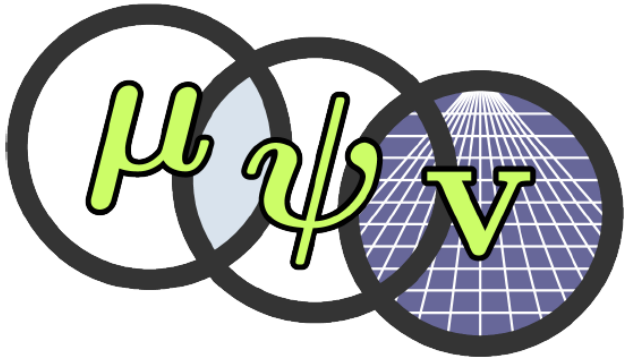

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Uncovering iconic patterns of syllogistic reasoning: A clustering analysis

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Syllogistic Reasoning

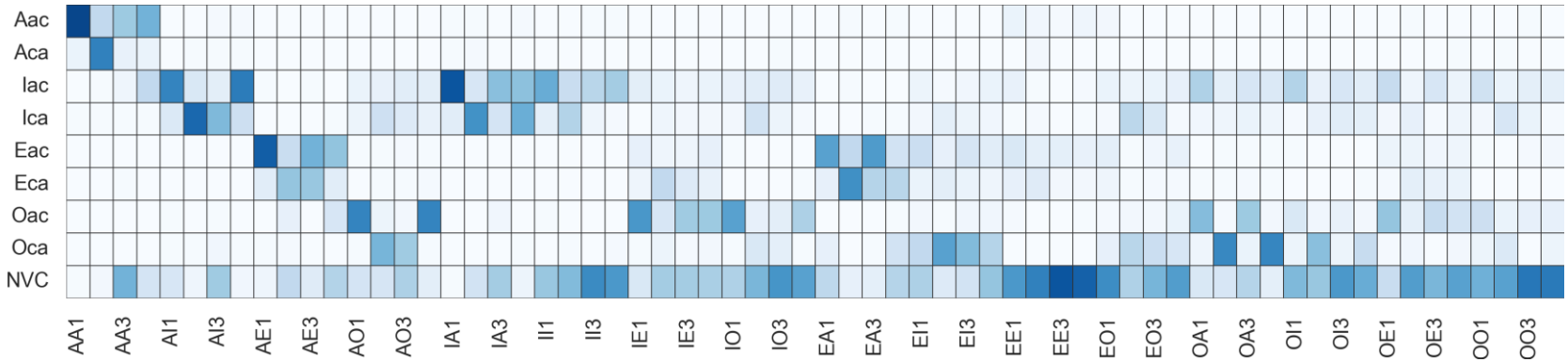
No B are A.
All B are C.

What, if anything, follows?

- Two quantified statements connected via a common term
- Task: Find a **conclusion** connecting the end terms
  - Or: conclude that **no valid conclusion exists** (NVC)
- Traditionally used with **first-order logic** quantifiers: All (A), None (E), Some (I), Some not(O)
- **Well-defined** domain with 64 tasks and 9 responses

