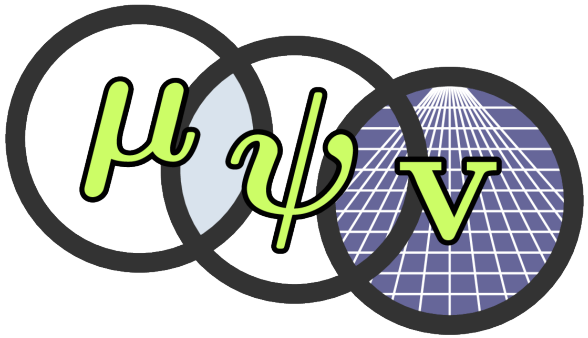

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Preferred Mental Models in Syllogistic Reasoning

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Introduction - Syllogisms

All red shapes are circles.
Some red shapes are marked with a star.

What, if anything, follows?

- Quantified premises describing relationships between three terms
- What kind of relation, if any, exists between the two end-terms?
  - 'circles' - subject
  - '(marked with a) star' - predicate
- Theories aim to explain and model processes behind human syllogistic reasoning
  - Mental Model Theory (MMT)[1]

[1] Johnson-Laird, P. (1975). Models of deduction. In *Reasoning: Representation and process in children and adults*, 7–54).

Introduction - Mental Models

All red shapes are circles.
Some red shapes are marked with a star.

What, if anything, follows?

- Given some observations, individuals create iconic representations – *mental models* – of possibilities
- Subjective mental representation of the information presented in a reasoning task
- Possible representations:

circles	[red]	[star]
circles		

circles		
circles	red	[star]
¬circles	¬red	

- Conclusion: "Some circles are marked with a star"
- To confirm validity all possible premise interpretations should be checked if they hold → difficult

Introduction - Preferred Mental Models

- Spatial relational reasoning domain - individuals have preferred mental models
- Model building process typically not addressed in syllogistic domain
  - Which models do individuals create?
  - Are the models correct?
  - Do they even have preferred models at all?
- Present experiment - visual responses showing representation of given syllogistic premises

RQ1: Can we examine what kind of models do individuals create from the premises of syllogistic tasks and do they have preferred mental models?

Introduction - Canonicality & mReasoner

- Canonical form - the minimal, simplest representation of an expression
- In mental models - which instances form a canonical set for a given syllogism?

All red shapes are circles

circle	red	canonical
¬circle	¬red	non-canonical

RQ2.1: How influential is the canonicality of mental models that individuals build for syllogistic premises on the correctness of derived conclusions?

Introduction - Canonicality & mReasoner

- mReasoner[2] - a LISP-based implementation of MMT for syllogistic reasoning
  - System 0: Create intensional representations of premises
  - System 1: Build and interpret an initial model
  - System 2: Perform search for counterexamples
- System 1 parameterizes number of entities and their canonicality

RQ2.2: Is the model building behavior observed in humans in line with the model building processes of mReasoner?

[2] Khemlani, S., & Johnson-Laird, P. (2013). The processes of inference. *Argument & Computation*, 4(1), 4–20.

Theoretical Background - Syllogisms

- The two syllogistic premises and conclusion are characterized by their mood and figure
- Quantifiers → Mood

A	*All* A are B	**E**	*No* A are B
I	*Some* A are B	**O**	*Some* A are *not* B

- Order of terms → Figure

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**
A - B	B - A	A - B	B - A
B - C	C - B	C - B	B - C

- Denoting syllogisms using abbreviations and figures:

  All red shapes are circles.
  Some red shapes are marked with a star. → AI4

- Denoting conclusions using quantifier and end-term order (*ac* or *ca*)
  - *Some* shapes marked with a star are *not* circles → *Oca*
- 'No valid conclusion' → NVC

