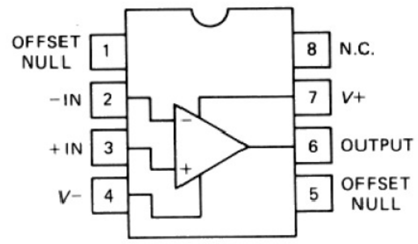
6 Amplification Circuits

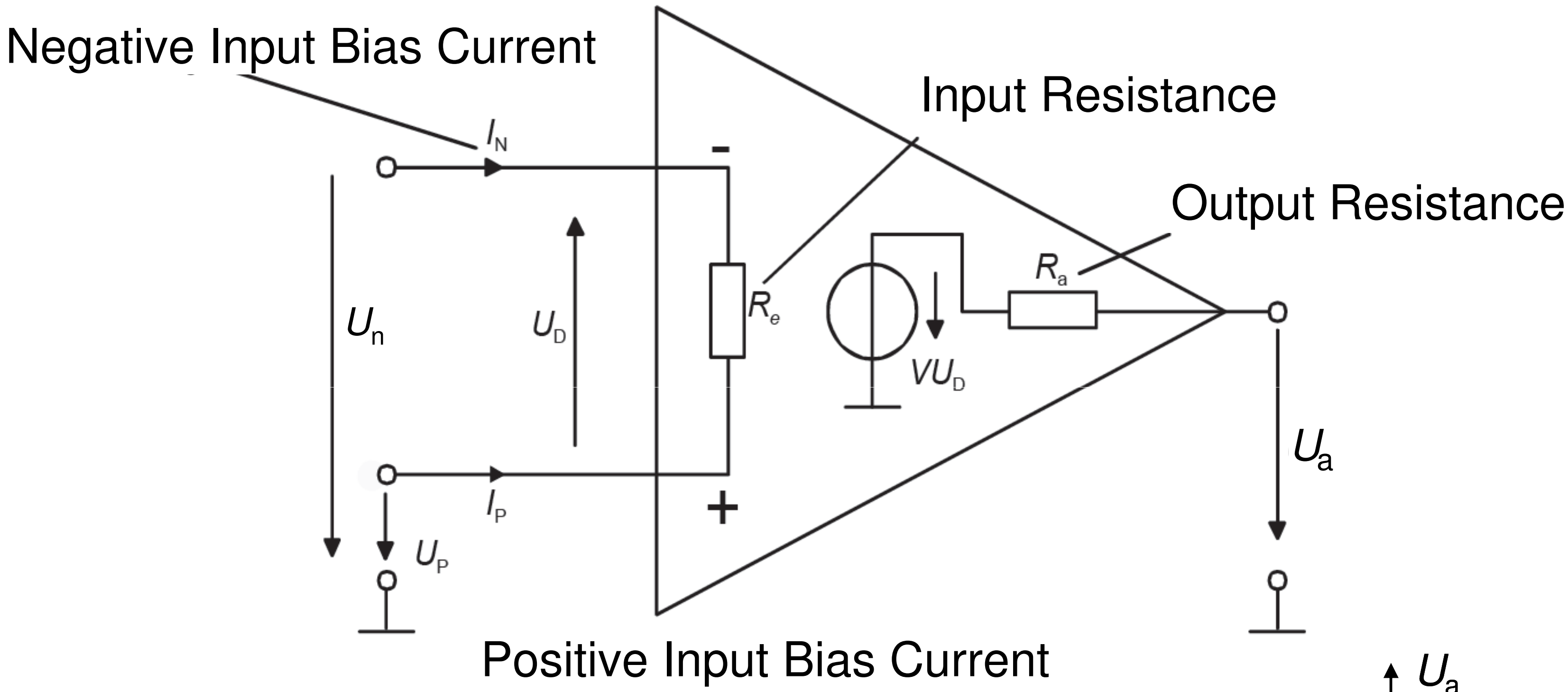
6.1 Requirements to Amplification

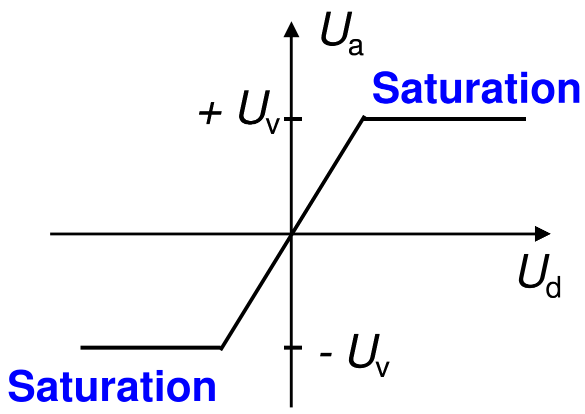
- defined transfer behaviour:
  - ✓Linearity
  - ✓Amplification independent on aging, fluctuations, environment influence and voltage supply
- high input sensitivity
- no influence on the measurand: → High input resistance
  - ✓for voltage amplifier very high
  - ✓for current amplifier very low
- stable output: → Low output resistance
- long life time
- low noise
- low energy consumption

6.2 Structure of Amplifiers

Equivalent Circuit and Transfer Characteristic







Important:
Ouput circuit is galvanically independent on the input circuit

6.2 Structure of Amplifiers
Amplification

$$U_d = U_p - U_n$$ ▷ $$U_a = V_0 \cdot U_d$$

V_0 open loop voltage gain (Differenzverstärkung)

Real $10^4 < V_0 < 10^7$
Ideal ∞

$$U_{CM} = U_p = U_n$$ ▷ $$U_d = 0\ V$$

$$U_a = V_{CM} \cdot U_{CM} \quad \text{with } U_{CM} = \left(\frac{U_p + U_n}{2}\right)$$

V_{CM} common mode amplification (Gleichtaktverstärkung)

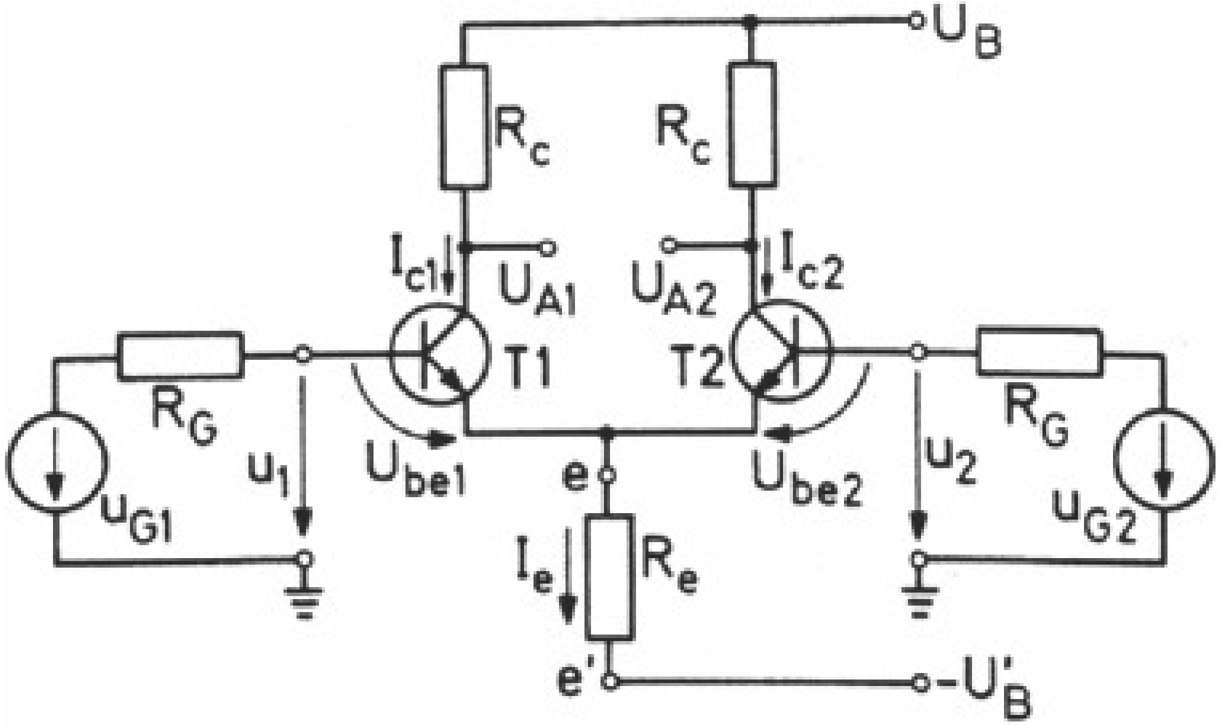
Ideal 0

Total Amplification

$$U_a = V_0 \cdot U_d + V_{CM} \cdot U_{CM}$$

with $\frac{V_0}{V_{CM}} \approx 10^4 \cdots 10^7$

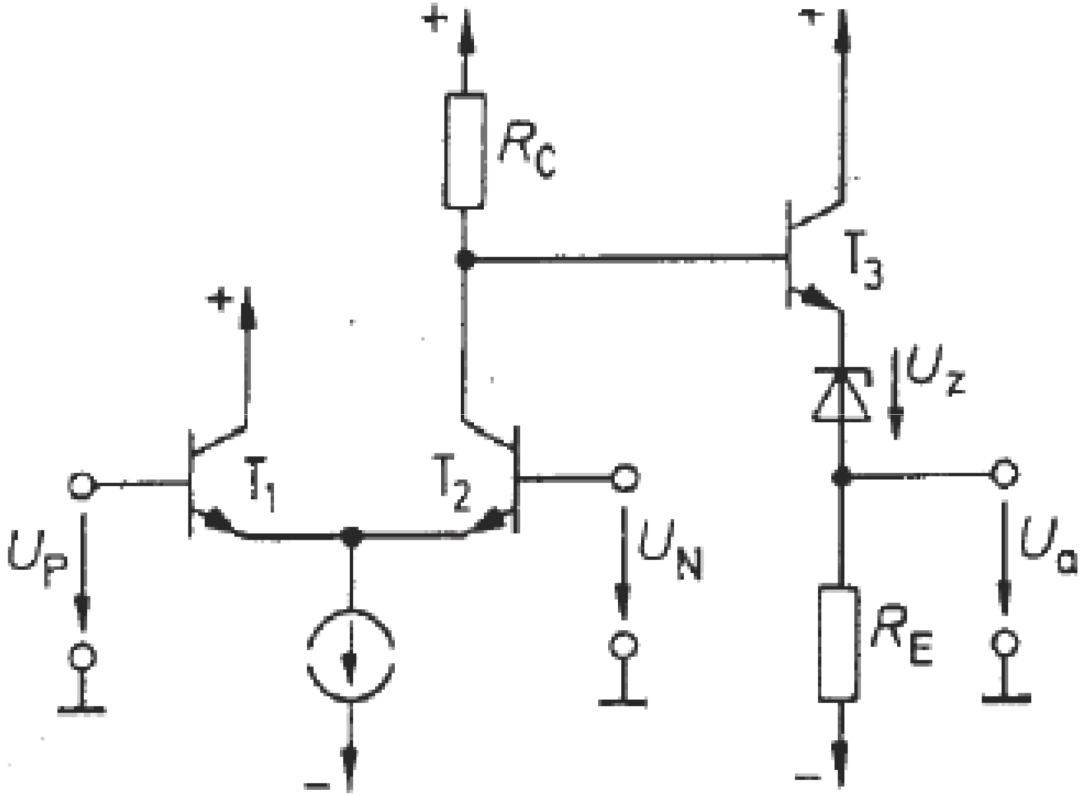
6.2 Structure of Amplifiers
Principle of the Difference Amplifier



$U_{be1} = U_{be2} \Rightarrow I_{c1} = I_{c2} \Rightarrow U_{A1} = U_{A2}$

$U_{be1} > U_{be2} \Rightarrow I_{c1} >> I_{c2} \Rightarrow U_{A1} >> U_{A2}$

6.2 Structure of Amplifiers
Simple Amplifier

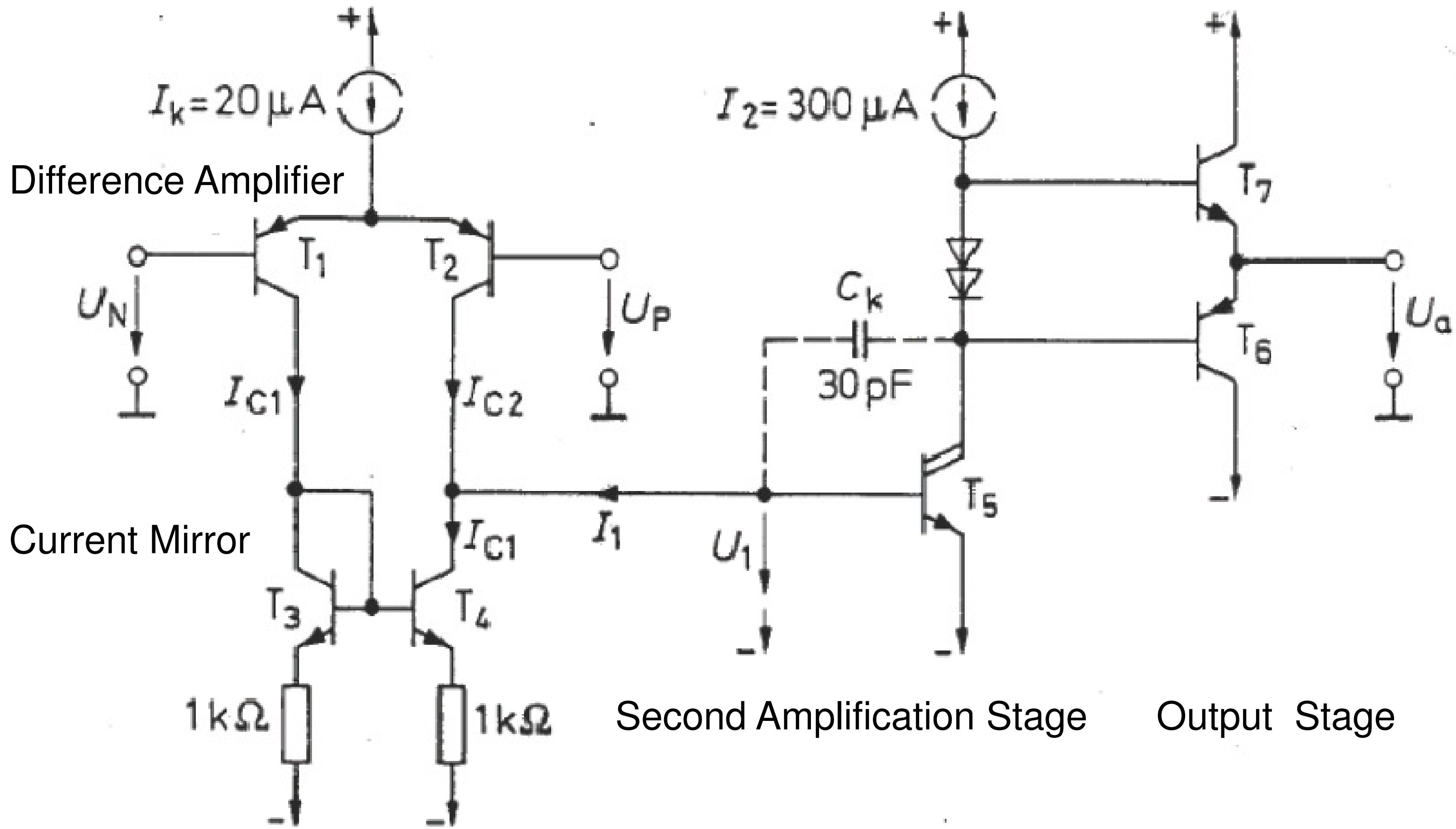


Difference Amplifier

Output stage

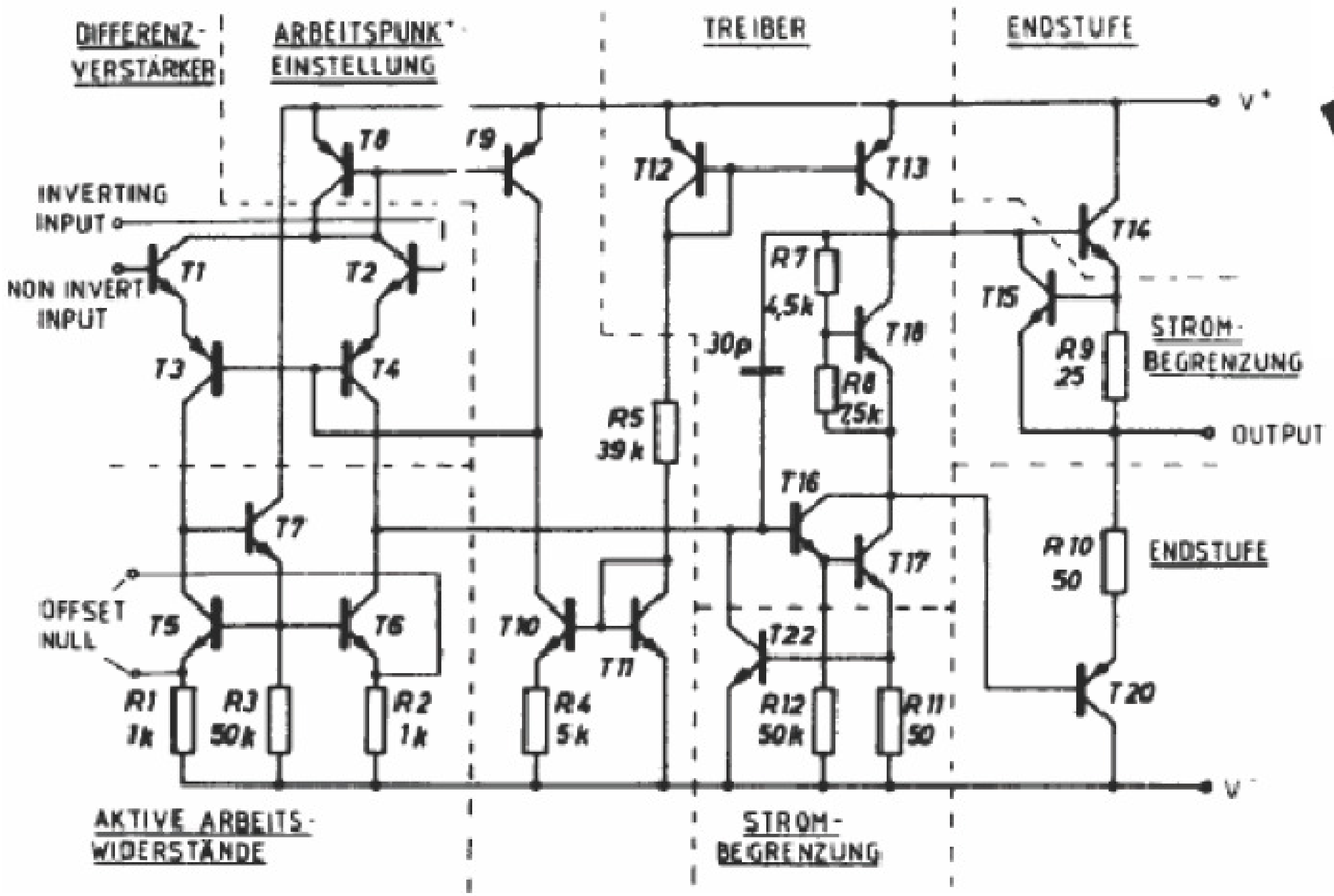
6.2 Structure of Amplifiers

Integrated Standard Operation Amplifier xx741 (Principle)