Reasoning Patterns

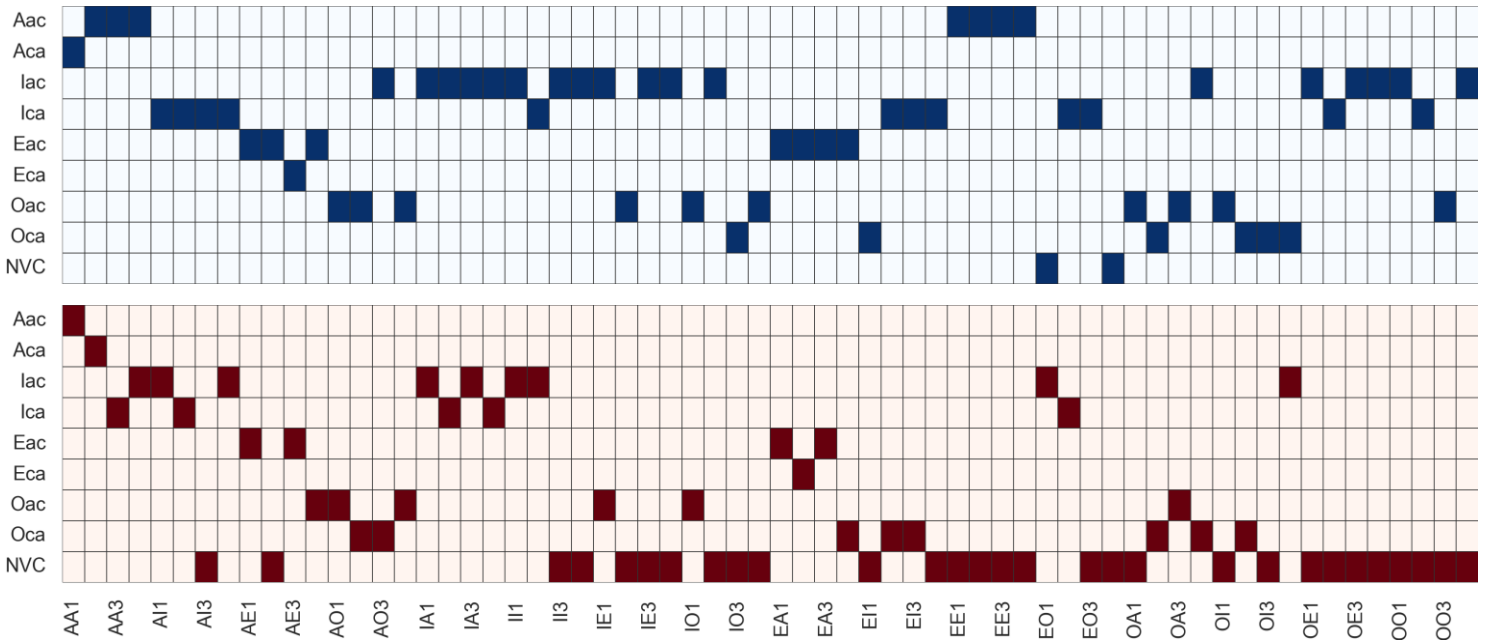
- Observations can be shown as 9×64 matrix
  → Reasoning pattern



- Cognitive models **generate** such reasoning patterns using their internal mechanisms
- Models perform well for **aggregated patterns**[1]
- This is not the case on the individual level! [2]

[1] Khemlani, S. S., & Johnson-Laird, P. N. (2012). Theories of the syllogism: A meta-analysis.
[2] Riesterer, N., Brand, D., & Ragni, M. (2020). Do models capture individuals? Evaluating parameterized models for syllogistic reasoning.