Theoretical Background - mReasoner

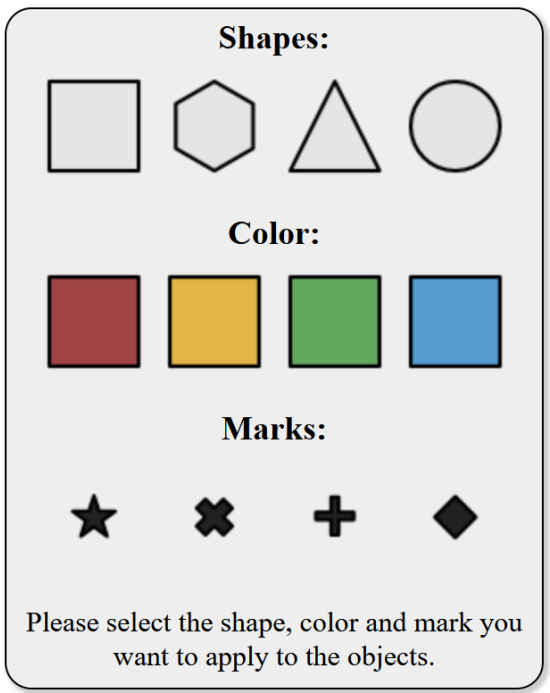
- MMT
  - Individuals represent entities described by quantifiers using mental models
  - → Aim to derive a conclusion
  - Before accepting - search for counterexamples
  - → If successful, reject and correct original conclusion or NVC
- mReasoner - four parameters
  - λ - model size
  - ϵ - canonicality
  - σ - counterexamples search
  - ω - NVC

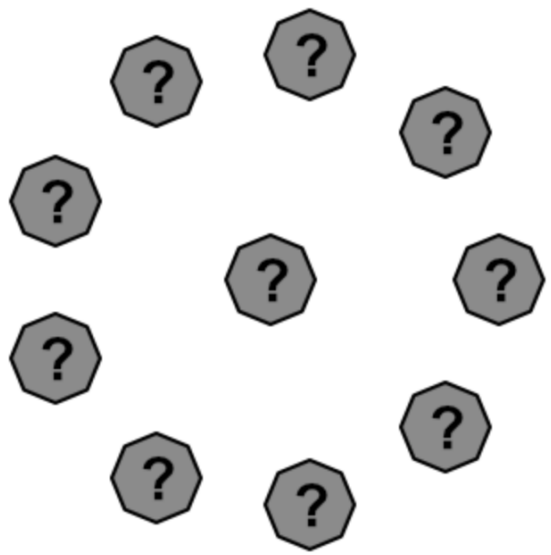
Experiment

No yellow shapes are hexagons

All shapes that are marked with a star are yellow

What do you imagine the ten objects below look like when thinking about the premises above?





