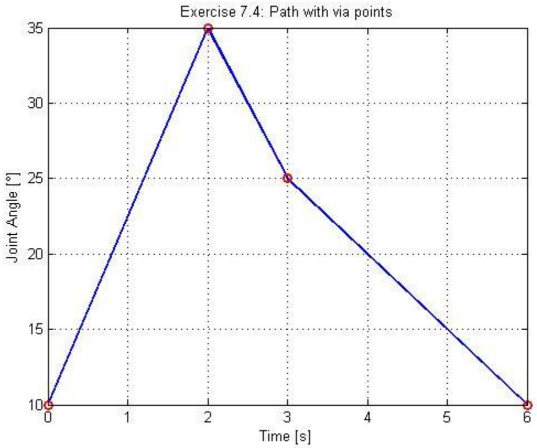
$$t_{23} = t_{d23} - \frac{1}{2}\left(t_2 + t_3\right)$$

$$= 1s - \frac{1}{2}(0.486s + 0.098s) = 0.708s$$

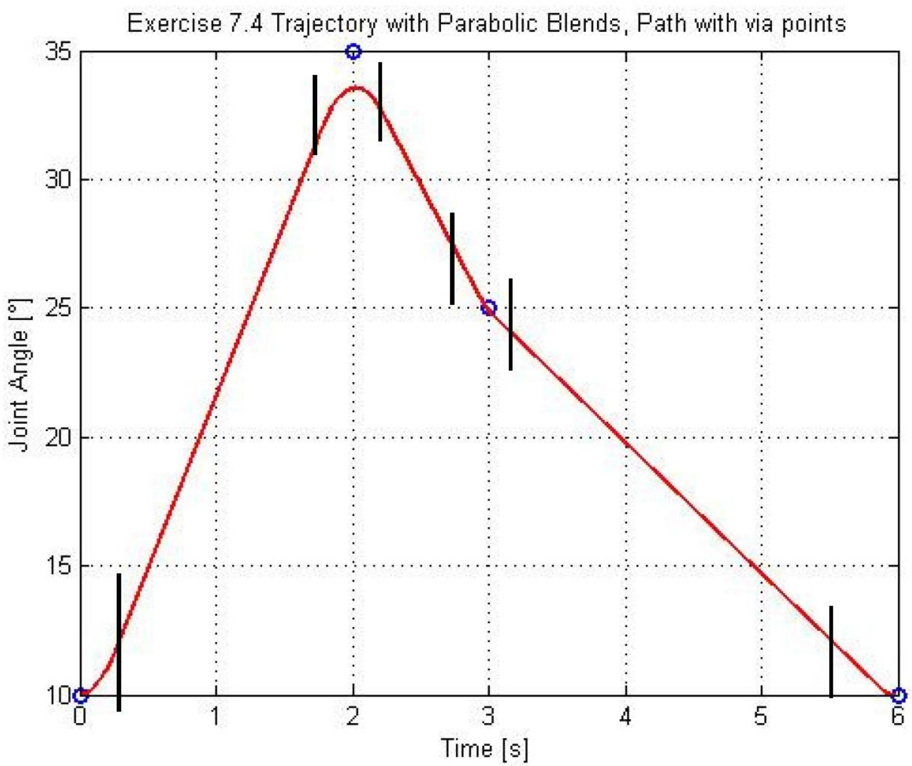
and

$$t_{34} = t_{d34} - t_4 - \frac{1}{2} t_3$$

$$= 3s - 0.102s - \frac{1}{2} 0.098s = 2.849s.$$



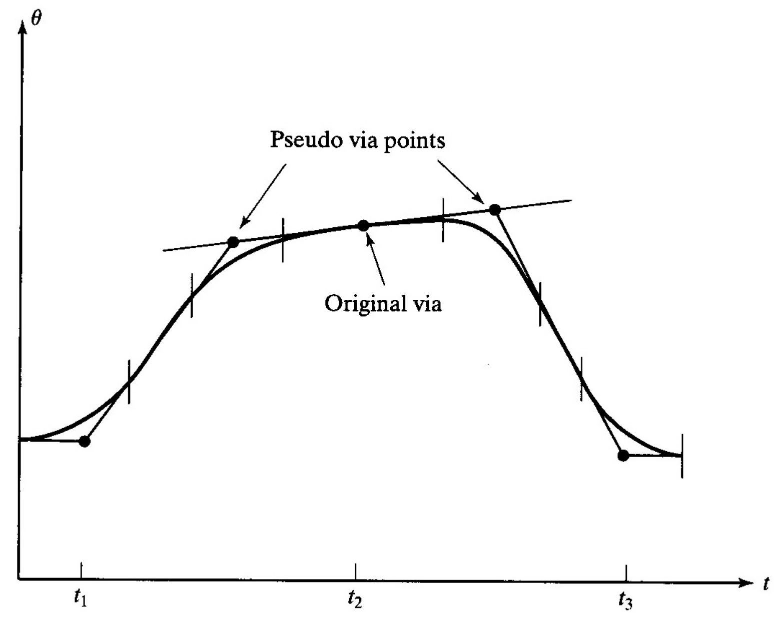
Example

