General Energy-Entropy-Momentum integration methods for nonlinear themomechanics

I. Romero
IMDEA Materials Institute, Madrid, Spain, ignacio.romero@imdea.org
Technical University of Madrid, Madrid, Spain

Energy-Entropy-Momentum (EMM) integration algorithms are a class of discretizations that preserve, exactly, the laws of thermodynamics when applied to mechanical systems, as well as certain of the symmetries of their equations [4]. The methods can be understood as an extension of the, now classical, Energy-Momentum method to the thermodynamic range, and both families share many of their features in the algorithmic design. In abstract terms, Energy-Momentum methods preserve the structure of symplectic mechanics, whereas Energy-Entropy-Momentum generalize to metriplectic geometry.

Initially obtained for thermoelastic problems in entropy variables [4, 5, 6], the methods have been extended to other *smooth* problems [2, 3] and for temperature variables [1]. In this work we present the theory of EEM extended to non-smooth problems, and with arbitrary variables. With this theory, we prove that structure preserving methods can be formulated for problems such as thermoplasticity, or frictional thermo-coupled contact, where the response is defined by kinetics formulated in terms of differential inclusions, instead of differential equations. Numerical examples will be provided to illustrate the performance and design of the methods.

References

[1] S. Conde Martín, P. Betsch, and J. C. García Orden, *A temperature-based thermodynamically consistent integration scheme for discrete thermo-elastodynamics*, Commun. Nonlinear. Sci. Numer. Simulat., 32 (2016), pp. 63–80.

[2] J. C. García Orden and I. Romero, *Energy-Entropy-Momentum integration of discrete thermo-visco-elastic dynamics*, Eur. J. Mech. A-Solid, 32 (2012), pp. 76–87.

[3] B. González-Ferreiro, H. Gomez, and I. Romero, *A thermodynamically consistent numerical method for a phase field model of solidification*, Commun. Nonlinear. Sci. Numer. Simulat., 19 (2014), pp. 2309–2323.

[4] I. Romero, *Thermodynamically consistent time stepping algorithms for nonlinear thermomechanical systems*, Int. J. Numer. Meth. Engng., 79 (2009), pp. 706–732.

[5] ——, *Algorithms for coupled problems that preserve symmetries and the laws of thermodynamics. Part I: monolithic integrators and their application to finite strain thermoelasticity*, Comput. Methods Appl. Mech. Engrg., 199 (2010), pp. 1841–1858.

[6] ——, *Algorithms for coupled problems that preserve symmetries and the laws of thermodynamics. Part II: fractional step methods*, Comput. Methods Appl. Mech. Engrg., 199 (2010), pp. 2235–2248.