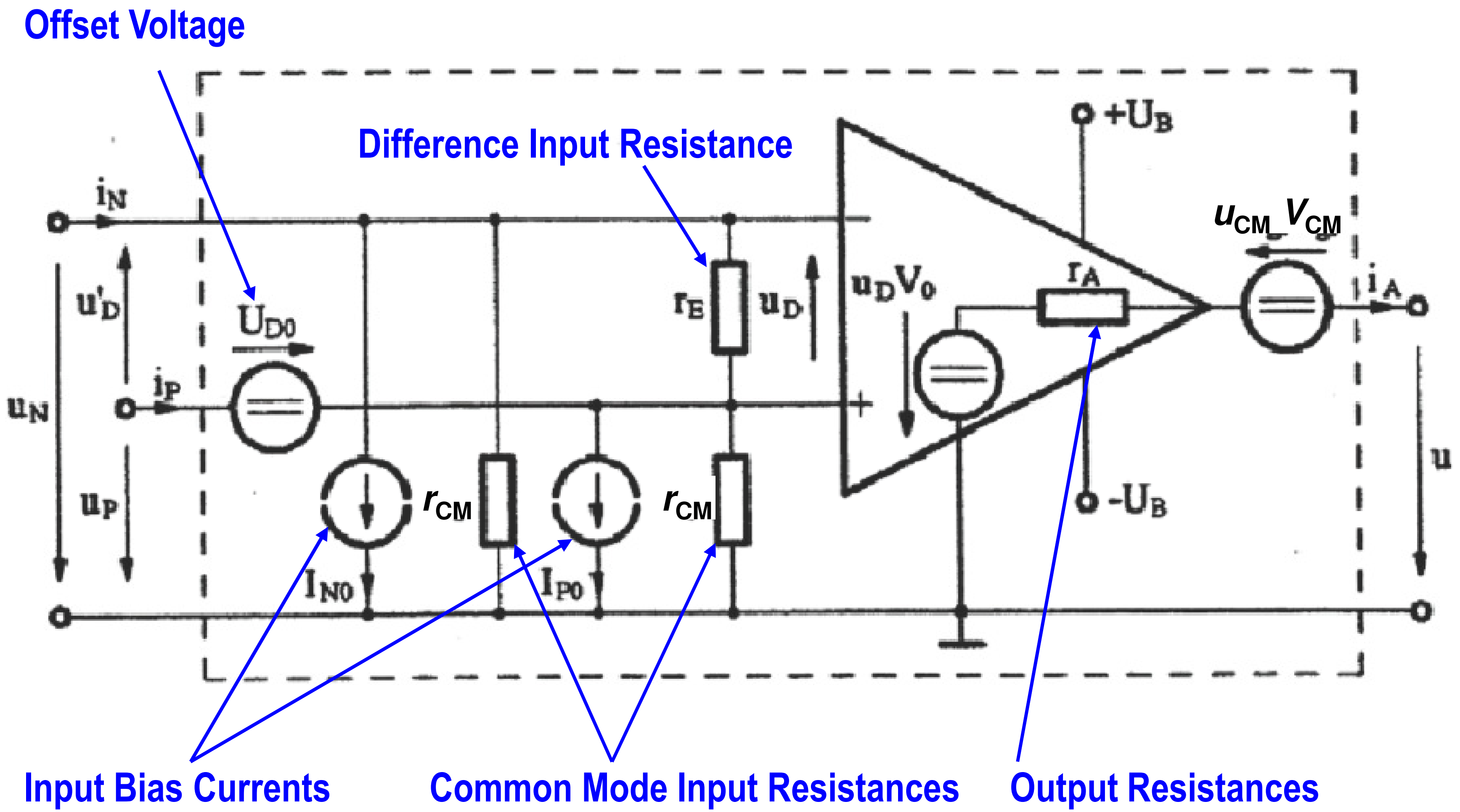


6.2 Structure of Amplifiers

Integrated Standard Operation Amplifier xx741 (Principle)



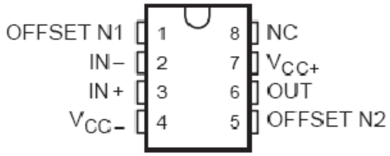
6.2 Structure of Amplifiers
Small Signal Equivalent Circuit of an Op-Amps(1)



6.3 Real Behaviour of Amplifiers

Data Sheets

μA741, μA741Y
GENERAL-PURPOSE OPERATIONAL AMPLIFIERS



SLOS094B – NOVEMBER 1970 – REVISED SEPTEMBER 2000

PARAMETER		TEST CONDITIONS	T_A†	μA741I, μA741M MIN	TYP	MAX	UNIT
V_{IO}	Input offset voltage	$V_O = 0$	25°C		1	5	mV
			Full range			6	
$\Delta V_{IO(adj)}$	Offset voltage adjust range	$V_O = 0$	25°C		±15		mV
I_{IO}	Input offset current	$V_O = 0$	25°C		20	200	nA
			Full range			500	
I_{IB}	Input bias current	$V_O = 0$	25°C		80	500	nA
			Full range			1500	
V_{ICR}	Common-mode input voltage range		25°C	±12	±13		V
			Full range	±12			
V_{OM}	Maximum peak output voltage swing	$R_L = 10\ k\Omega$	25°C	±12	±14		V
		$R_L \geq 10\ k\Omega$	Full range	±12			
		$R_L = 2\ k\Omega$	25°C	±10	±13		
		$R_L \geq 2\ k\Omega$	Full range	±10			
A_{VD}	Large-signal differential voltage amplification	$R_L \geq 2\ k\Omega$	25°C	50	200		V/mV
		$V_O = \pm 10\ V$	Full range	25			
r_i	Input resistance		25°C	0.3	2		MΩ
r_o	Output resistance	$V_O = 0$, See Note 5	25°C		75		Ω
C_i	Input capacitance		25°C		1.4		pF

6.3 Real Behaviour of Amplifiers

Data Sheet OPA 365 [Texas Instruments]

2.2V, 50MHz, Low-Noise, Single-Supply Rail-to-Rail

Rail-To-Rail-inputs

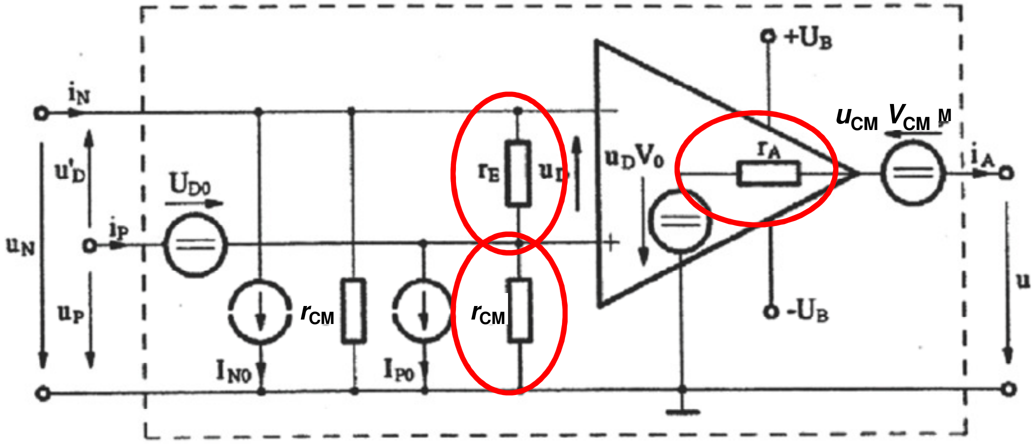
→ Input voltages are amplified until the level of the supply voltage distortion-free

CMRR	Common-mode rejection ratio	$V_{IC} = V_{ICR}min$	25°C	70	90		dB
			Full range	70			
k_{SVS}	Supply voltage sensitivity ($\Delta V_{IO}/\Delta V_{CC}$)	$V_{CC} = +9$ V to $+15$ V	25°C		30	150	µV/V
			Full range			150	
I_{OS}	Short-circuit output current		25°C		±25	±40	mA
I_{CC}	Supply current	$V_O = 0$, No load	25°C		1.7	2.8	mA
			Full range			3.3	
P_D	Total power dissipation	$V_O = 0$, No load	25°C		50	85	mW
			Full range			100	

operating characteristics, $V_{CC\pm} = \pm 15$ V, $T_A = 25°C$

PARAMETER		TEST CONDITIONS		µA741I, µA741M			UNIT
				MIN	TYP	MAX	
t_r	Rise time	$V_I = 20$ mV, $C_L = 100$ pF,	$R_L = 2$ kΩ, See Figure 1		0.3		µs
	Overshoot factor				5%		
SR	Slew rate at unity gain	$V_I = 10$ V, $C_L = 100$ pF,	$R_L = 2$ kΩ, See Figure 1		0.5		V/µs

6.3 Real Behaviour of Amplifiers



Differential Input Resistance
(Differenzeingangswiderstand)

$$r_E = \frac{\partial u_D}{\frac{1}{2}\partial(i_P - i_N)} \qquad r_E \approx 1\mathrm{G\Omega} \ldots 100\mathrm{T\Omega} \quad (\text{ideal} \infty)$$

Common Mode Input Resistance
(Gleichtakteingangswiderstand)

$$r_{CM} = \frac{\partial u_{CM}}{\frac{1}{2}\partial(i_P + i_N)} \qquad r_{CM} \approx 1\mathrm{M\Omega} \ldots 1\mathrm{T\Omega} \quad (\text{ideal} \infty)$$

Output Resistance
(Ausgangswiderstand)

$$r_A = -\left.\frac{\partial u_A}{\partial i_A}\right|_{u_D = const.} \qquad r_A \approx 2\Omega \ldots 100\Omega \quad (\text{ideal } 0)$$

6.3 Real Behaviour of Amplifiers

Common Mode Input Resistances R_{GIP} R_{GIN}
(Gleichtakteingangswiderstände)

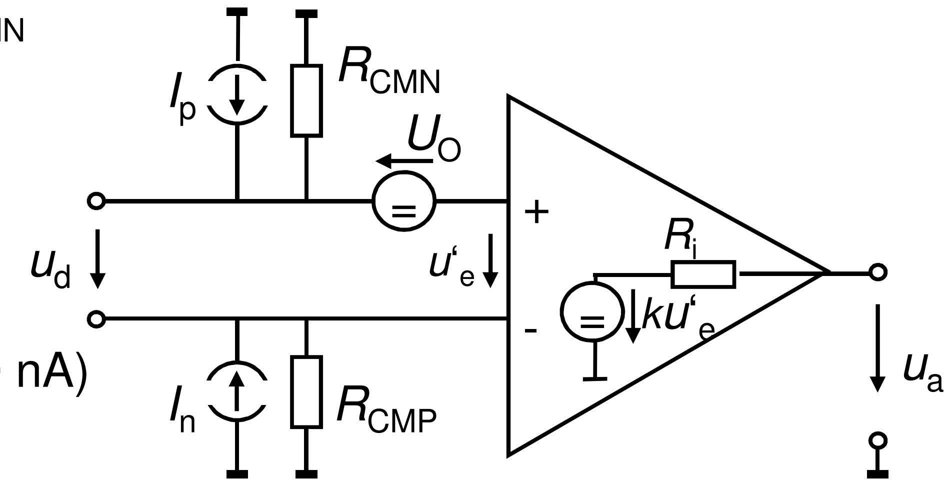
Input Bias Current (Eingangsruhesströme):

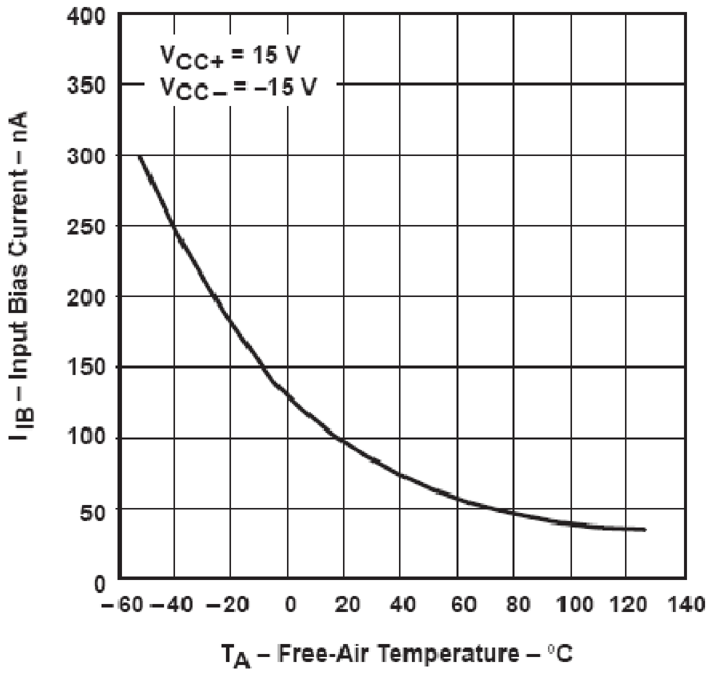
(typ. 1 nA...500 nA)

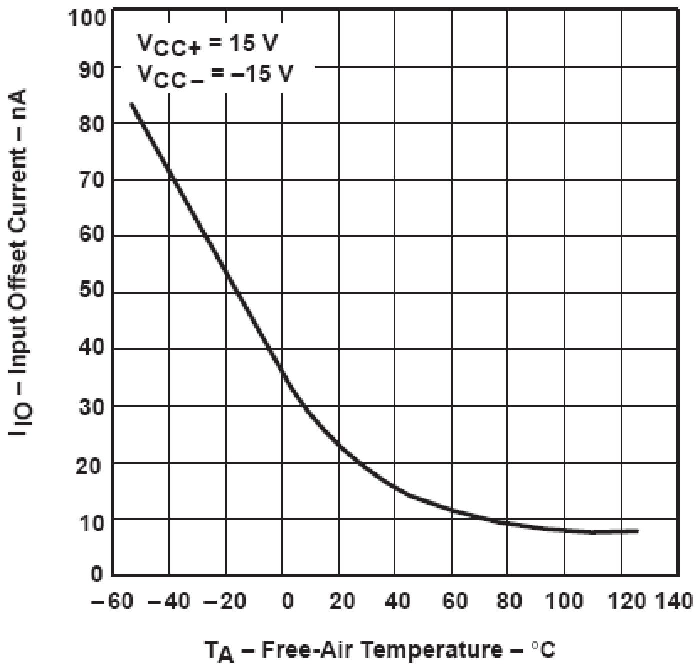
$I_P = I_b + I_O$ *mit* I_b Bias Current

$I_N = I_b - I_O$ I_O Offset Current (typ. 20 fA...20 nA)

$I_b \approx 50$ fA (FET); $I_b \approx 1\,\mu$A (Bipolar)







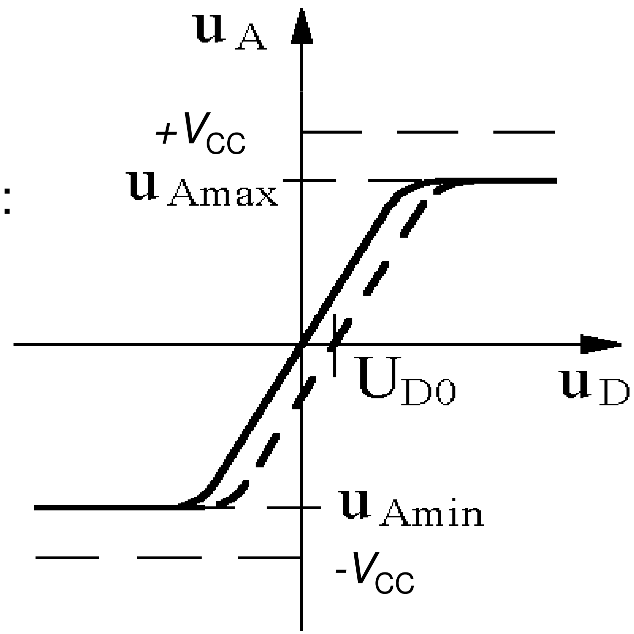
6.3 Real Behaviour of Amplifiers

Differential Input Voltage (Differenz-Eingangsspannung):
U_d (typ. ± 3V.... ± 30 V)