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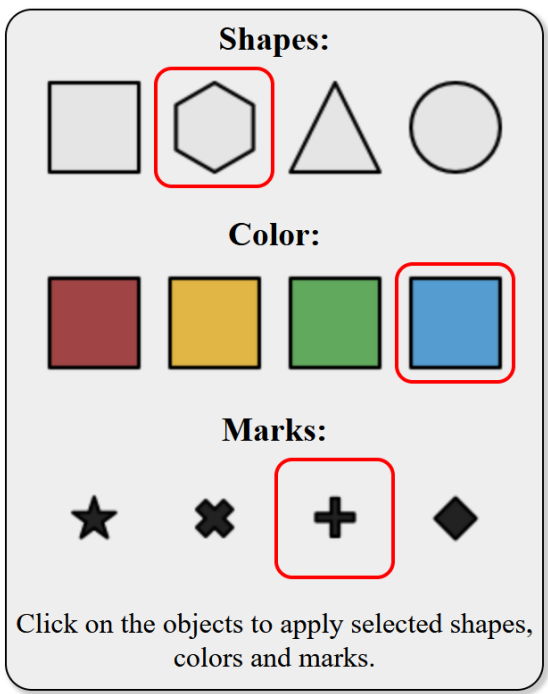
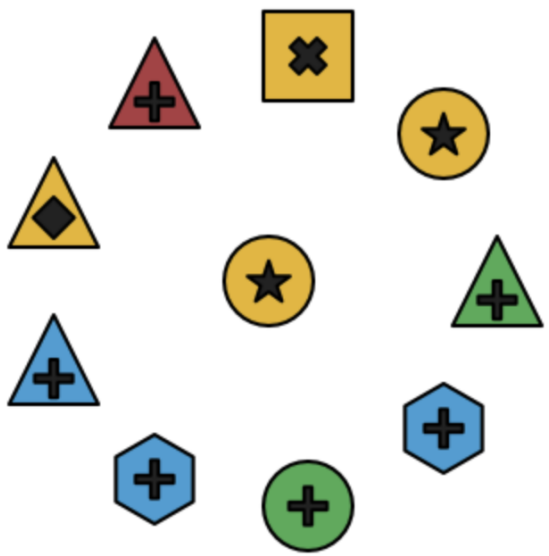
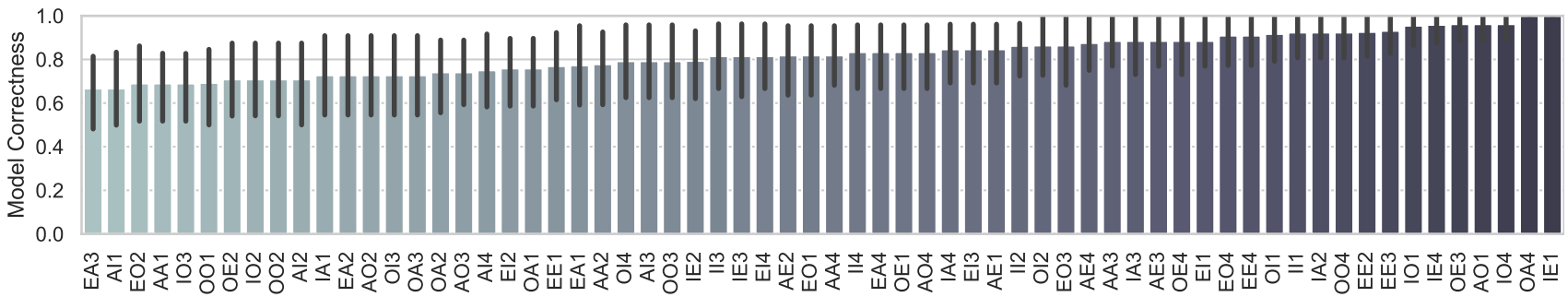