Individual Patterns

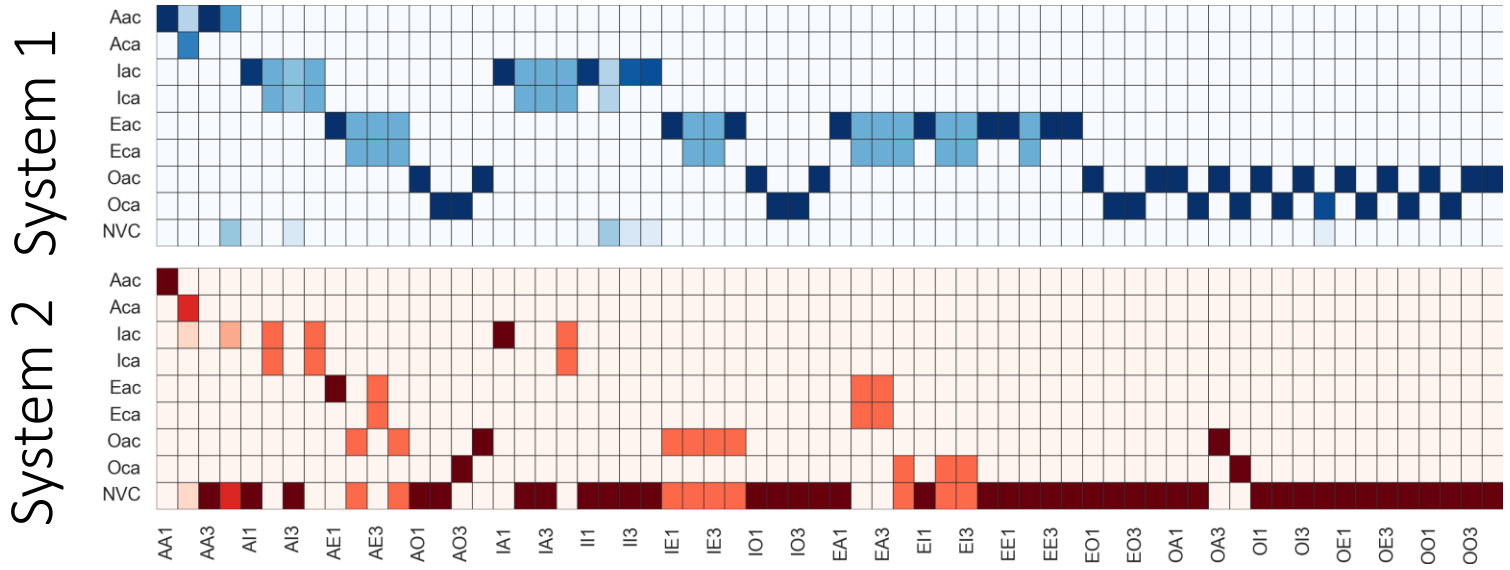


- Substantial **inter-individual differences**
  - → Likely that **no single inferential process** is sufficient

→**Multiple patterns** are necessary

Modeling Individuals

- Models incorporate multiple processes
- Prominent approach: **Dual-processing** accounts[1]
  - System 1: Fast-and-frugal heuristics
  - System 2: deliberative, more logical
- Incorporated by the Mental Model Theory[2] (MMT)



[1] Evans, J. S. B. T. (2008). Dual-processing accounts of reasoning, judgment, and social cognition.
[2] Khemlani, S. S., & Johnson-Laird, P. N. (2013). The processes of inference.

Research Questions

- **How many patterns are necessary?**
  - Is the distinction into two main processes justified?
- **How do optimal patterns look like?**
  - How well could they account for individual data?

A Data-driven Perspective

- Cognitive models **are constraint** by theoretical assumptions

→ Search patterns in the data **directly**

→ **Clustering**

  - Find *k* patterns that are **representative** for a respective group of individuals
  - Which clustering method?
  - What is the best *k*?

Clustering Methods

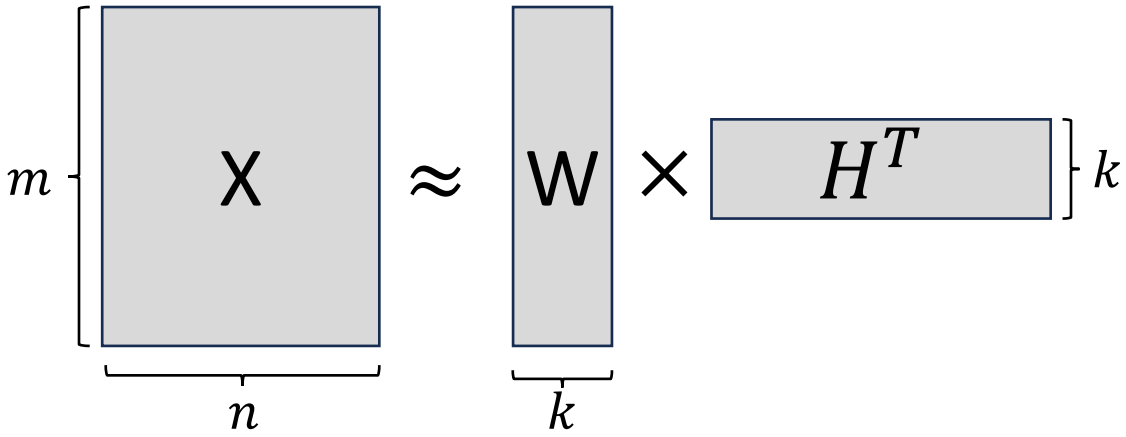
- k-Means
  - Common method for clustering
  - Uses the **mean of a cluster** as a centroid