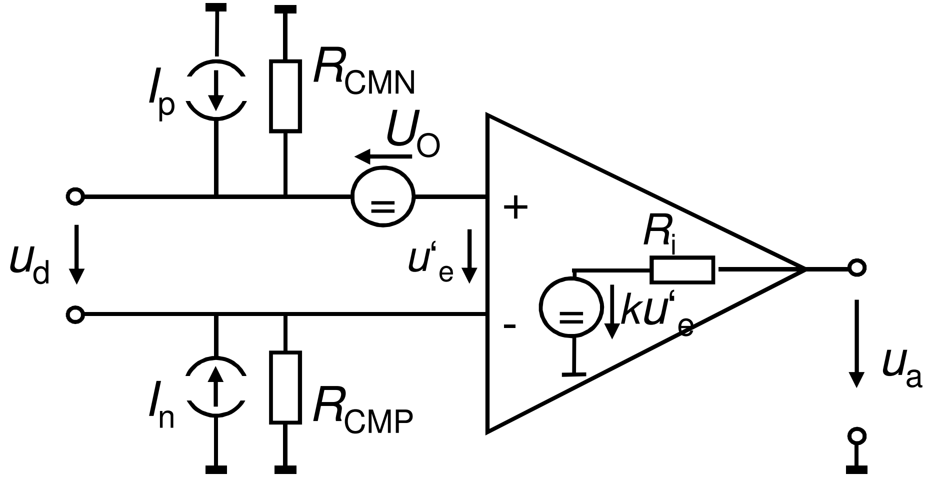
Common Mode Input Voltage
(Gleichtakteingangsspannung):
U_{CM} (typ. ±13V... ±16V)
Input voltage relative to ground

Output Voltage Swing (Ausgangsspannungshub):
U_{Amax} (typ. 27 V.... 32 V)
Maximal value of the output voltage without amplitude limitation

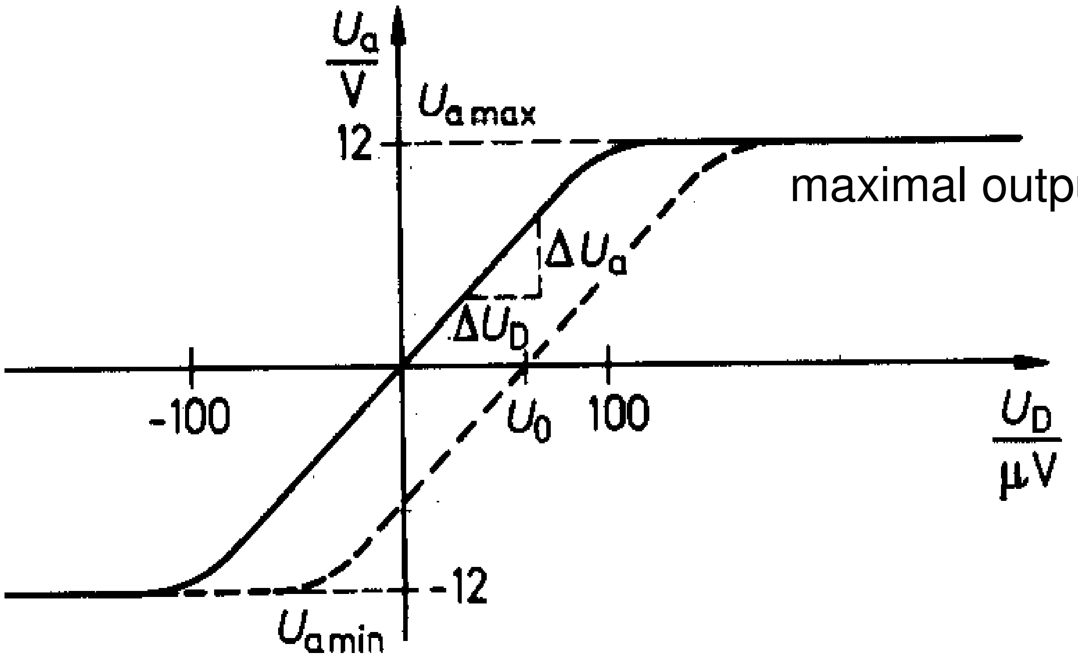
Rail-to-Rail
→ output voltage swing next the voltage supply V_{CC}



Input-Offset Voltage U_0 (Offset-Spannung):
(typ. 0,5 µV... 5 mV)
Voltage between difference inputs, so that the output voltage 0V is reached



$$U_a(U_d = U_0) = 0V$$



maximal output voltage

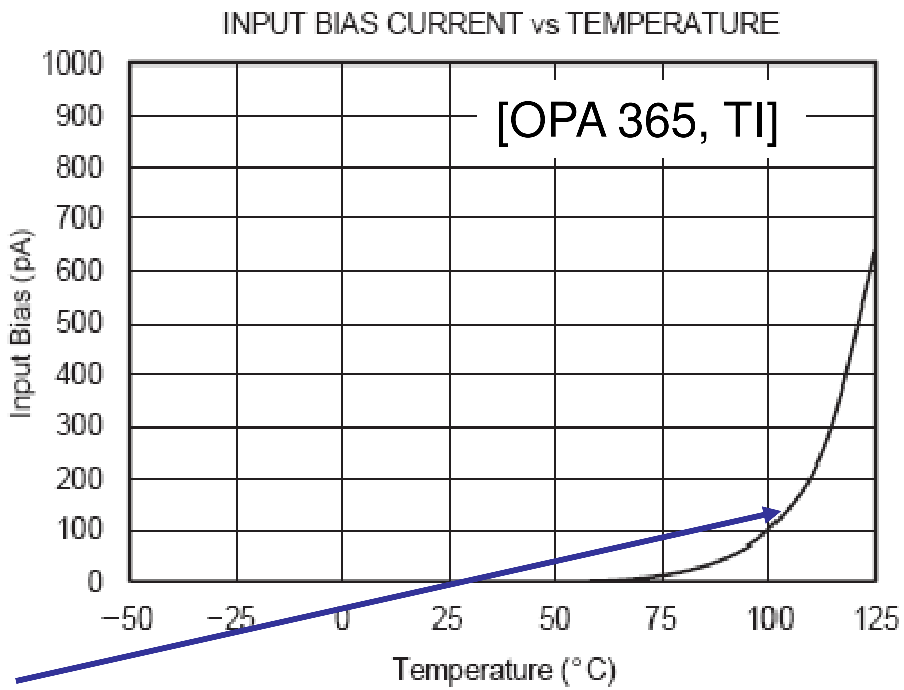
6.3 Real Behaviour of Amplifiers

Offset Voltage Drift (Offsetspannungsdrift):
(typ. 0,01 µV/°C ... 15 µV/°C)

$$\Delta U_0 = \frac{\partial U_O}{\partial \vartheta}\Delta\vartheta + \frac{\partial U_O}{\partial U_V}\Delta U_V + \frac{\partial U_O}{\partial t}\Delta t$$

Input Current Drift (Eingangsstromdrift):
(typ. 10 fA/°C ... 1 µA/°C)

$$\left.\frac{\partial\left(i_p, i_n\right)}{\partial \vartheta}\right|_{U_N=const., U_p=const.}$$

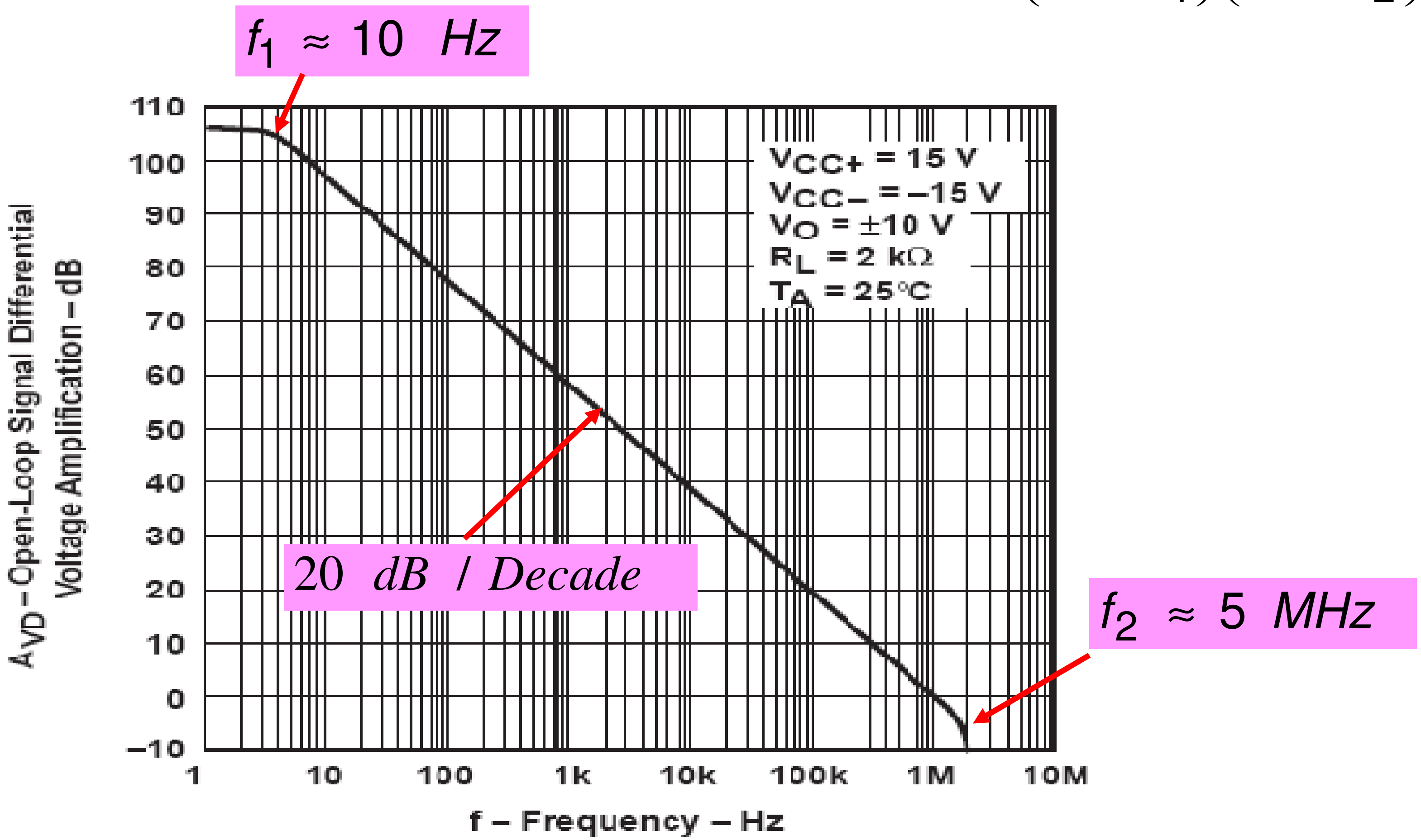


Different behaviour compared to µA 741 (see Page 13)

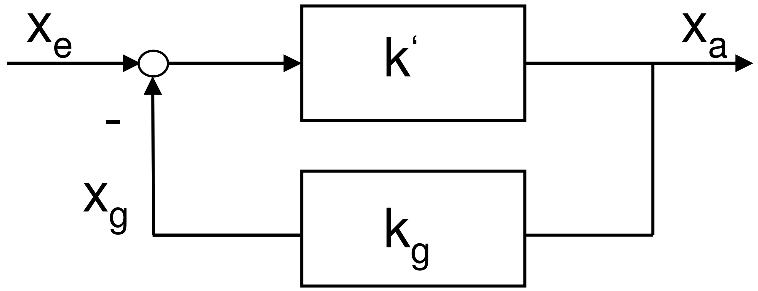
Transfer Function

Corresponds to the complex amplification

$$\underline{G}(\omega) \approx \frac{\underline{U}_A(\omega)}{\underline{U}_D(\omega)} = \frac{V_0}{\left(1 + j\frac{\omega}{\omega_1}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$



6.3 Real Behaviour of Amplifiers
Feedback (Closed *Loop*)



$$x_a = k'\left(x_e - x_g\right) = k'\left(x_e - k_g x_a\right)$$

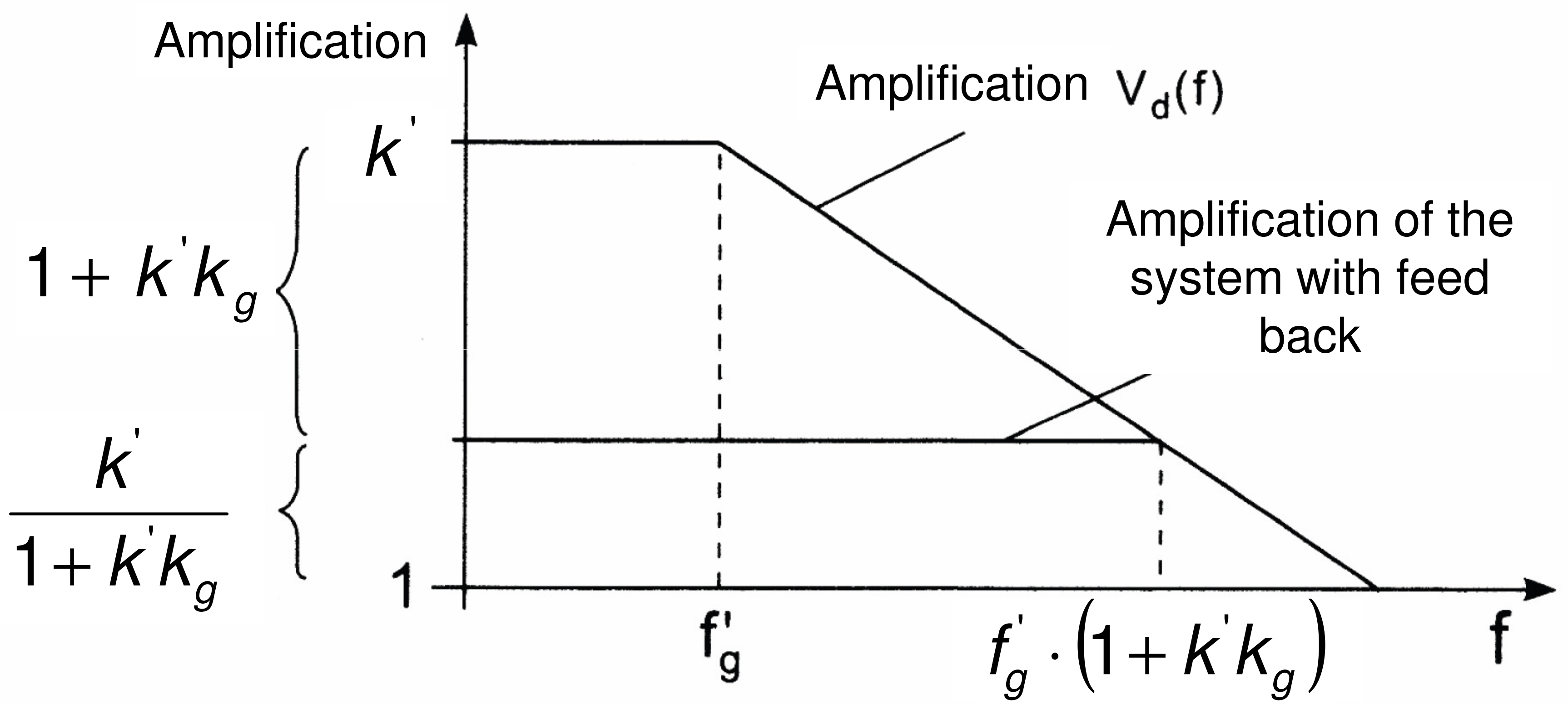
$$x_a = \frac{k'}{1 + k' k_g}\, x_e = \frac{1}{\frac{1}{k'} + k_g}\, x_e$$

$$\approx \frac{1}{k_g}\, x_e \quad \text{für} \quad k' \to \infty$$

Advantages:

- Smaller amplification, but just dependent on k_g if k' is sufficently high
  - → Selection of stabile circuit elements
  - → independence on changes of amplifier properties
- Band width (Frequenzbereich) becomes higher.
- Input resistance of voltage Amplifiers $\nearrow$ and by Current Amplifiers $\searrow$
- Output resistance $\searrow$ by voltage output and and $\nearrow$ by current output

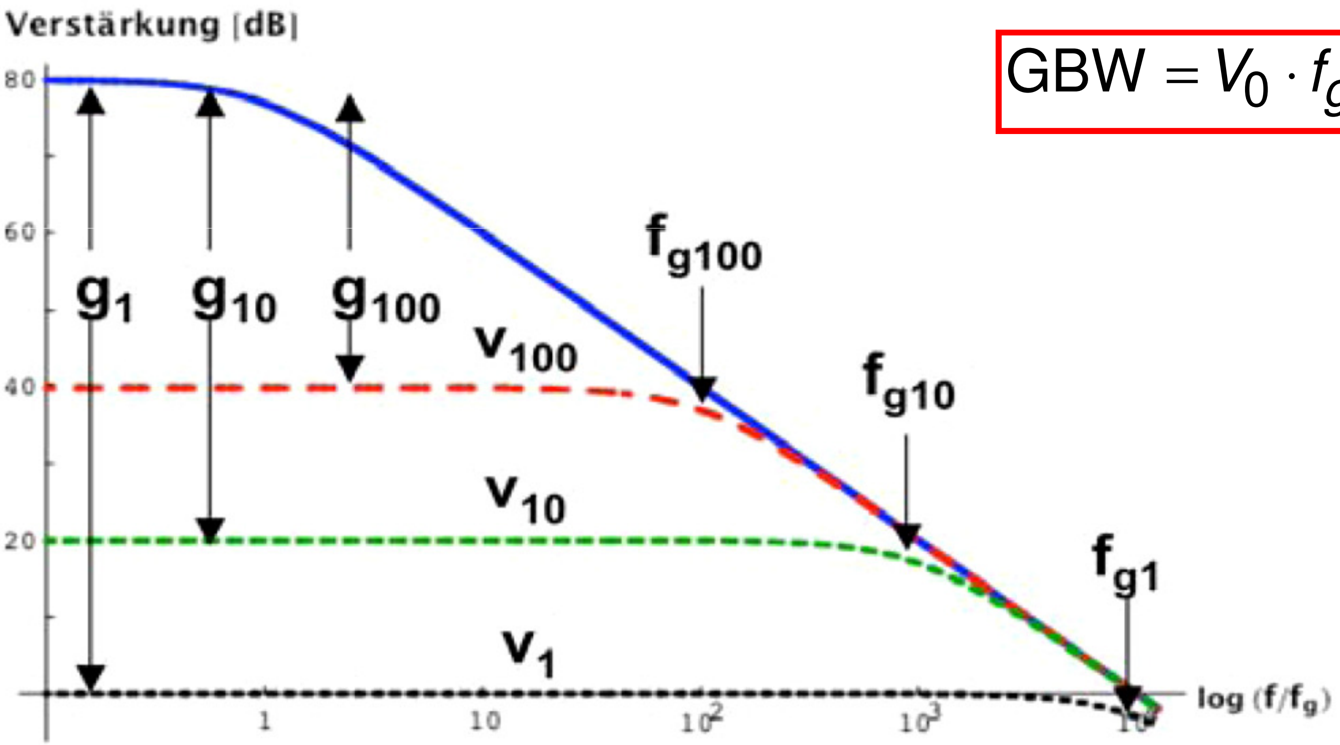
6.3 Real Behaviour of Amplifiers
Feedback (Closed *Loop*)