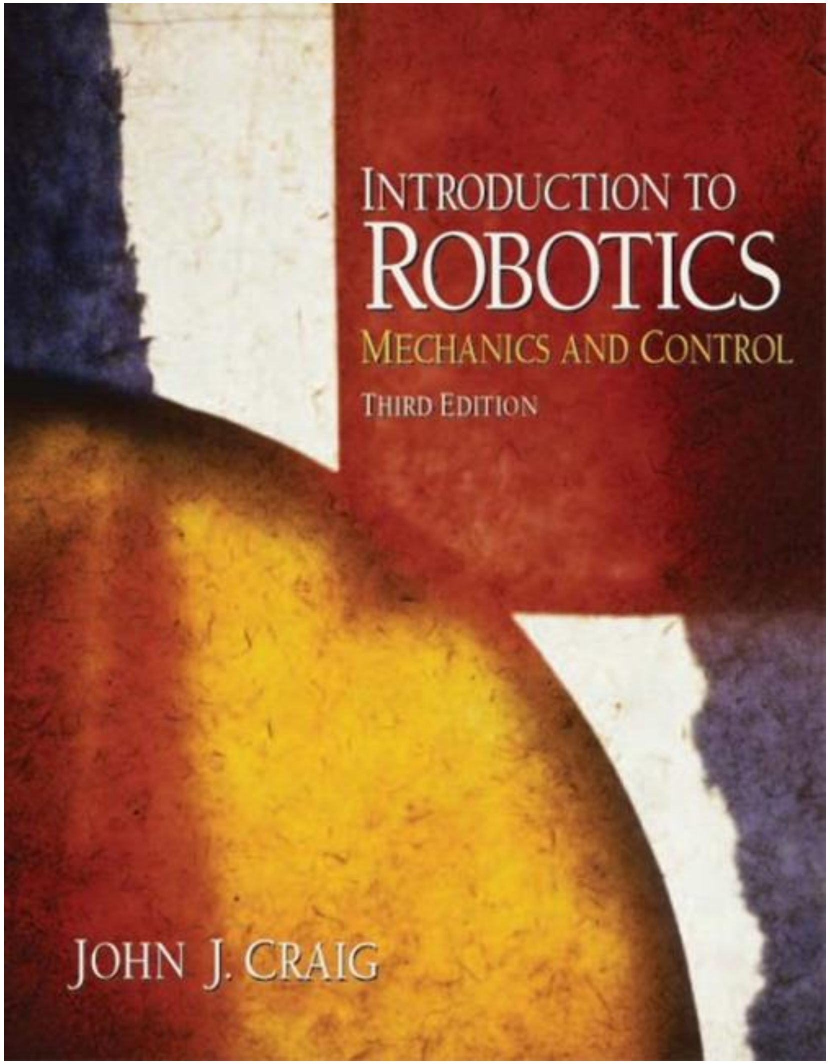


Note that in these linear-parabolic-blend splines the via points are not actually reached unless the manipulator comes to a stop.

Pseudo Via Points

If we wishes to specify that the manipulator pass exactly through a via point without stopping, we should introduce “Pseudo Via Points”

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