  → Direct extension to **aggregated** patterns
- k-Medoids
  - Uses **actual data points** instead of the mean

  → Patterns are not *artificial*
- Nonnegative Matrix Factorization
  - Formally, clustering can be understood as a special case of matrix decomposition[1]
  - Finds latent patterns using **dimensionality reduction**
  - Usually good interpretability of resulting patterns

[1] Kim, J., & Park, H. (2008). Sparse nonnegative matrix factorization for clustering.

Nonnegative Matrix Factorization



- $m \times n$ matrix *X with m*-dimensional patterns from *n* reasoners
- NMF factorizes *X* into two matrices *W* and *H*:
  - *W* $(m \times k)$: **Contains *k* patterns**
  - *H* $(n \times k)$: **Contains assignment** to the *k* patterns
  - **Clustering**: Assignments in *H* must be **unique**

Analysis

- Task: Select **best clustering method** and **optimal *k***

→ Analysis based on cross-validation

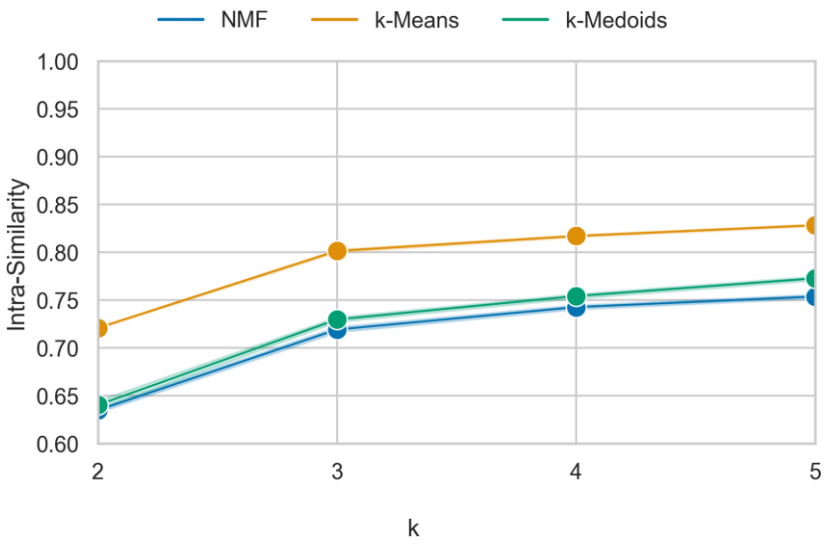
  - Dataset[1] contains the responses of 106 participants to all syllogisms (**full patterns**)
  - Use different clustering methods and values of *k* to **find patterns** in the training set
  - **Test quality** of the patterns on the test set