6.3 Real Behaviour of Amplifiers

Gain-Bandwidth-Product (Verstärkungs-Bandbreite-Produkt): GBW
(typ. 0.8 MHz..3 MHz)

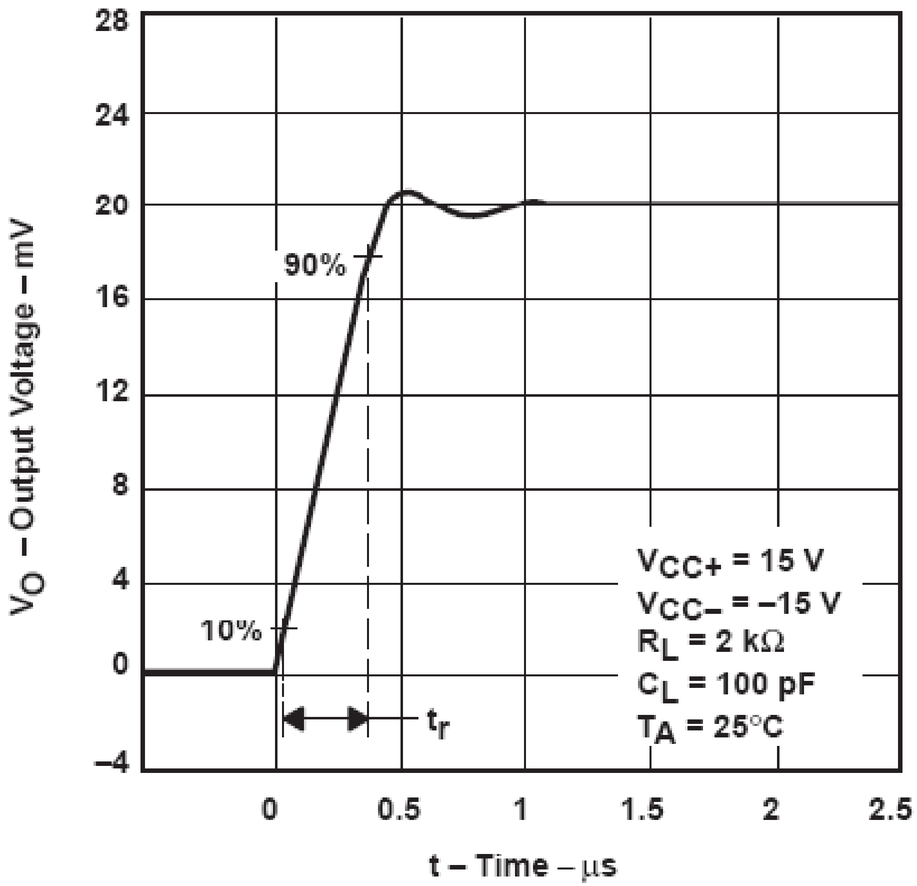
In the sector in which the amplification is sinking with 20 dB/Dekade, the product of frequency and corresponding amplification is constant.



$$\boxed{GBW = V_0 \cdot f_{g0}}$$

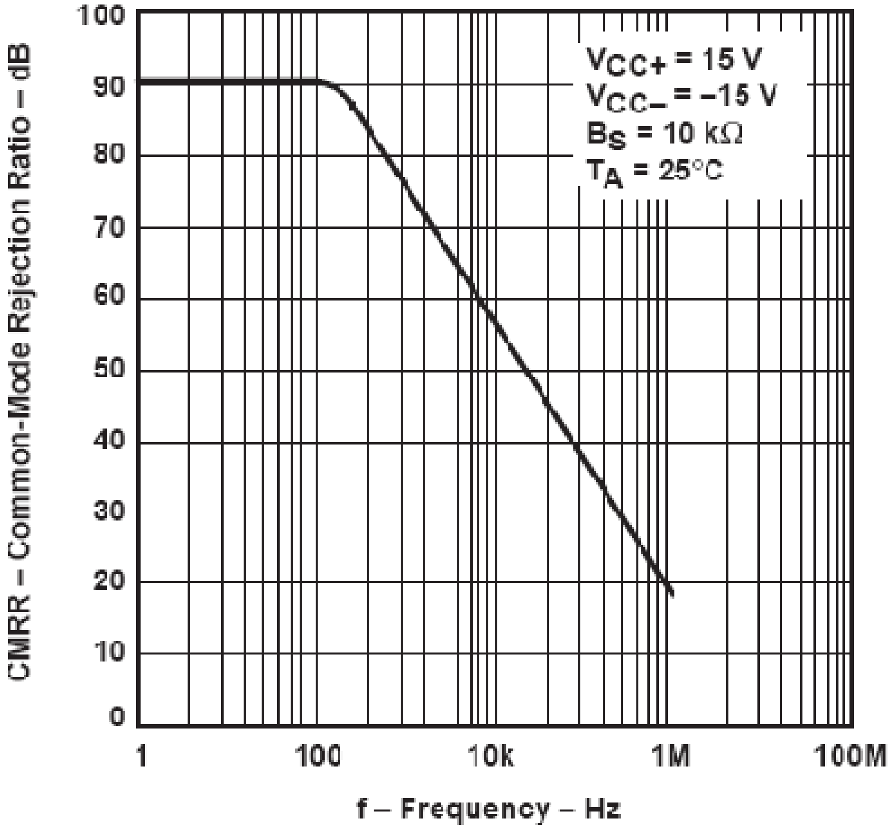
Slew Rate (Maximale Anstiegssteilheit):
SWR (typ. 0.5 V/µs...50 V/µs)
SWR is the maximum possible change of the output voltage pro Time Unit

$$\boxed{SWR = \left(\frac{\partial u_A}{\partial t}\right)_{max}}$$



Common Mode Rejection Ratio (Gleichtaktunterdrückung): CMRR (typ. 80dB...120dB)
Proportion of the open voltage gain to the common mode gain

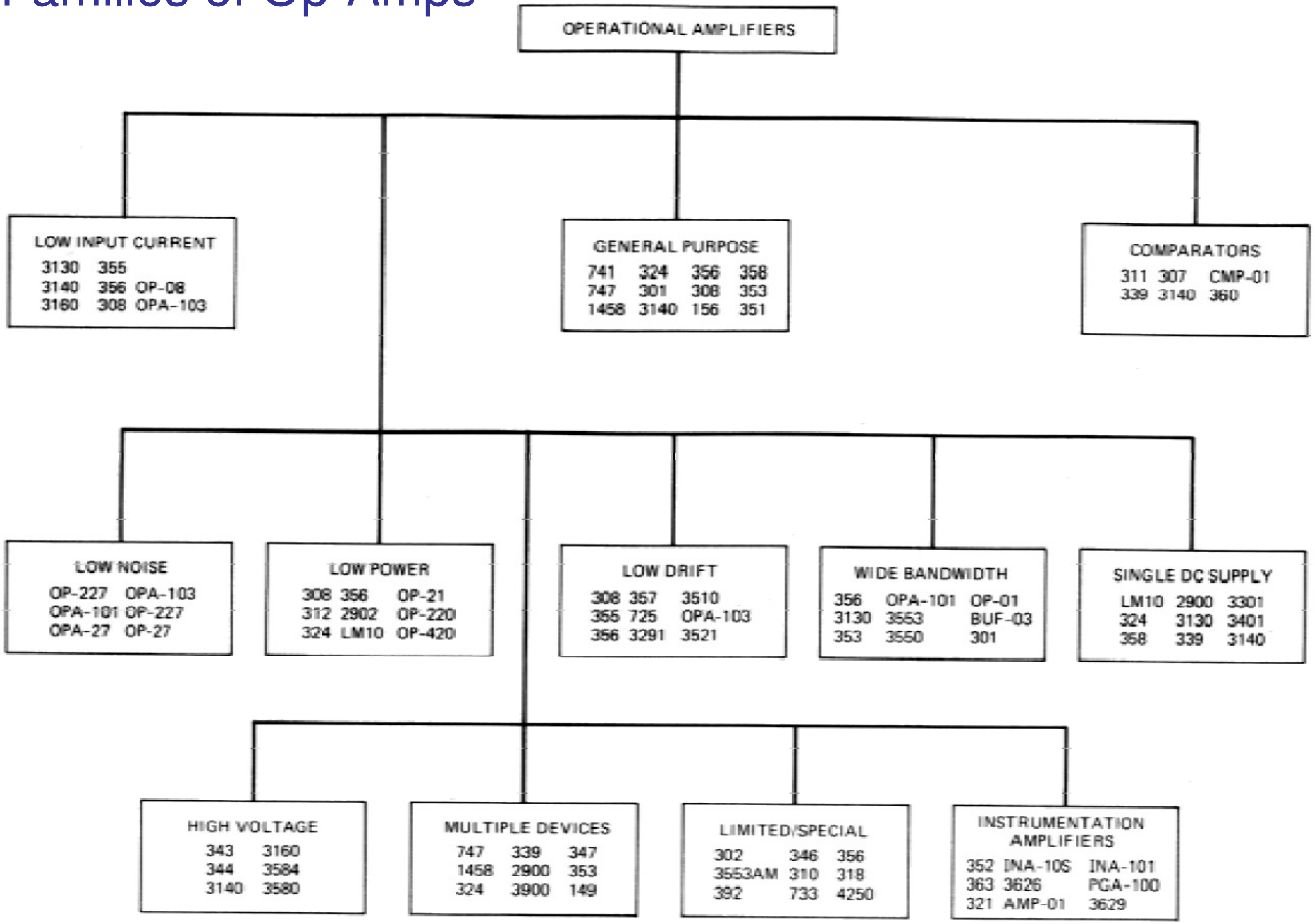
$$\boxed{CMR = 20 \cdot \lg\left(\frac{V}{V_{CM}}\right) = 80 \cdots 120\,dB}$$



Supply Voltage Rejection (Betriebsspannungsunterdrückung): SVR (typ. -60dB..-100dB)
Proportion of the offset voltage change related to the changes of the voltage supply

Signal-Noise-Ratio: SNR

6.3 Real Behaviour of Amplifiers
Families of Op-Amps

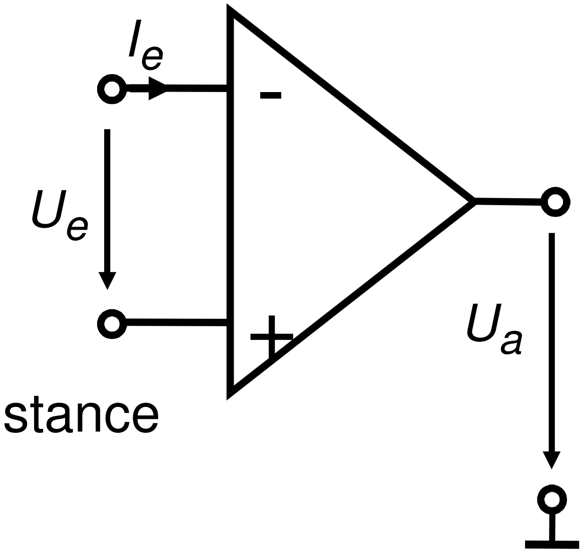


6.3 Real Behaviour of Amplifiers
Ideal Amplifier

„is a useful Model for amplification circuits“

- $I_e \rightarrow 0$
- $U_e \rightarrow 0$
- High input resistance

  $R_e \rightarrow \infty$



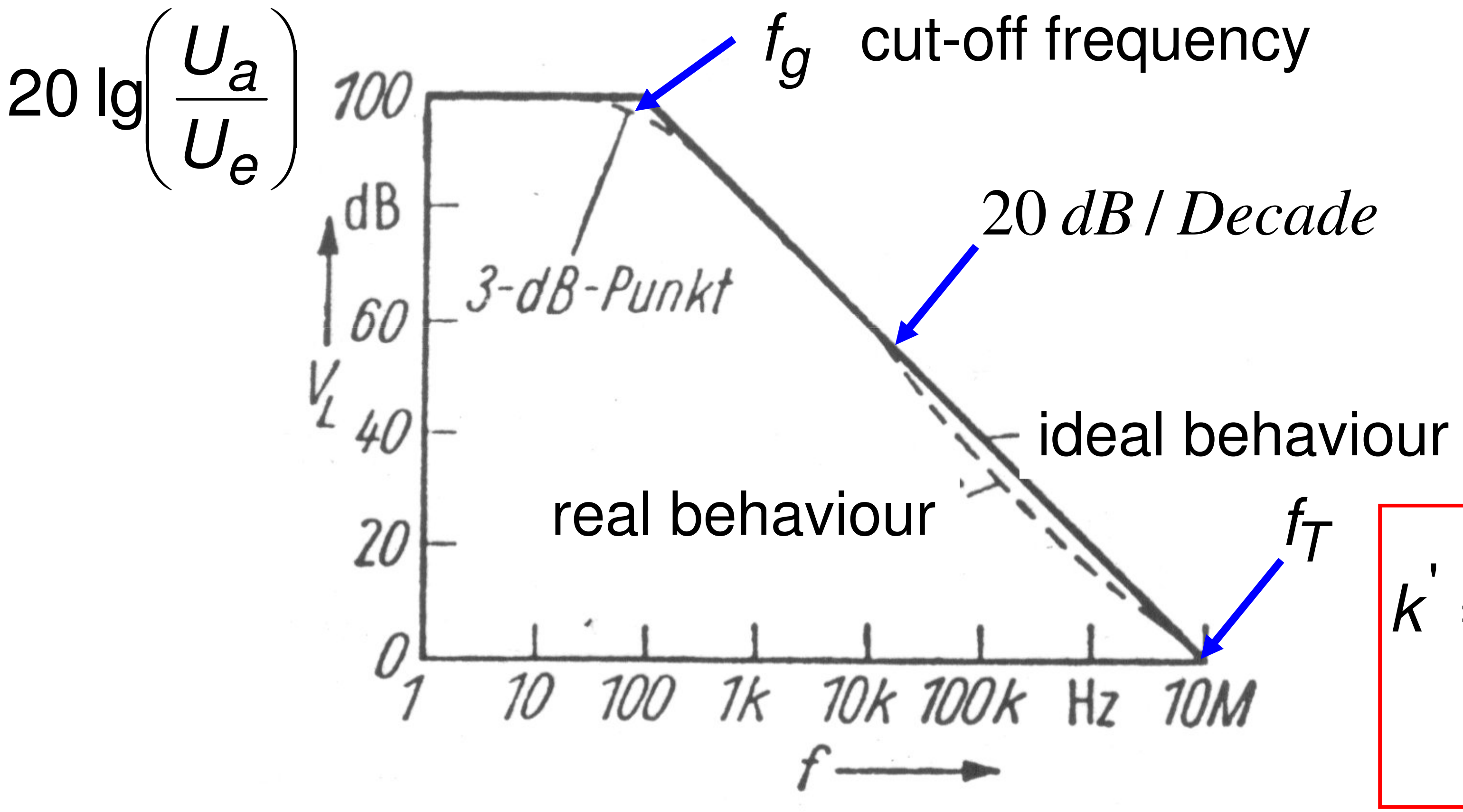
- *any* I_a, U_a are possible
- very high difference amplification

  $V \rightarrow \infty$
- Common mode amplification
  $V_{Gl} \rightarrow 0$
- low output resistance
  $R_a \rightarrow 0$
- no cut-off frequency