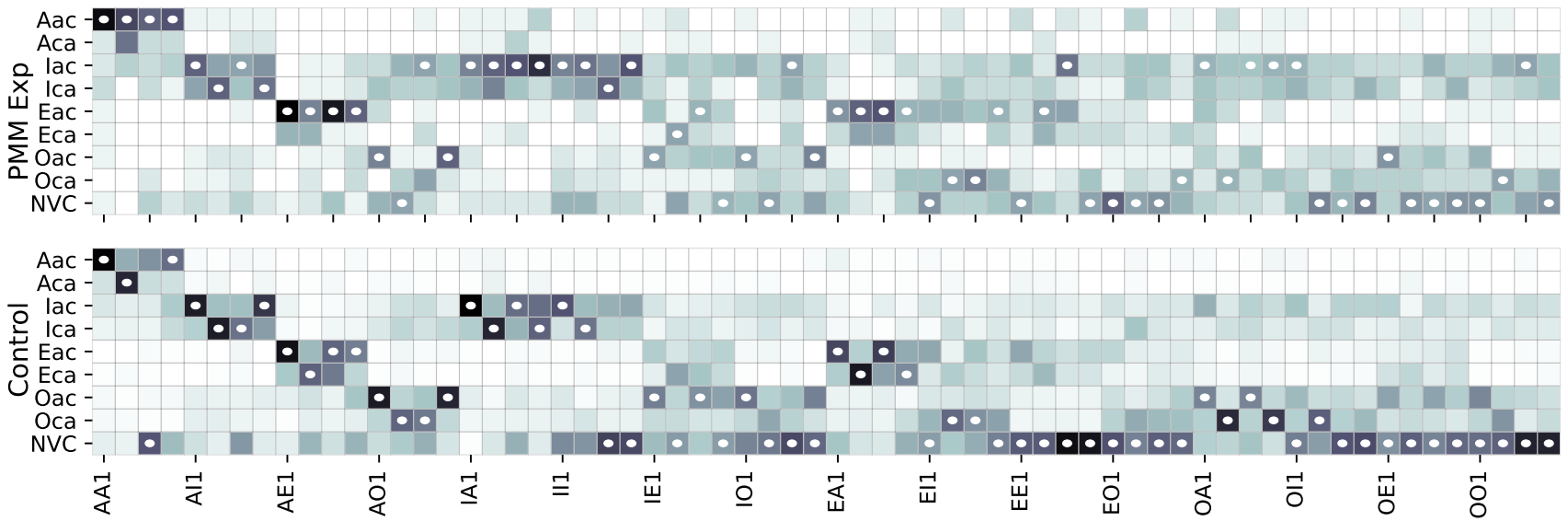
Analysis - Experimental Data

- Correct representation $\rightarrow$ 82.12%
  - Not affected by negativity of quantifiers, particularity or validity
  - Figure 2 $\rightarrow$ More incorrect
  - Figure 4 $\rightarrow$ Potentially easier



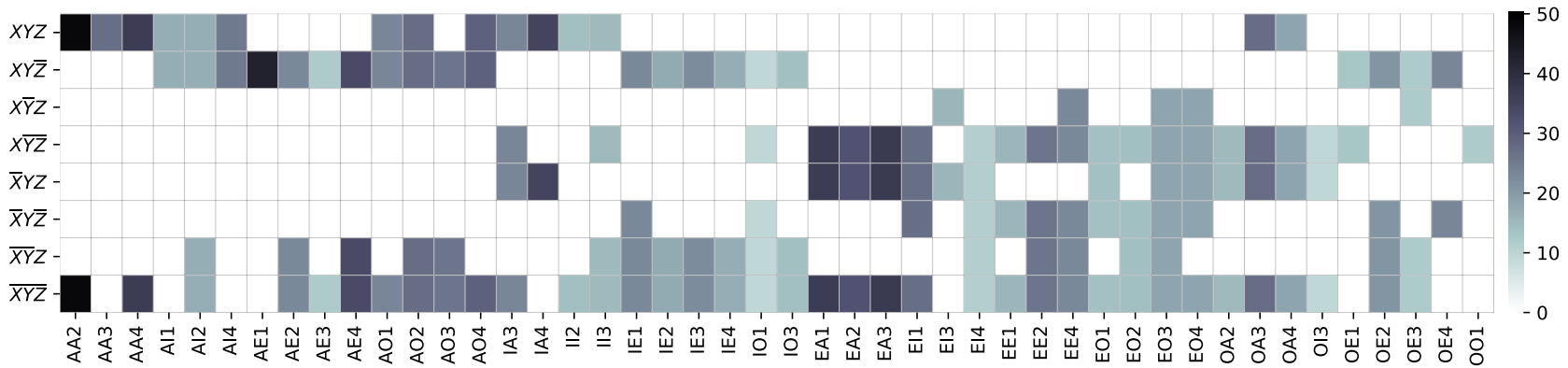
Analysis - Experimental Data

- Correct response → 31.06%
- Correct representation *and* response → 25.50%
  - No significant correlation



Control: `Ragni-2016` dataset in CCOBRA (`https://orca.informatik.uni-freiburg.de/ccobra/`)

Analysis - Preferred Mental Models



- *All* $\rightarrow$ non-canonical instances
- Particular quantifiers (*Some*, *Some not*) $\rightarrow$ weaker preference
- AA1 and EA4 $\rightarrow$ two models with equal preference
- Otherwise $\rightarrow$ no preferred models