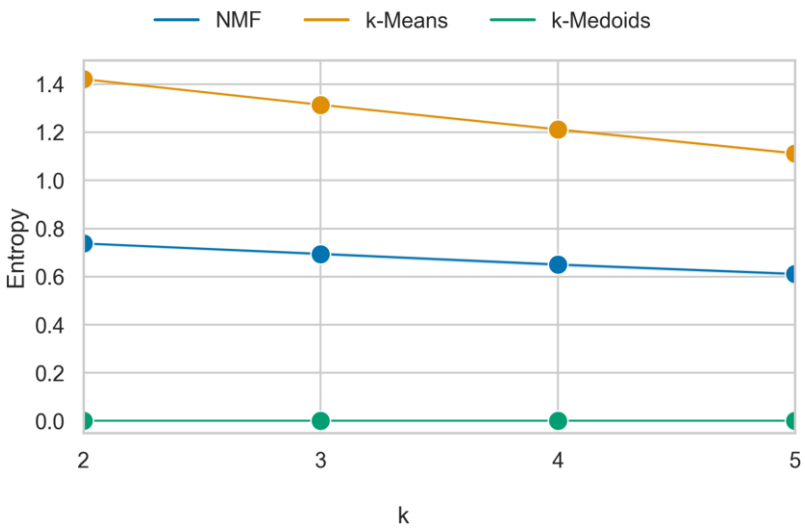
[1] Dames, H., Klauer, K. C., & Ragni, M. (2022). The stability of syllogistic reasoning performance over time.

Quality Metrics

- Distinct patterns:
  - All *k* patterns should **not be similar to each other**
  - Patterns should make precise predictions
- Stable patterns:
  - Found patterns should be stable and not be dependent on the **specific composition** of the training data
- Test-Accuracy:
  - Patterns found in the training data should be **predictive** for individuals in the test data

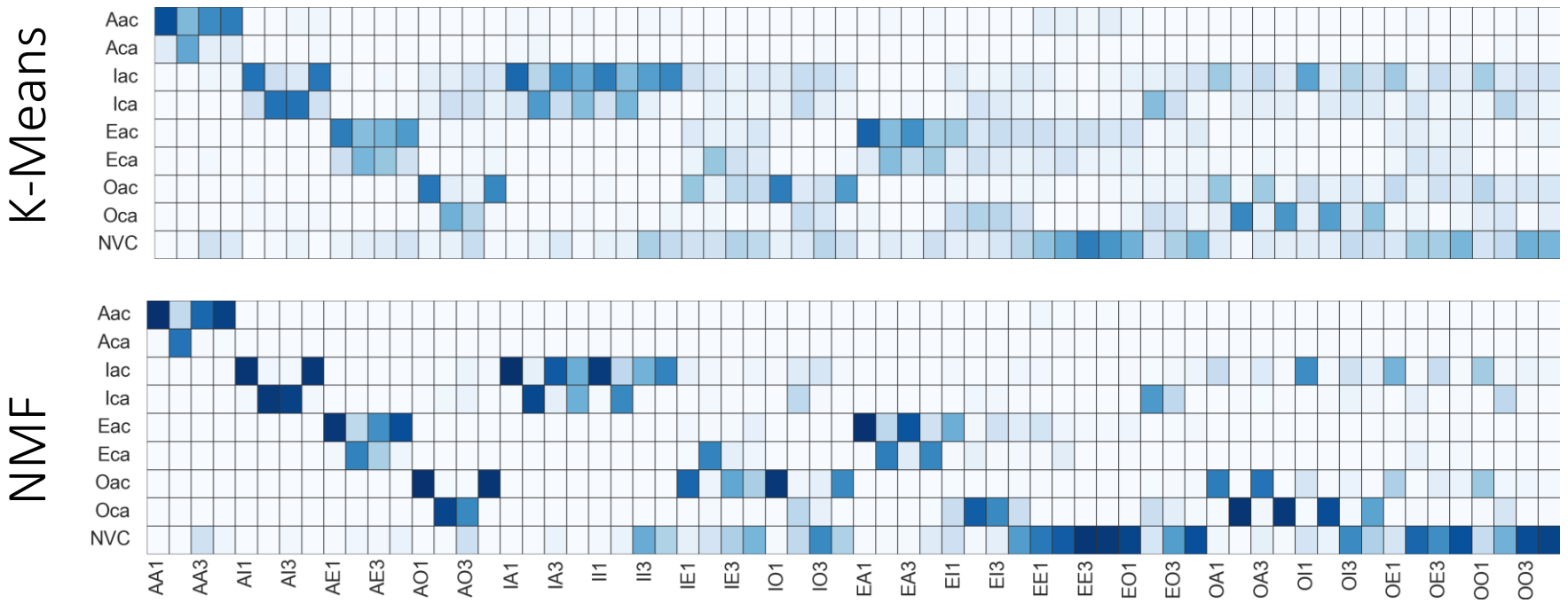
Distinct Patterns





- Maximum cosine similarity **between** all *k* patterns
- Cosine similarity: $sim(w_1, w_2) = \frac{w_1 w_2}{\|w_1\|\|w_2\|}$
- **Within** patterns, entropy is used
- Entropy: H= $-\sum_i p_i * log_2 p_i$