6.4 Correction of the Real Behavior

Frequency Behaviour

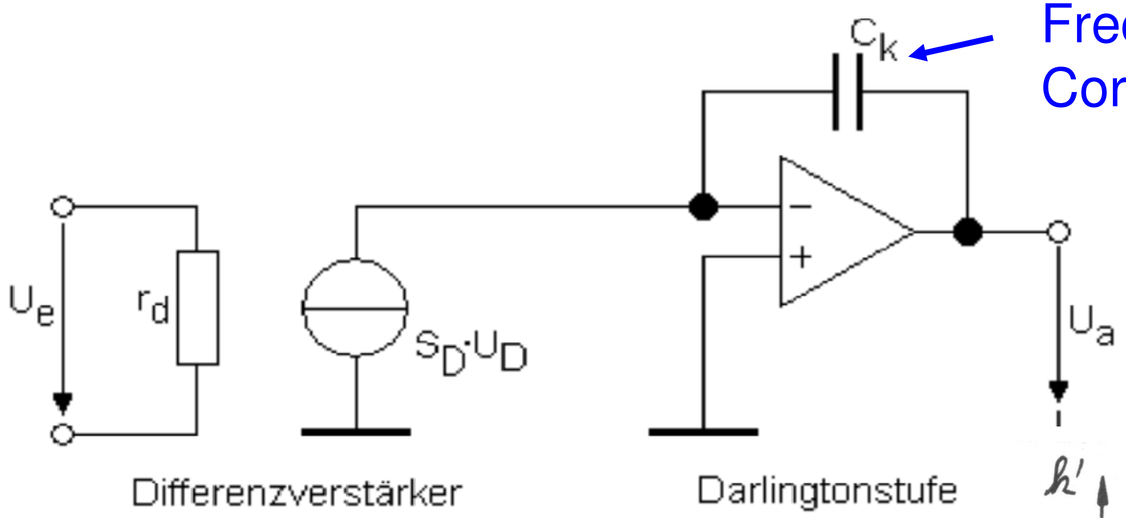
Open Loop Amplification



$$k' = \frac{k_0'}{1 + j\frac{f}{f_g}}$$

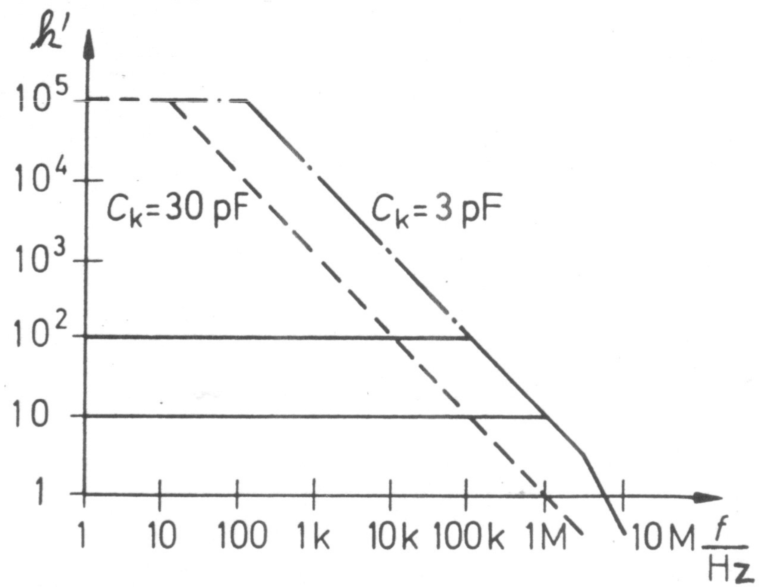
6.4 Correction of the Real Behavior

Frequency Behaviour



Frequency Behaviour Correction

- Smaller band width
- Slew-Rate reduction



angepaßte Frequenzgangkorrektur

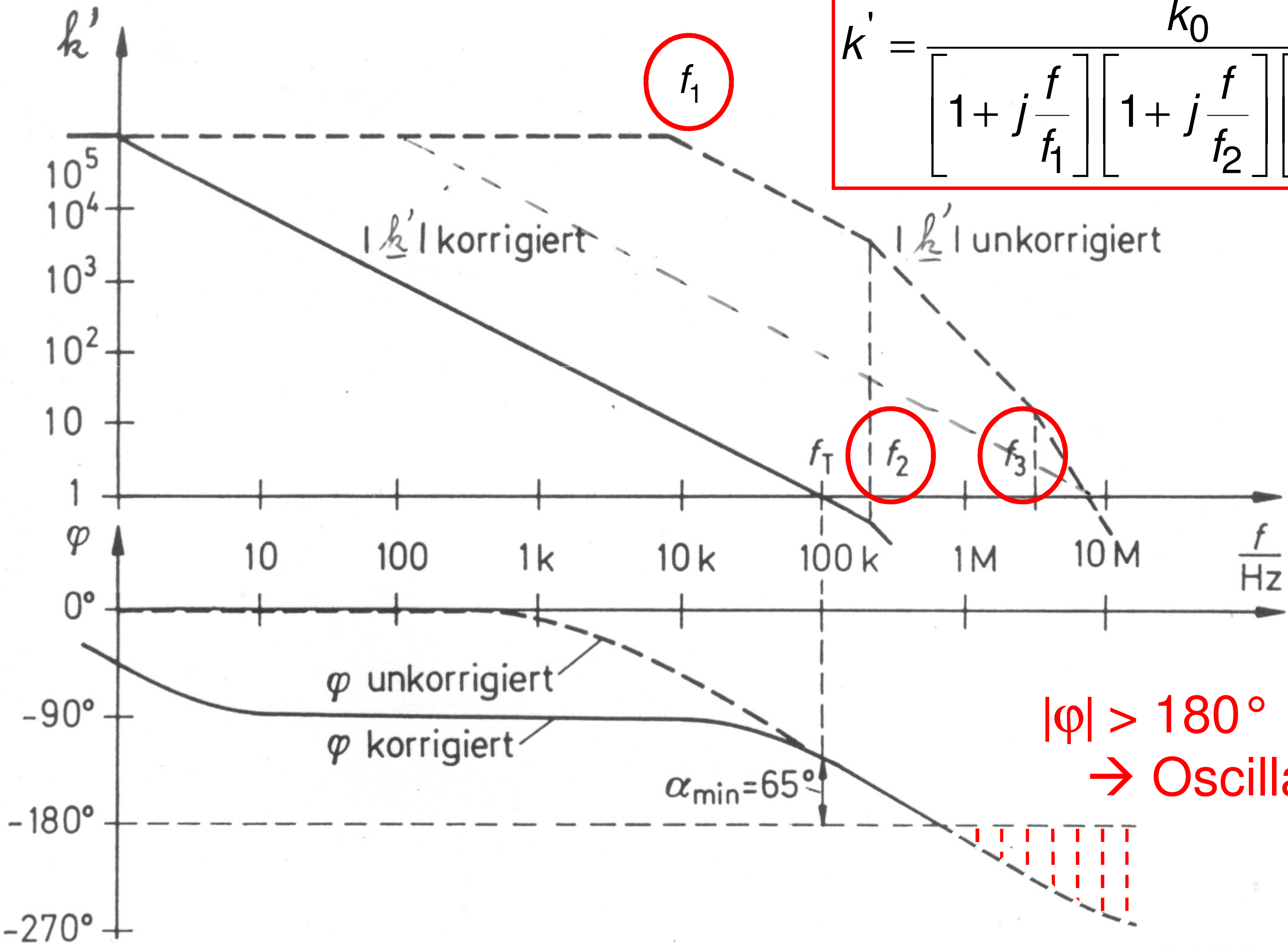
6.4 Correction of the Real Behavior

Frequency Behaviour

Real amplifier

→ Many amplification stages

$$k' = \frac{k_0'}{\left[1 + j\frac{f}{f_1}\right]\left[1 + j\frac{f}{f_2}\right]\left[1 + j\frac{f}{f_3}\right]}$$

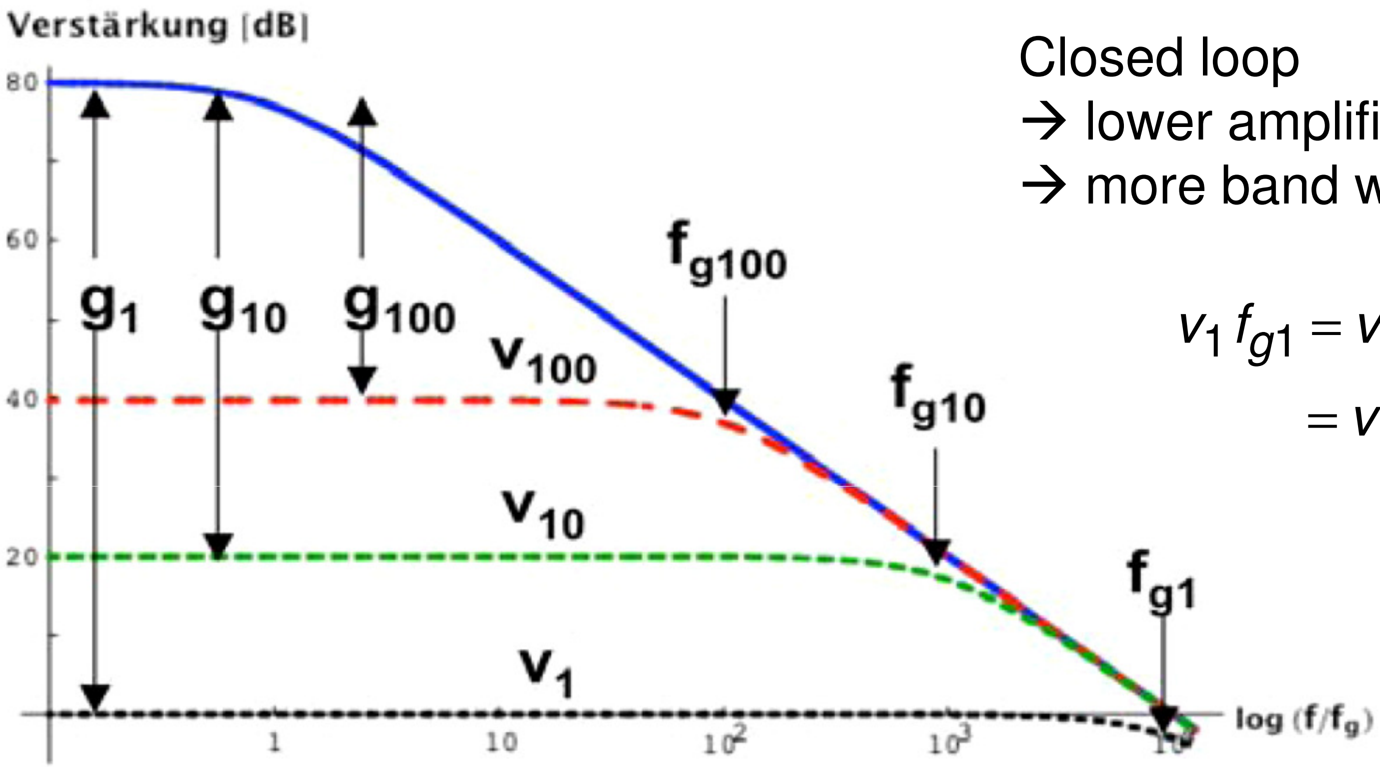


|φ| > 180°

→ Oscillation

6.4 Correction of the Real Behavior

Frequency Behaviour



Closed loop
→ lower amplification
→ more band width

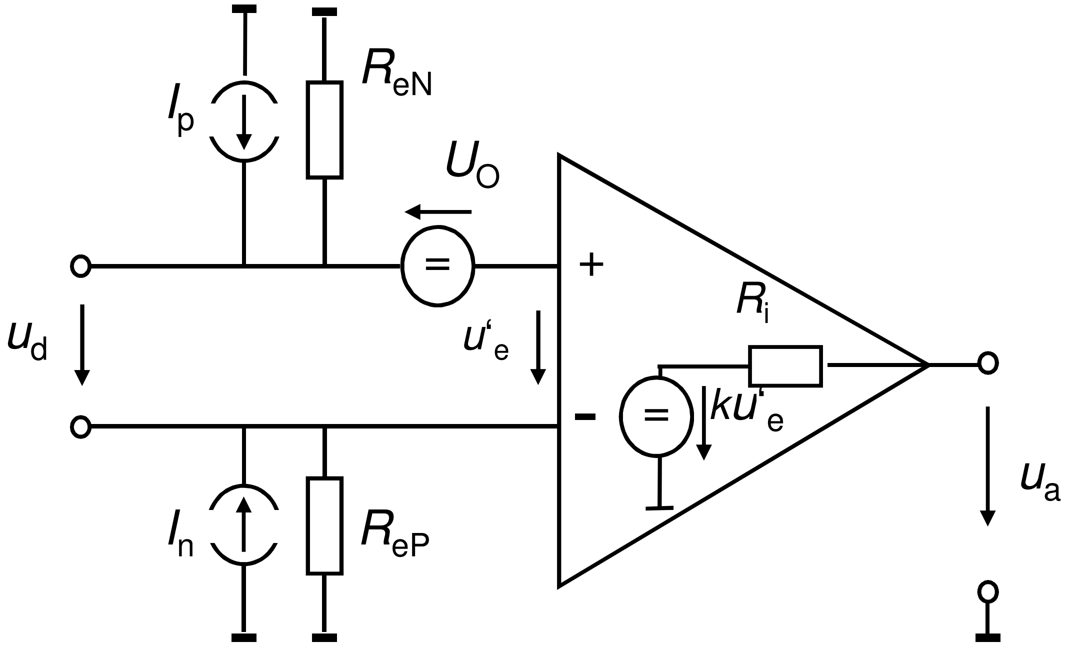
$$v_1 f_{g1} = v_{10} f_{g10}$$
$$= v_{100} f_{g100} = f_T$$

For every amplification level, a different compensation is necessary

- A transimpedance amplifier is a special amplifier with very high band width and a variable amplification

6.4 Correction of the Real Behavior
Zero Point Error

How can we treat offset voltages and input bias currents?



Application of the superposition principle

Example: u/u amplifier

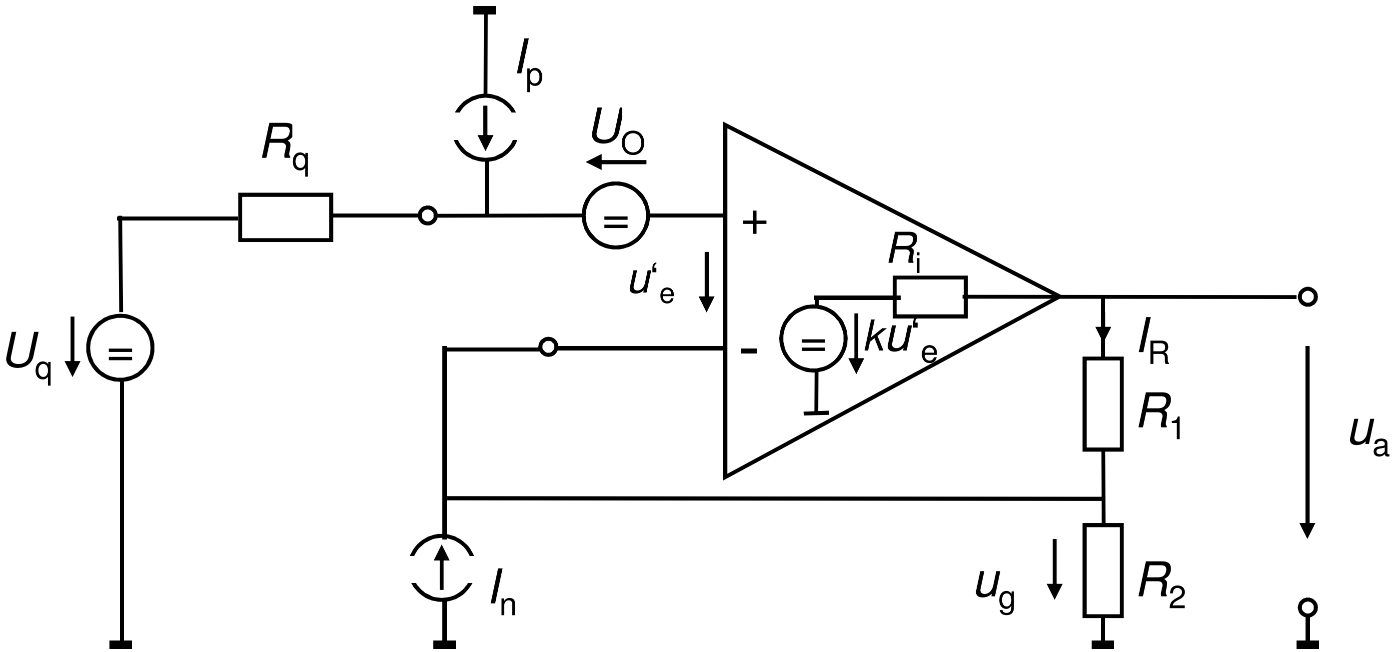
6.4 Correction of the Real Behavior

Zero Point Error

Superposition Principle

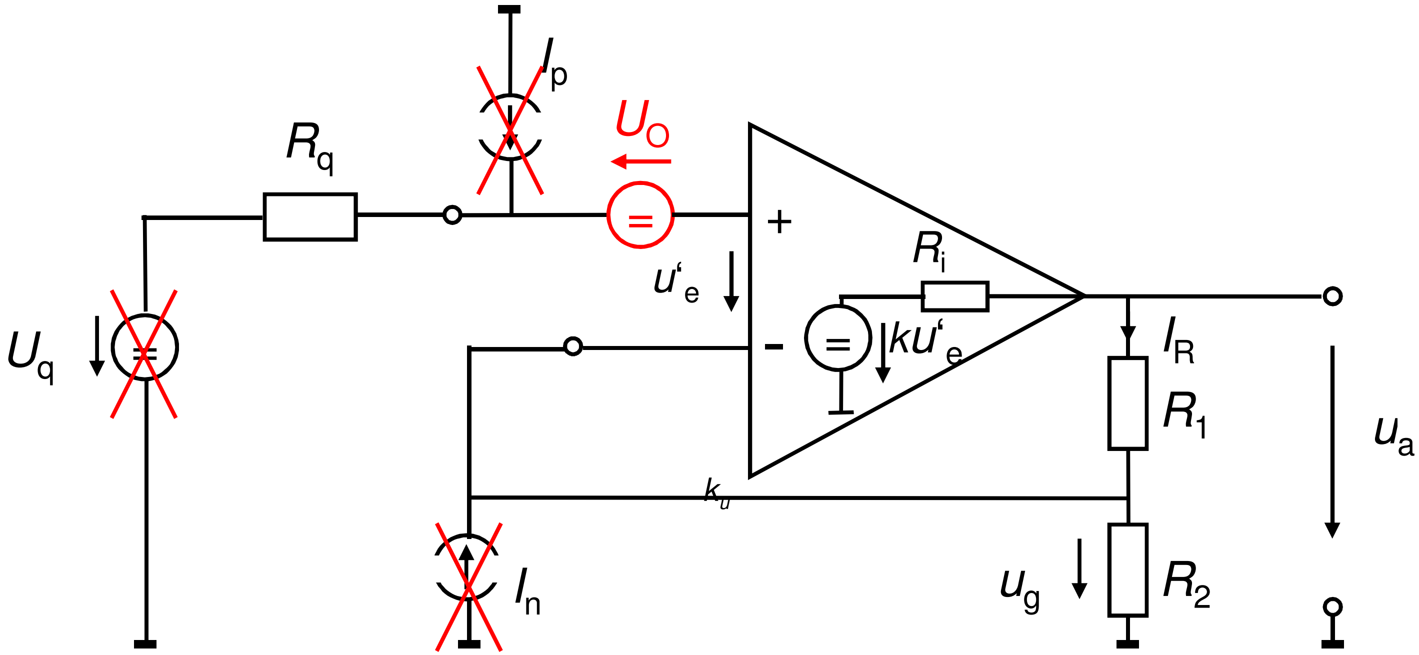
- The reaction on every source of errror is considererd alone:
  ...... additionally available voltage sources are short-circuited
  ...... additionally available current sources are broken