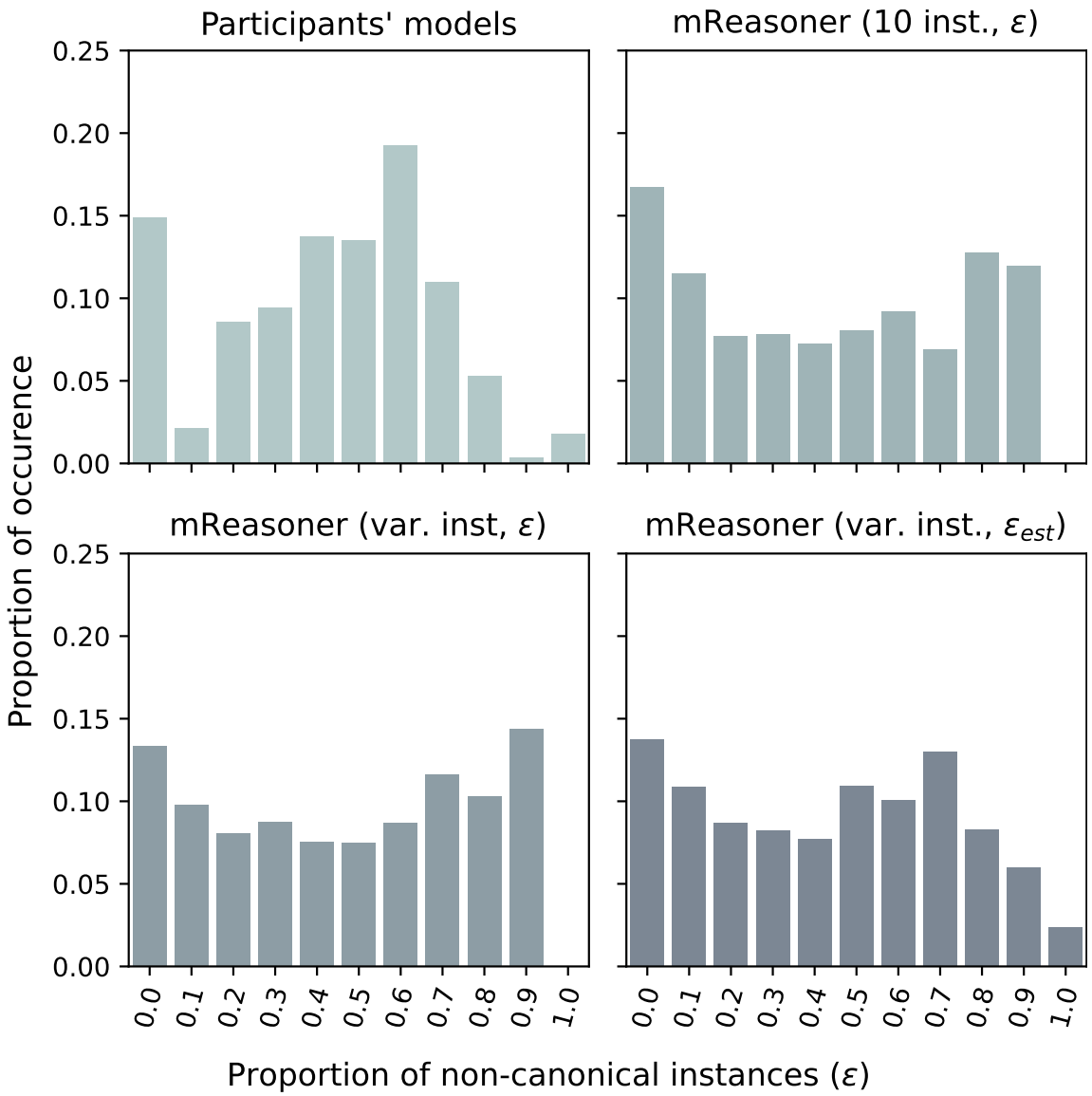
Analysis - mReasoner

- Relevant model building parameters:
  - λ - number of instances in the model
  - ϵ - likelihood that instances are from the full set

Quantifier	Canonical	Noncanonical
All	X Y	¬X Y ¬X ¬Y
Some	X Y X ¬Y	¬X Y ¬X ¬Y
No	¬X Y X ¬Y	¬X ¬Y
Some not	X Y X ¬Y ¬X Y	¬X ¬Y

- Derived ϵ values based on participants' responses
- Fit mReasoner to task responses
  - Fixed $\lambda = 10$
  - Free λ

Analysis - mReasoner



Discussion

RQ1 What kind of mental models do individuals create and do they have preferred models?

- Designed and conducted and experiment
- Found a belief bias tendency
- 82% correct visual representations
- Preferred mental models for 46 syllogisms

RQ2 Does model canonicality have influence on correctness? Is the mReasoner model building process in line with the one observed in humans?

- No significant correlation in any scenario
- Lack of relevance of the models for the responses?
- Many ϵ values lead to the same answer
- Assumption of correct representation mostly in line
- NVC not possible with one model

Conclusion

- Individuals do have preferred mental models
- Initially built mental model not substantial for finding conclusions
- Instances chosen correctly in line with premises
- Model building $\rightarrow$ rather easy task for humans
- Solving tasks by repeated construction of models?