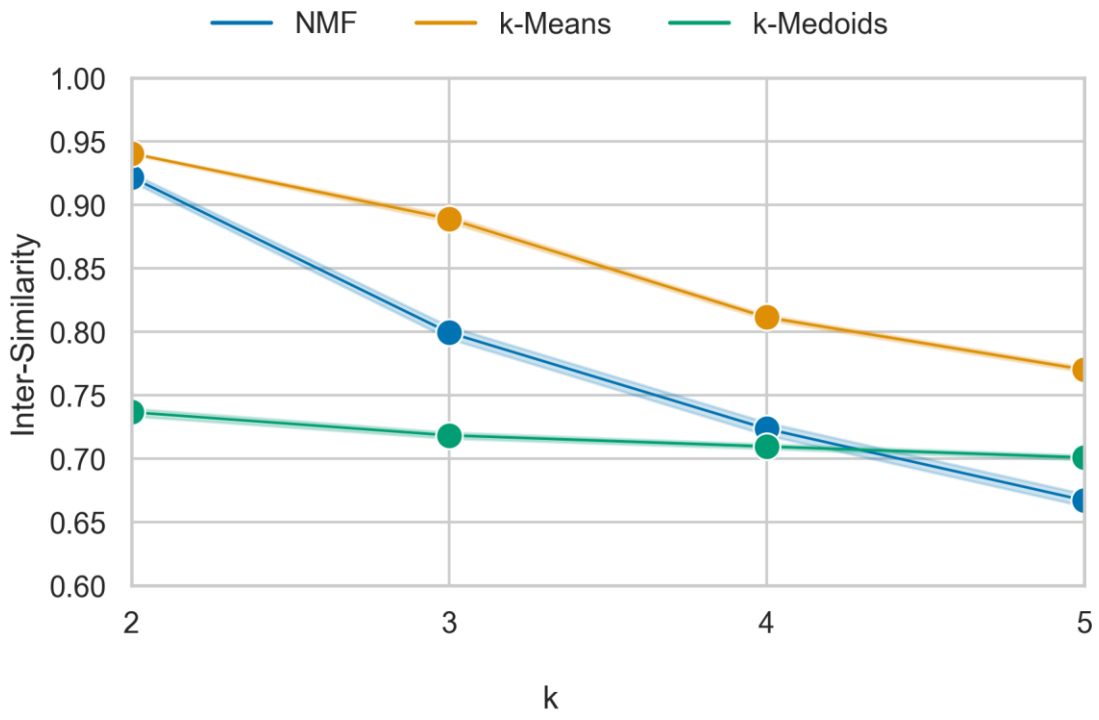
Example: k-Means vs. NMF



- NMF pattern is sharper
- k-Means yields no clear predictions on an individual level