- The results are added to each other



6.4 Correction of the Real Behavior
Zero Point Error

Influence of Off-Set Voltage



$$U_q = 0, \quad I_p = I_n = 0$$

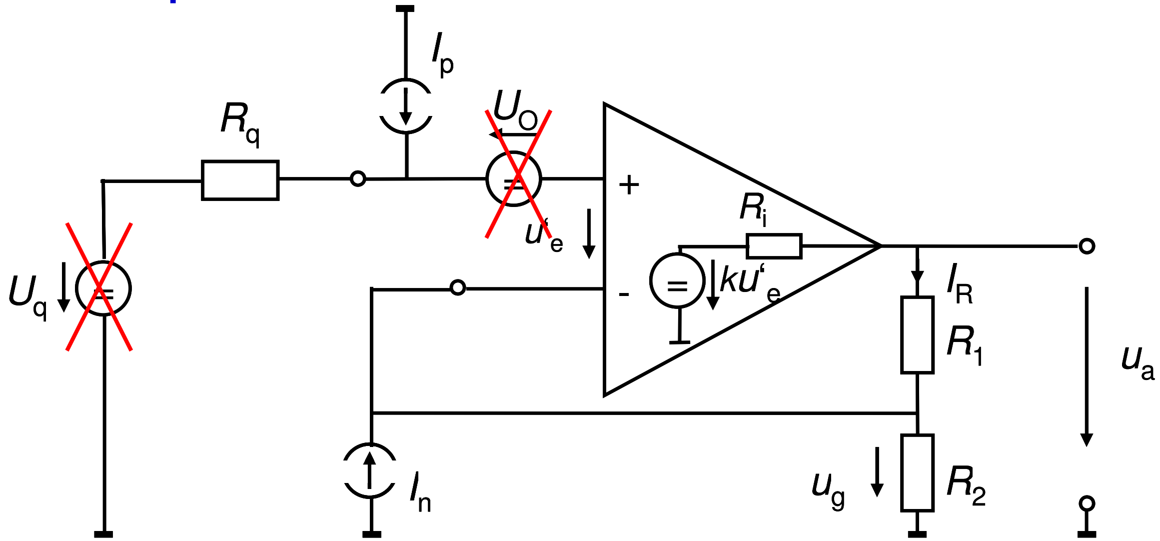
Input Mesh: $U_O - U_g = 0$

$$\boxed{U_a(U_O) = \frac{R_1 + R_2}{R_2} \cdot U_O}$$

6.4 Correction of the Real Behavior

Zero Point Error

Influence of Input Bias Current



Influence of I_p $\quad U_q = 0, \quad U_O = 0, \quad I_n = 0$

Input Mesh: I_p flows over R_q

$$\boxed{U_a(I_p) = \frac{R_1 + R_2}{R_2} \cdot R_q \cdot I_p}$$

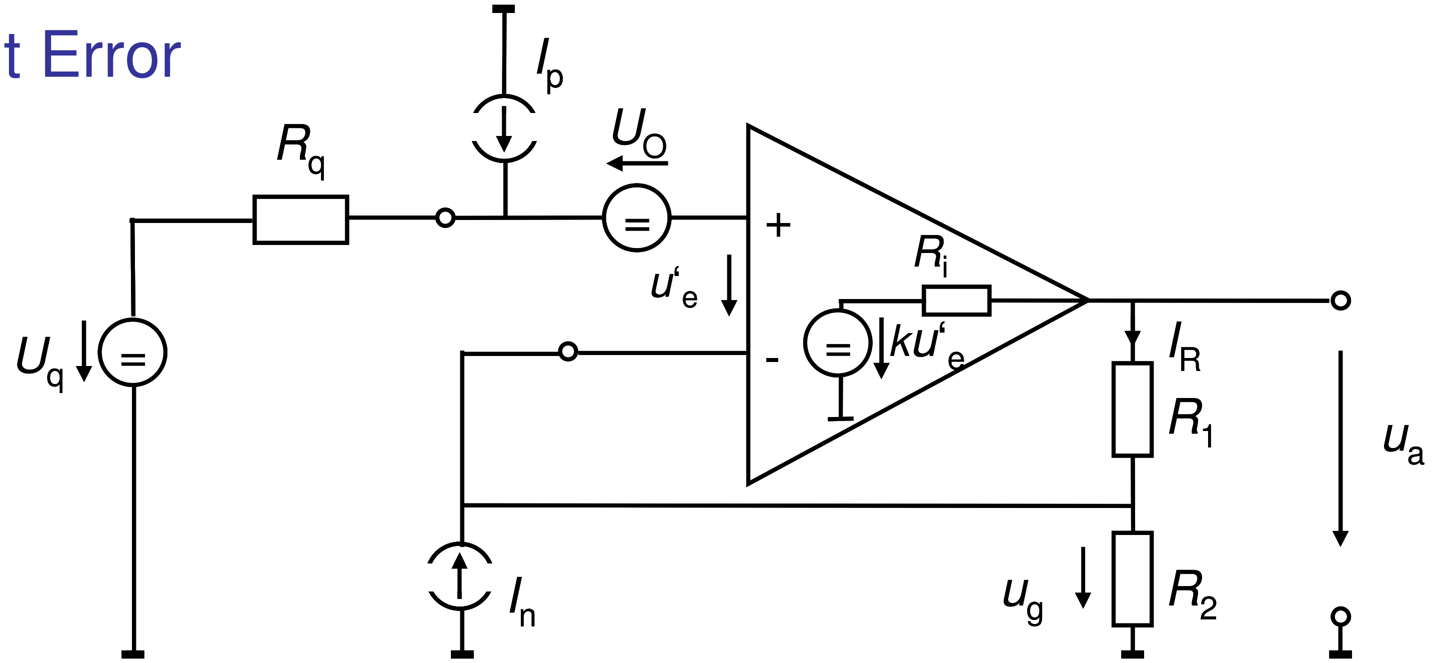
Influence of I_n $\quad U_q = 0, \quad U_O = 0, \quad I_p = 0$

I_n flows to the node between R_1 und R_2 $\quad U_g = (I_n + I_R) \cdot R_2$

Input Mesh: $\quad U_g = 0 \quad I_n = -I_R$

$$\boxed{U_a(I_n) = -R_1 \cdot I_n}$$

6.4 Correction of the Real Behavior
Zero Point Error



Superposition principle

$$U_a(U_q, U_O, I_p, I_n) = \frac{R_1 + R_2}{R_2} \cdot \left(U_q + U_O + R_q \cdot I_p - \frac{R_1 R_2}{R_1 + R_2} I_n \right)$$

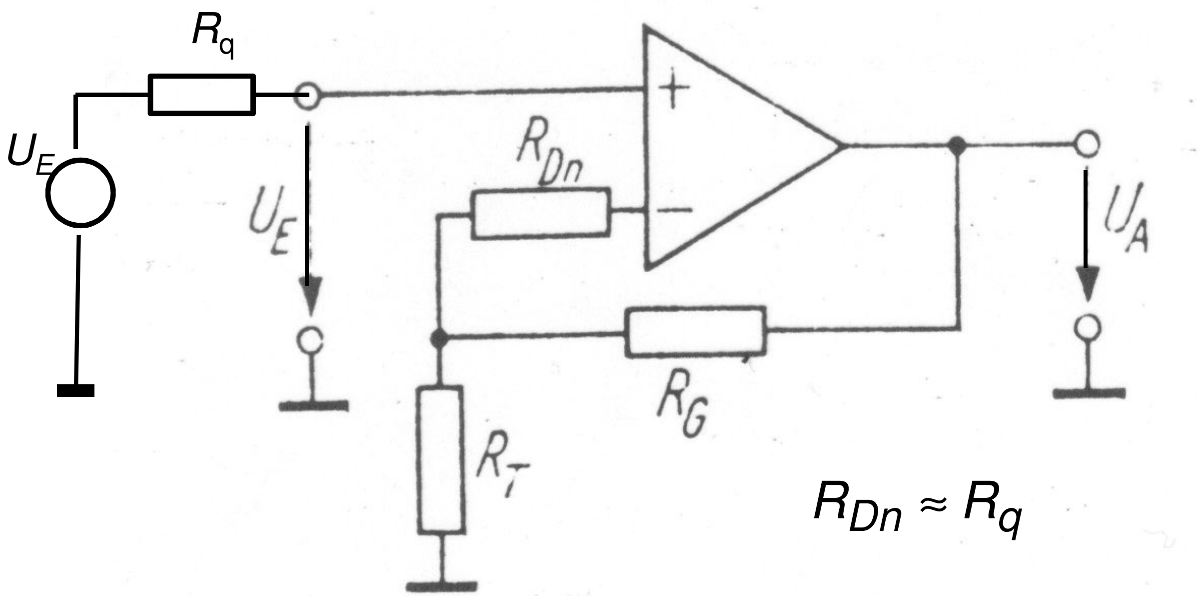
Compensation for $R_q = R_1 \| R_2$

$$U_a = \frac{R_1 + R_2}{R_2} \cdot \left(U_q + U_O + R_q \cdot \left(I_p - I_n \right) \right) = \frac{R_1 + R_2}{R_2} \cdot \left(U_q + U_O + R_q \cdot I_O \right)$$

6.4 Correction of the Real Behavior
Zero Point Error

Noninverting Amplifier

- Input bias currents flowing through the resistances at the input are acting like an off-set voltage
- If the resistances are equal to each other, no difference voltage is amplified

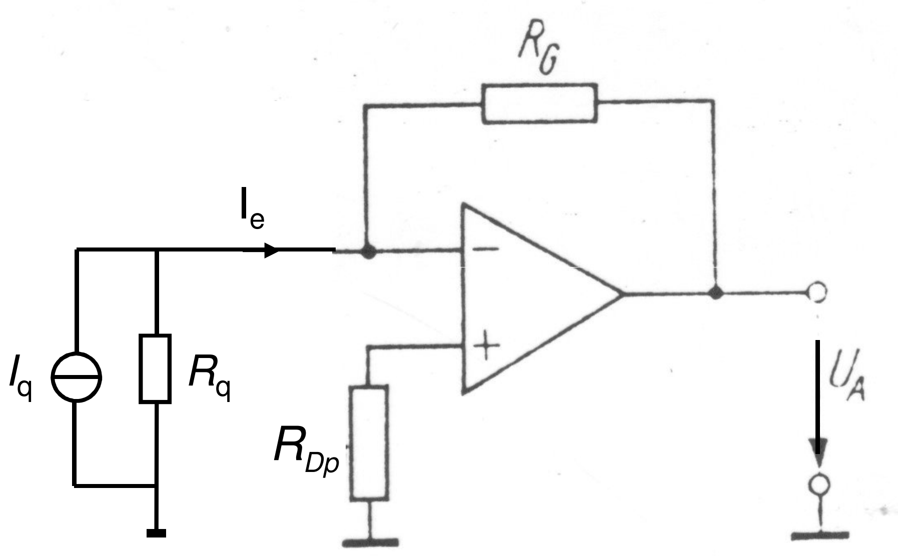


$$R_{Dn} \approx R_q$$

$$U_a = \frac{R_T + R_G}{R_G} \cdot U_e$$

6.4 Correction of the Real Behavior
Zero Point Error

Inverting Amplifier



$$U_a(U_O) = \left(1 + \frac{R_G}{R_E}\right) \cdot U_O$$

→ The Offset voltage is not amplified!

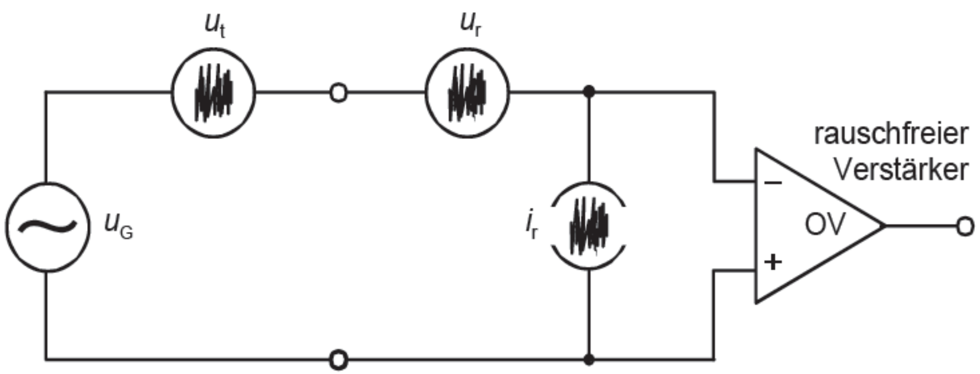
$$U_a(I_n) = -R_G I_n$$

I_p over R_{Dp} has a similar influence to U_O

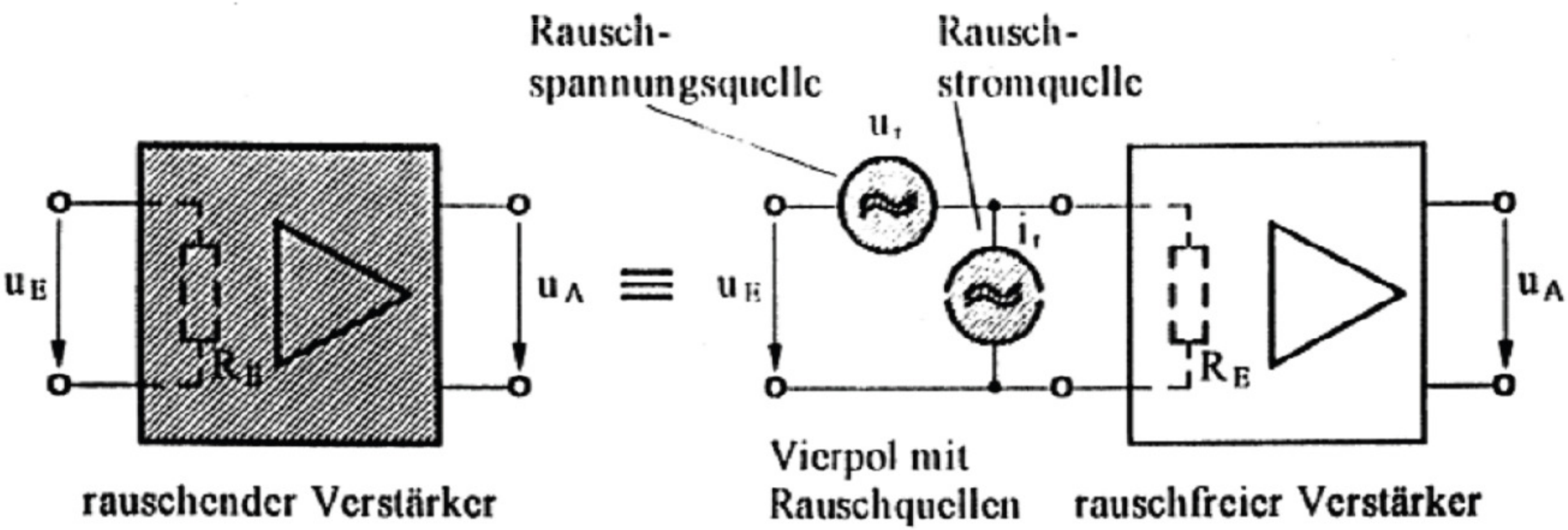
$$U_a(I_p, I_n) = \left(1 + \frac{R_G}{R_q}\right) \cdot R_{Dp} I_p - R_g I_n$$

$$R_{Dp} = \frac{R_G R_q}{R_q + R_G}$$ ▷ $$\boxed{U_a(I_p, I_n) = R_g I_O}$$

6.4 Correction of the Real Behavior
Noise



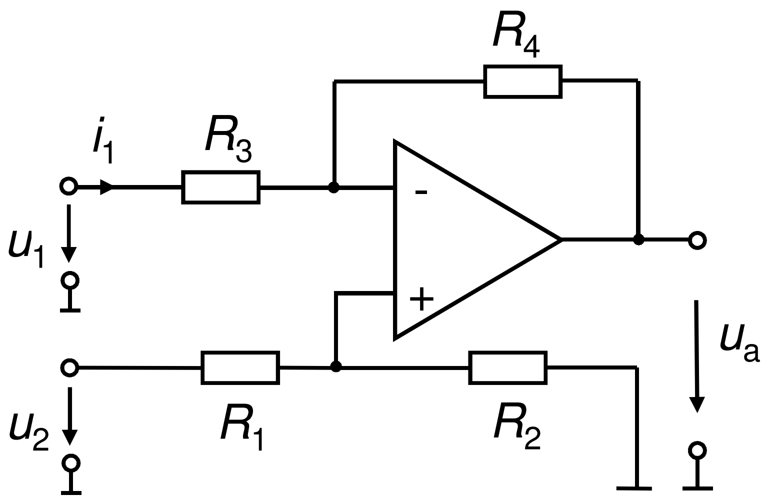
Reduction of the Signal-to-Noise- Ratio



6.5 Instrumentation Amplifier (1)

... Is a precision amplifier
with difference input and
an output related to the ground

Subtraction



$$V_- = -\frac{R_4}{R_3}$$

$$V_+ = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_3 + R_4}{R_3}\right)$$

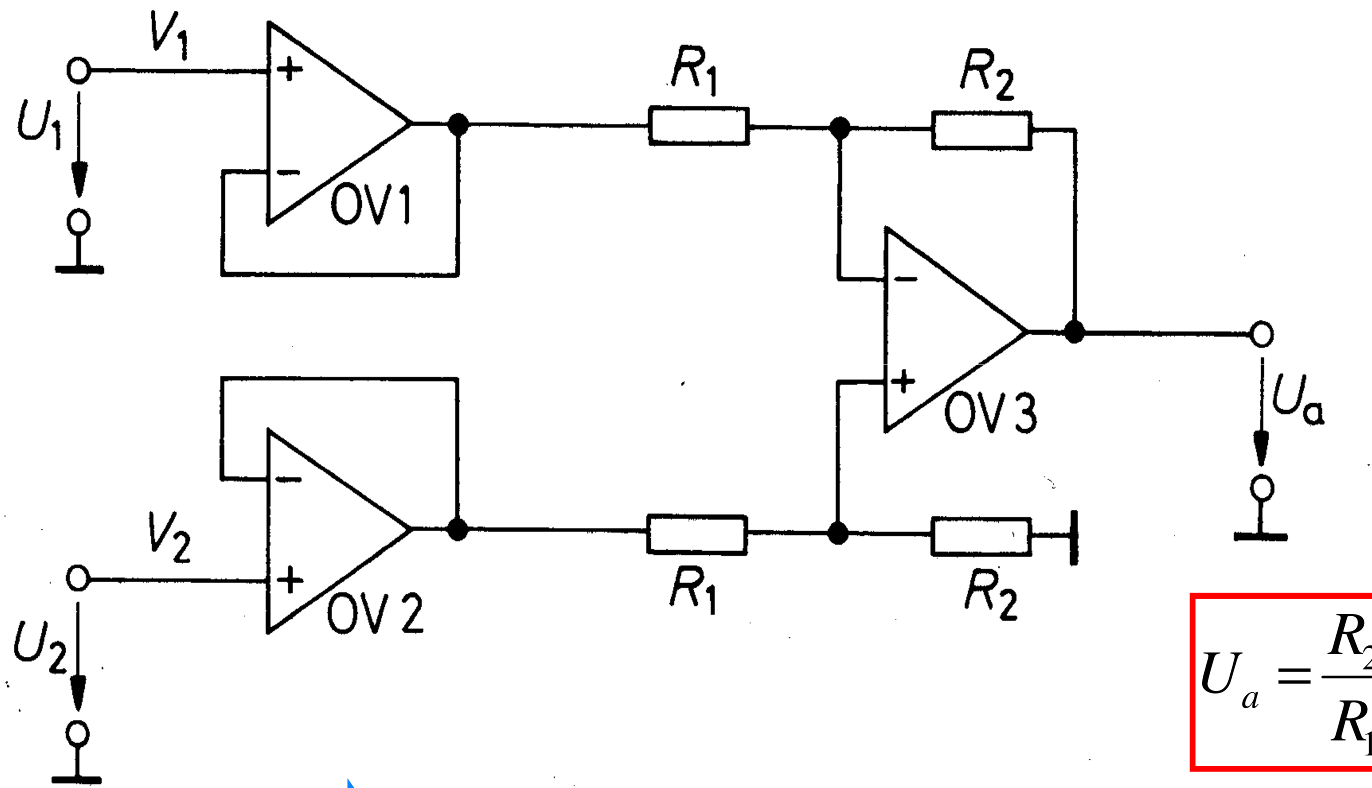
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \triangleright \quad V = \frac{R_4}{R_3}$$

But a high input impedance is not easy to realise!

6.5 Instrumentation Amplifier (2)

Impedance converter for amplification of difference voltages

For example for Bridges

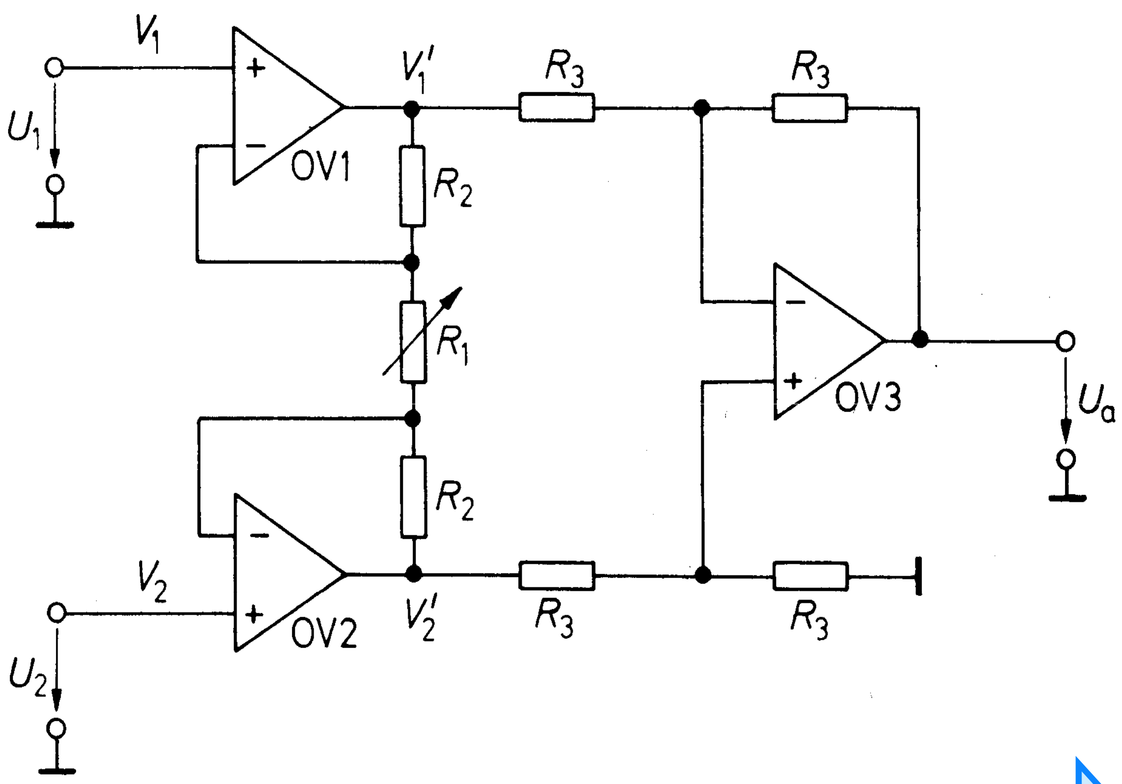


$$U_a = \frac{R_2}{R_1}\left(U_2 - U_1\right)$$

Amplification changes only by changing two resistances!

6.5 Instrumentation Amplifier (3)

Impedance Converter



$$I_{R1} = \frac{U_1 - U_2}{R_1}$$

$$V_1' = I_{R1}\left(R_1 + R_2\right) + U_2$$

$$V_2' = U_2 - I_{R1} R_2$$

$$U_a = V_2' - V_1'$$

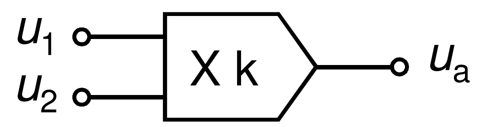
$$U_a = -I_{R1}\left(R_1 + 2R_2\right)$$

$$\boxed{U_a = \left(1 - \frac{2 \cdot R_2}{R_1}\right)\left(U_2 - U_1\right)}$$

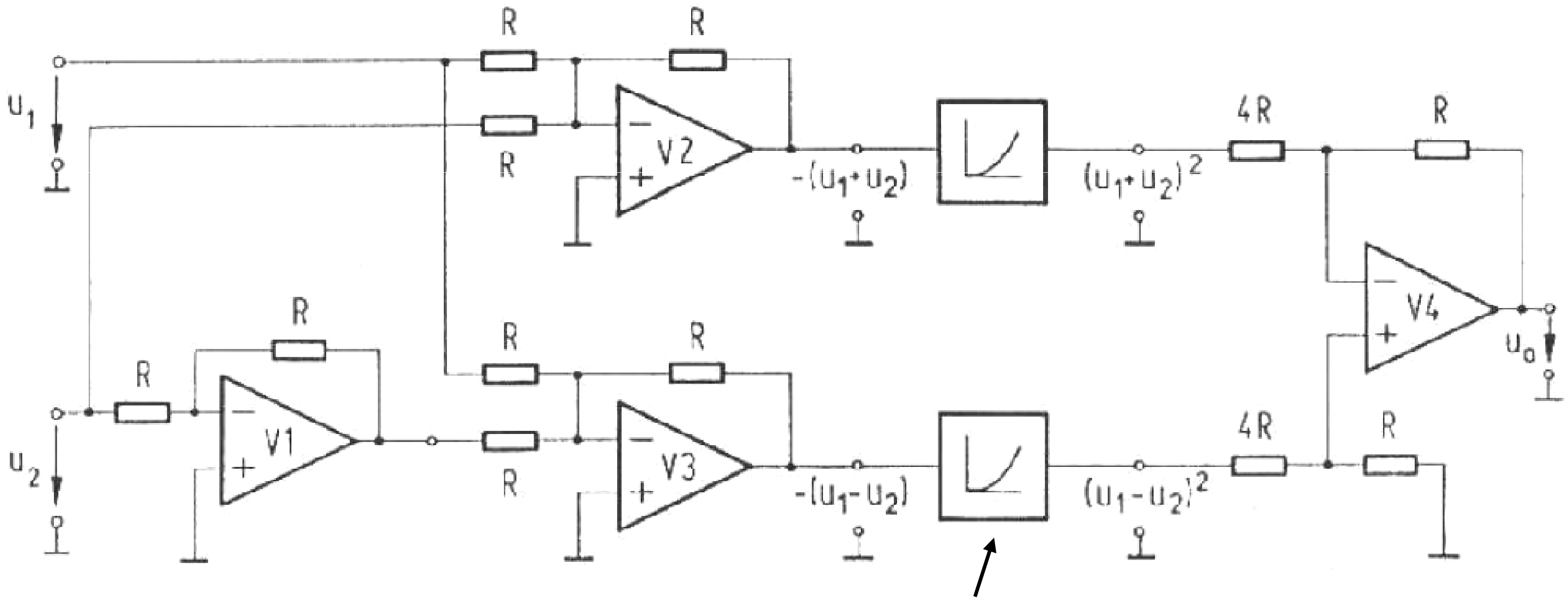
6.6 Applications of Inverting Amplifier

Multiplication

$$u_1 \cdot u_2 = \frac{1}{4}\left[(u_1 + u_2)^2 - (u_1 - u_2)^2\right]$$



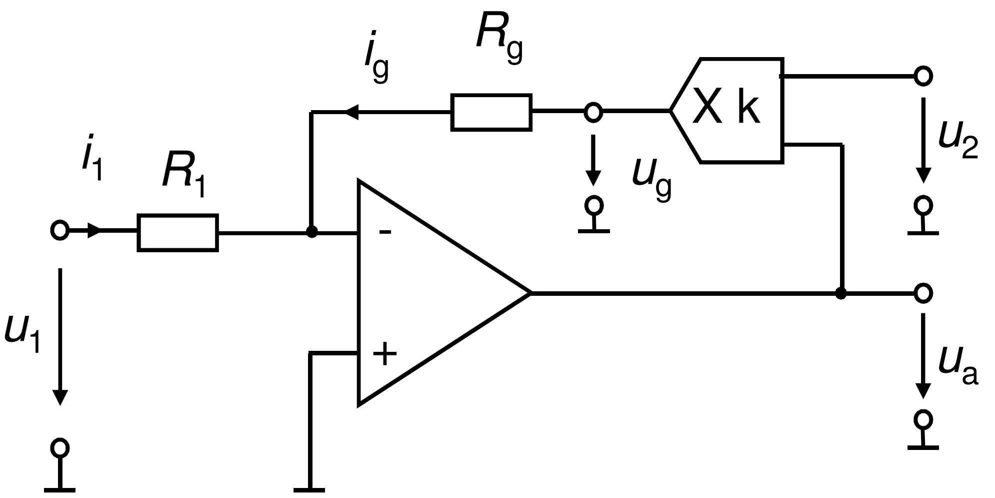
$$u_a = k \cdot u_1\, u_2$$



e. g. Thermal Transducer

6.6 Applications of Inverting Amplifier

Division



$$u_g = k\, u_a u_2$$

$$i_g = \frac{u_g}{R_g} = \frac{k u_a u_2}{R_g} = -i_1 = -\frac{u_1}{R_1}$$

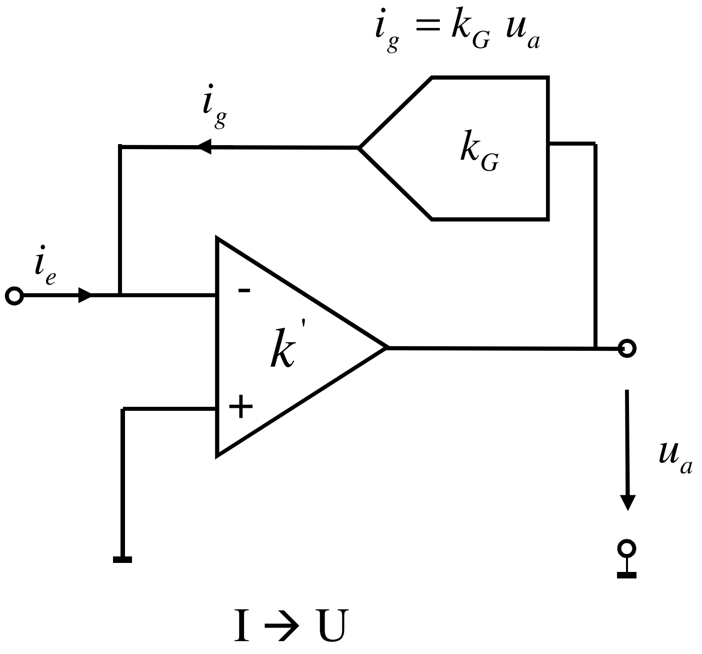
$$u_g = -\frac{R_g}{R_1} u_1$$

$$\boxed{u_a = \frac{u_g}{k \cdot u_2} = -\frac{R_g}{k\, R_1} \cdot \frac{u_1}{u_2}}$$

6.6 Applications of Inverting Amplifier

Realisation of arithmetic operations by closed loop

(Generalization)



$$i_g = -i_e$$

$$i_g = k_G\, u_a$$

$$u_a = -k' i_e$$

$$u_a = -k' i_e = -\frac{1}{k_G} i_e$$

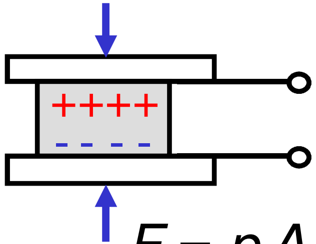
$$k' = \frac{1}{k_G}$$

The amplification in the forward direction should be always the inverse operation to the element in the feedback

Forward Amplification		Feedback Amplification
divide	→	Multiply
square root	→	square
integrate	→	differenciate
logarithmize	→	exponentiate

6.6 Applications of Inverting Amplifier

Charge Amplifier for Piezoelectric Sensors



$F = p\,A$

[N]

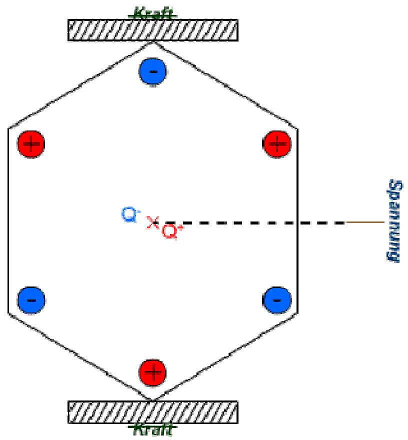
$Q = k\,F$

[As]

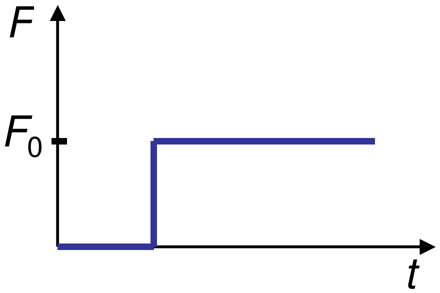
Q Charge

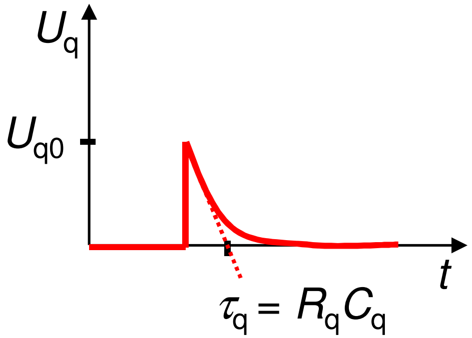
F Force

k Transfer Faktor

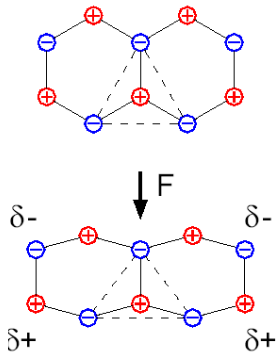


Electric polarized Crystals, e.g. SiO_2 (Quarz





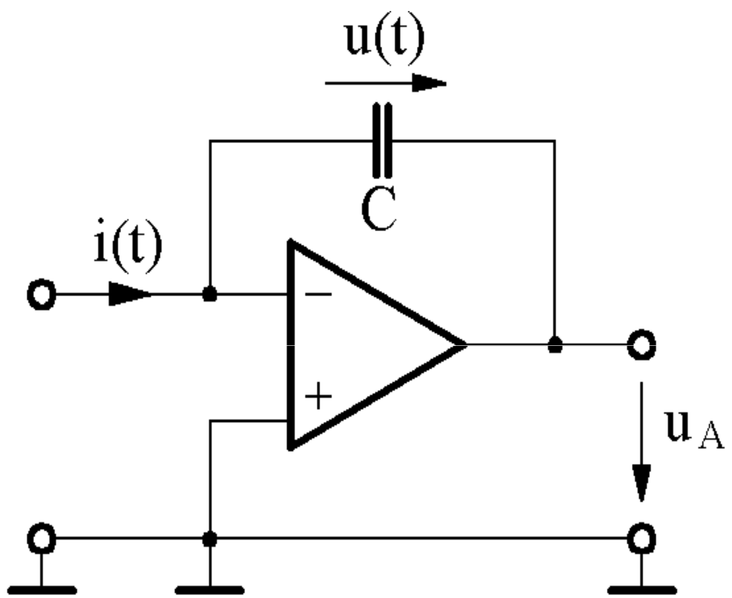
~1 s



Output voltage should be integrated!