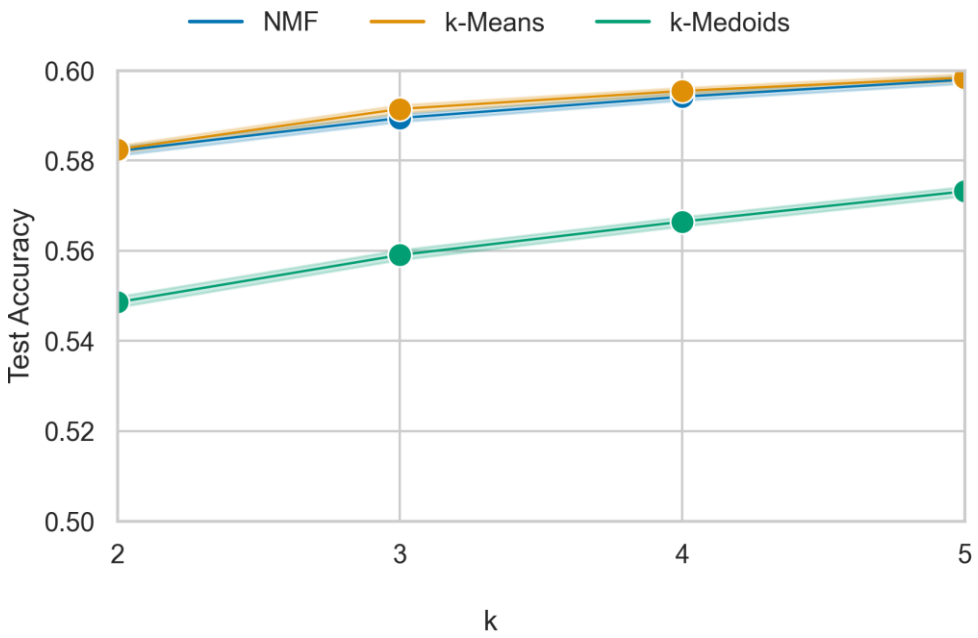
Stable Patterns: Inter-Similarity

- Patterns should not depend on the **specific composition** of the training data
- Cosine similarity between **corresponding patterns** from multiple CV-splits



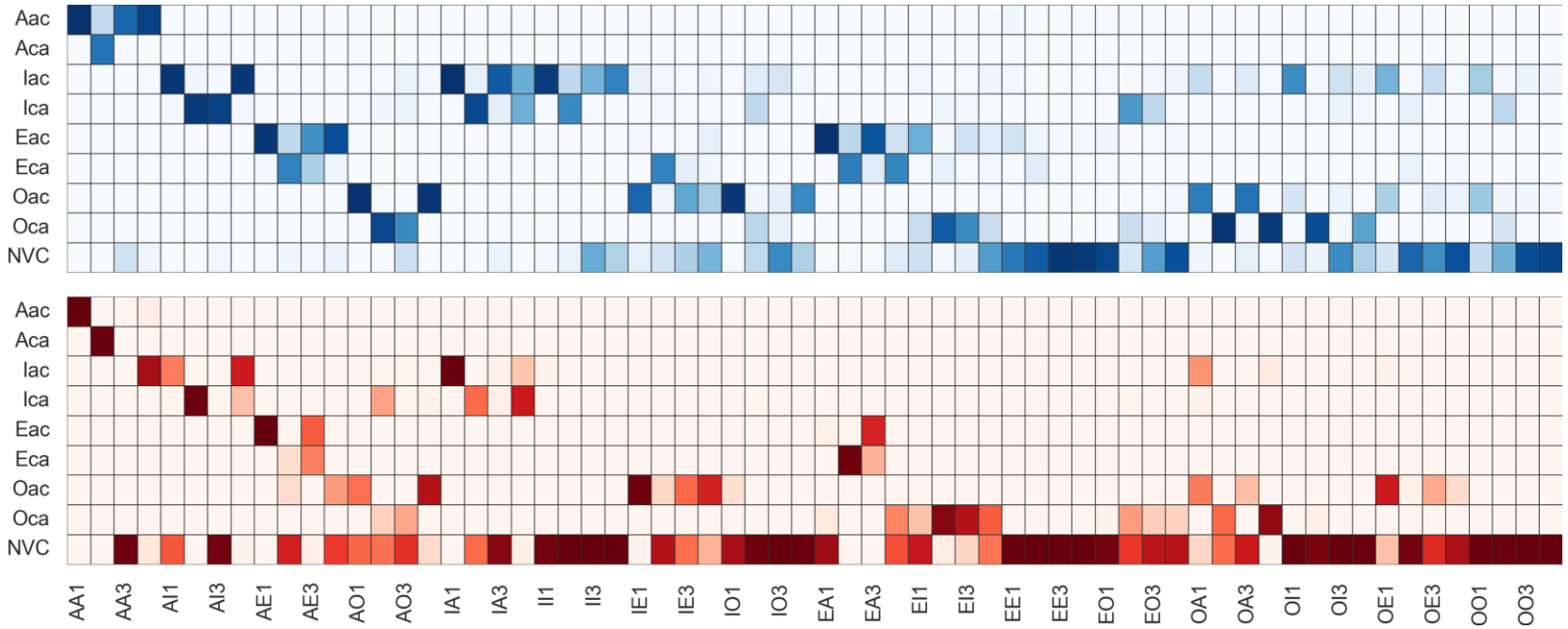
Test Accuracy

- Patterns found in the training data should be good predictive models for individuals in the test data
- Accuracy on the test set of the best fitting pattern



Results

- The overall best configuration was $k = 2$ with NMF



- Patterns roughly resemble S1/S2 patterns
- $k = 2$ and NVC differences

  → Support for dual-process accounts?

Support for Dual-Process

- We tested correlations for:
  - Need for Cognition (NFC)
  - Cognitive Reflection Task (CRT) performance
  - Response times (RT)
- Should be **higher** for participants associated with the „system-2 pattern “

Support for Dual-Process

	Mean S1	Mean S2	p	U
NFC	4.65	4.73	.536	1224.5
CRT	.47	.7	< .001	747.0
RT	15803	13468	.001	1697.0

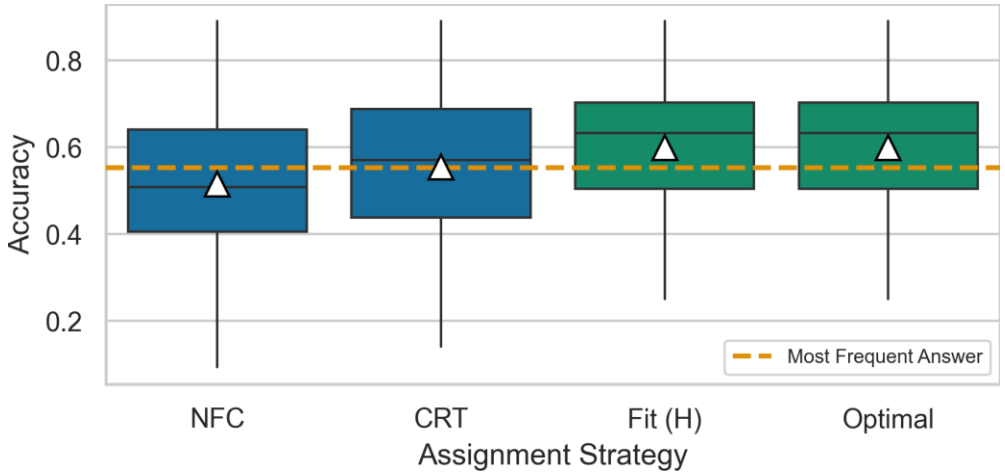
- No significant result for NFC
- **CRT** performance significantly higher for S2!
- Response times significantly higher for S1?

→Mixed results: No support for dual-processes

→CRT could be a good predictor

Using Patterns as Models

- Assign individuals to patterns based on CRT & NFC



- CRT allows to surpass the most frequent answer
- Performance is far behind the optimal assignment to the two patterns

Conclusions

- Clustering analysis showed that only **two patterns** seem to be sufficient
  → Explains the convergence of models
- Dual-process assumption was **not supported**
  → Different explanations for NVC necessary
- Iconic patterns have a **high predictive accuracy**
  →Good assignment strategies valuable
- Methodology is applicable in all domains where datapoints form well-defined patterns
- Optimal *k* heavily depends on the **quality criteria**:
  - We focused on stable patterns that are likely to exist in most datasets

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