6.6 Applications of Inverting Amplifier

Charge Amplifier



Charge $Q(t) = \int i(t)dt$

$$i(t) = \frac{dQ(t)}{dt} = -C \cdot \frac{du_A(t)}{dt}$$

$$\Rightarrow \boxed{u_A(t) = -\frac{1}{C} \cdot Q(t)}$$

<u>Problem</u>: Input bias currents will be also integrated!

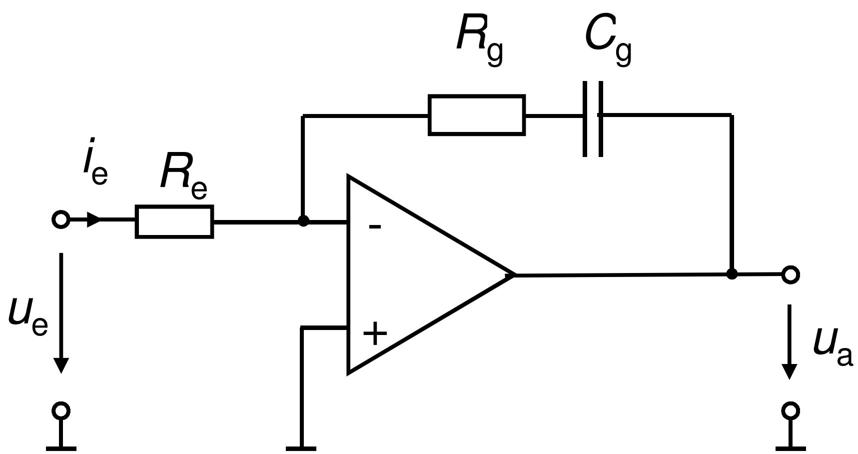
6.6 Applications of Inverting Amplifier

Regulator Circuits (1)

P-Regulator Simple Amplifier

I-Regulator Integration

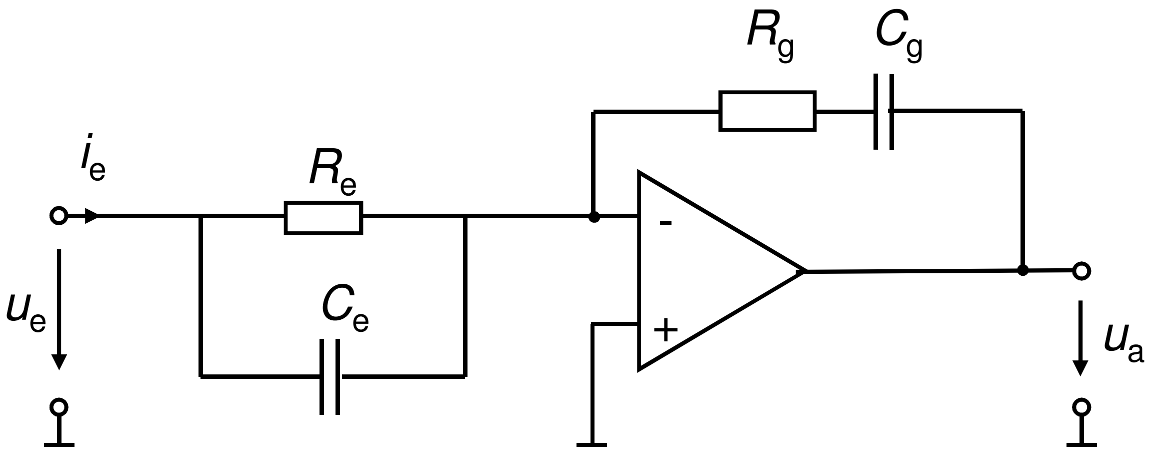
PI-Regulator



$$u_a = -\frac{R_g}{R_e} u_e - \frac{1}{R_e C_g} \int u_e \, dt$$

Regulator Circuits (2)

PID- Regulator



$$u_a = -\left(\frac{R_g}{R_e} + \frac{C_e}{C_g}\right) u_e - R_g C_e \frac{du_e}{dt} - \frac{1}{R_e C_g} \int u_e \, dt$$

6.6 Applications of Inverting Amplifier

Time Domain

$g(t)$: pulse response $\qquad X_a(t) = X_e(t) * g(t)$

$h(t)$: Step response $\qquad g(t) = \frac{\partial}{\partial t} h(t)$

Frequency Domain

Transfer Function

$$G(p) = \ell(g(t)) = \frac{X_a(p)}{X_e(p)} = \frac{\ell(X_a(t))}{\ell(X_e(t))} = \frac{\int_0^\infty x_a(t) \cdot e^{-pt} dt}{\int_0^\infty x_e(t) \cdot e^{-pt} dt}$$

$$X_a(p) = G(p) \cdot X_e(p)$$

$$h(t) = \ell^{-1}\left(\frac{G(p)}{p}\right) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} \frac{G(p)}{p} \cdot e^{pt} dt$$

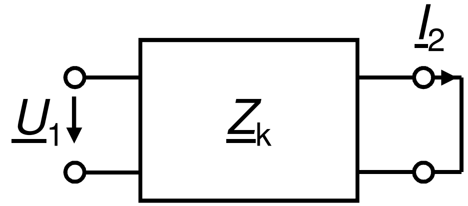
Frequency Response (Fourier-Transformed)

$$F(j\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt = Fourier(g(t))$$

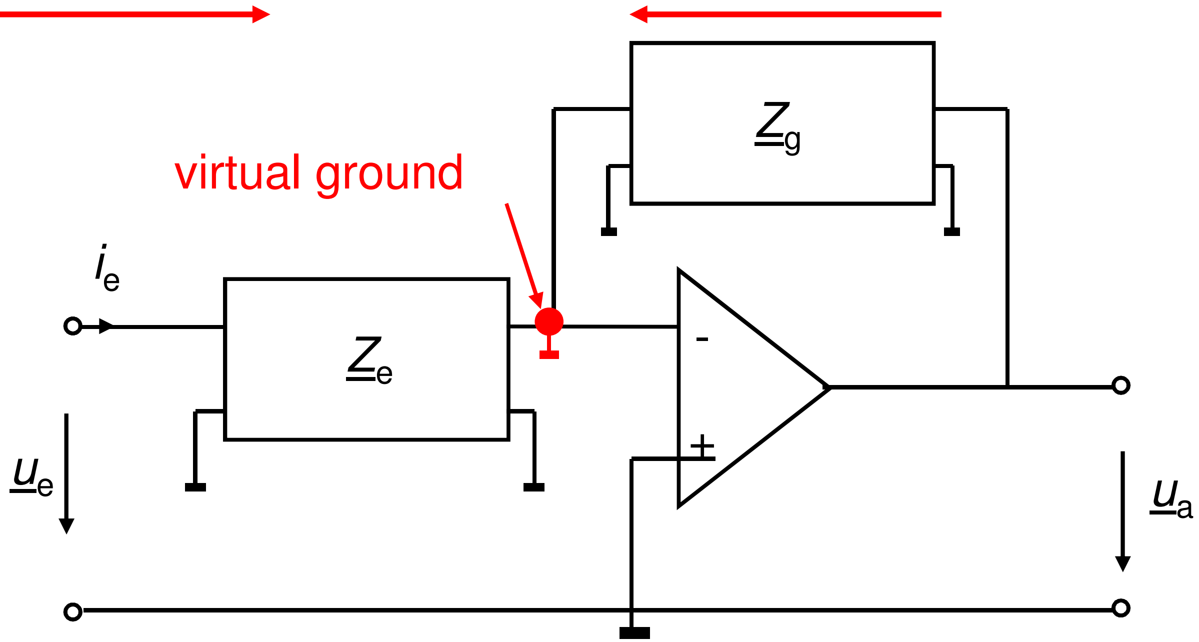
6.6 Applications of Inverting Amplifier

Realisation of Defined Transfer Functions (1)

Short Circuit Kernel Impedance



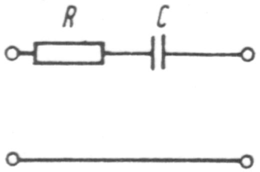
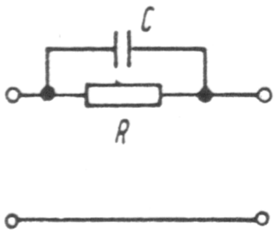
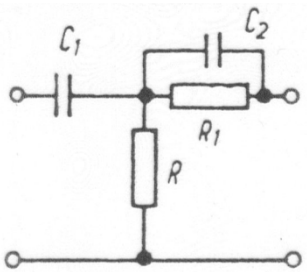
Short Circuit Kernel Impedance Short Circuit Kernel Impedance



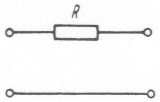
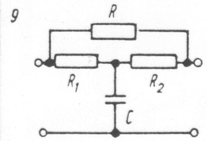
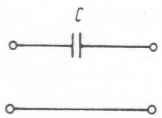
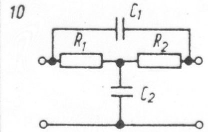
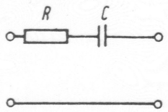
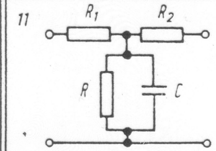
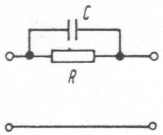
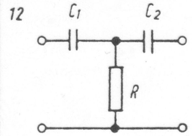
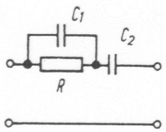
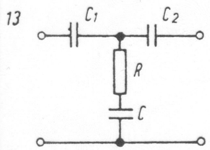
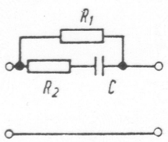
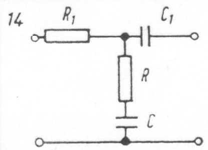
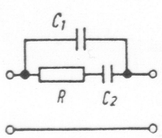
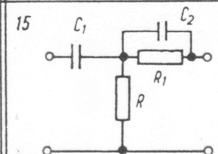
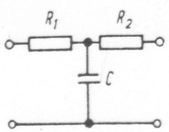
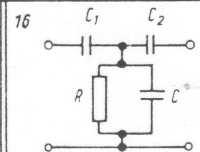
$$\boxed{\underline{u}_a = -\frac{\underline{Z}_g}{\underline{Z}_e}\underline{u}_e}$$

6.6 Applications of Inverting Amplifier

Realisation of Defined Transfer Functions (2)

Circuit	Short Circuit Kernel Impedance
3 	$R\dfrac{s+\dfrac{1}{RC}}{s}$
4 	$\dfrac{1}{C}\cdot\dfrac{1}{s+\dfrac{1}{RC}}$
15 	$\dfrac{C_1+C_2}{C_1C_2}\cdot\dfrac{\left(s+\dfrac{R+R_1}{RR_1(C_1+C_2)}\right)}{s\left(s+\dfrac{1}{R_1C_2}\right)}$

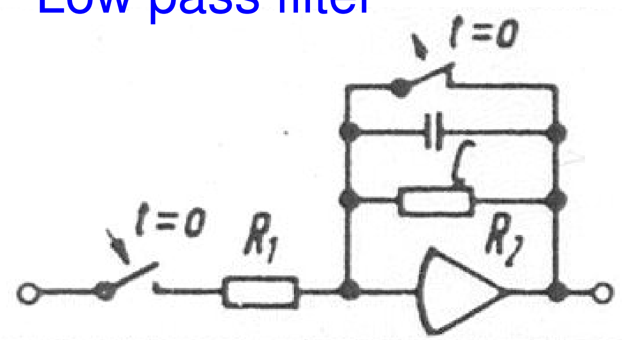
s=p

Schaltung	Kurzschlußkernimpedanz	Schaltung	Kurzschlußkernimpedanz
1 	R	9 	$R\,\dfrac{\left(s+\dfrac{R_1+R_2}{R_1R_2C}\right)}{\left(s+\dfrac{R_1+R_2+R}{R_1R_2C}\right)}$
2 	$\dfrac{1}{C}\cdot\dfrac{1}{s}$	10 	$\dfrac{\left(s+\dfrac{R_1+R_2}{R_1R_2C_2}\right)}{C_1\left(s^2+s\,\dfrac{R_1+R_2}{R_1R_2C_2}+\dfrac{1}{R_1C_1R_2C_2}\right)}$
3 	$R\,\dfrac{s+\dfrac{1}{RC}}{s}$	11 	$R_1+R_2+R_1R_2C\left(s+\dfrac{1}{RC}\right)$
4 	$\dfrac{1}{C}\cdot\dfrac{1}{s+\dfrac{1}{RC}}$	12 	$\dfrac{C_1+C_2}{C_1C_2}\cdot\dfrac{\left(s+\dfrac{1}{R\,(C_1+C_2)}\right)}{s^2}$
5 	$\dfrac{(C_1+C_2)\left(s+\dfrac{1}{R\,(C_1+C_2)}\right)}{C_1C_2\,s\left(s+\dfrac{1}{RC_1}\right)}$	13 	$\dfrac{C_1+C_2}{C_1C_2}\cdot\dfrac{\left(s+\dfrac{C+C_1+C_2}{RC\,(C_1+C_2)}\right)}{s\left(s+\dfrac{1}{RC}\right)}$
6 	$\dfrac{R_1R_2}{R_1+R_2}\cdot\dfrac{\left(s+\dfrac{1}{R_2C}\right)}{s+\dfrac{1}{(R_1+R_2)\,C}}$	14 	$R_1\left[\left(s+\dfrac{1}{R_1C_1}\right)+\dfrac{s}{RC\left(s+\dfrac{1}{RC}\right)}\right]$
7 	$\dfrac{\left(s+\dfrac{1}{RC_2}\right)}{C_1\,s\left(s+\dfrac{1}{RC_2}\right)+\dfrac{1}{R}s}$	15 	$\dfrac{C_1+C_2}{C_1C_2}\cdot\dfrac{\left(s+\dfrac{R+R_1}{RR_1\,(C_1+C_2)}\right)}{s\left(s+\dfrac{1}{R_1C_2}\right)}$
8 	$R_2+R_1R_2C\left(s+\dfrac{1}{R_2C}\right)$	16 	$\dfrac{C+C_1+C_2}{C_1C_2}\cdot\dfrac{\left[s+\dfrac{1}{R\,(C_1+C_2+C)}\right]}{s^2}$

6.6 Applications of Inverting Amplifier

Realisation of Defined Transfer Functions (3)

Low pass filter



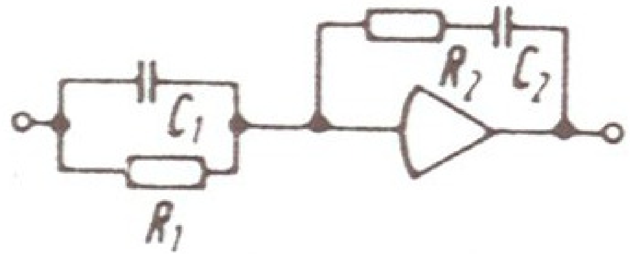
Transfer Function

$$-\frac{1}{R_1C}\frac{1}{\left(p+\frac{1}{R_2C}\right)}$$

Transfer Function

$$U\frac{R_2}{R_1}\left(1-e^{-\frac{1}{R_2C}t}\right)$$

PID-Regulator



Transfer Function

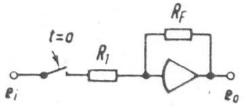
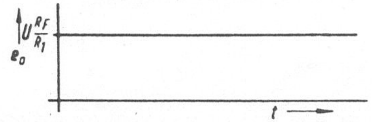
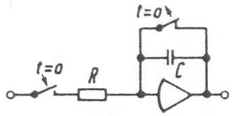
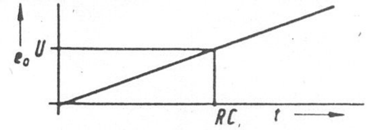
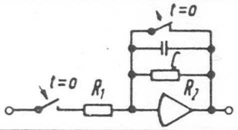
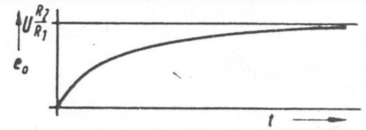
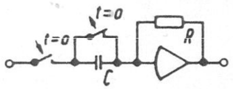
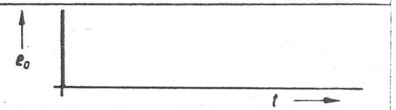
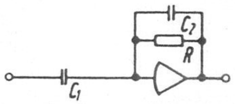
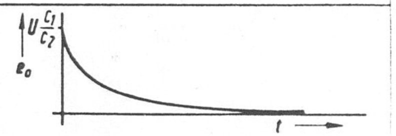
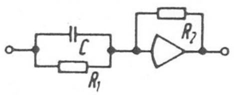
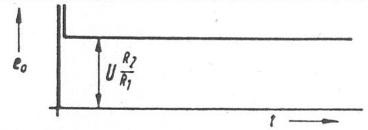
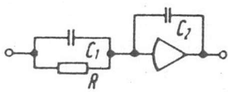
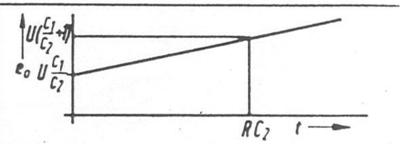
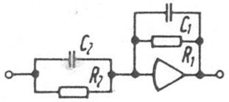
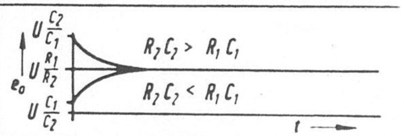
$$-R_2C_2\frac{\left(p+\frac{1}{R_2C_2}\right)\left(p+\frac{1}{R_1C_1}\right)}{p}$$

Transfer Function

$$U R_2\, C_1\left[\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}\right)+\delta(t)+\frac{1}{R_1C_1R_2C_2}t\right]$$

6.6 Applications of Inverting Amplifier

Realisation of Defined Transfer Functions (3)

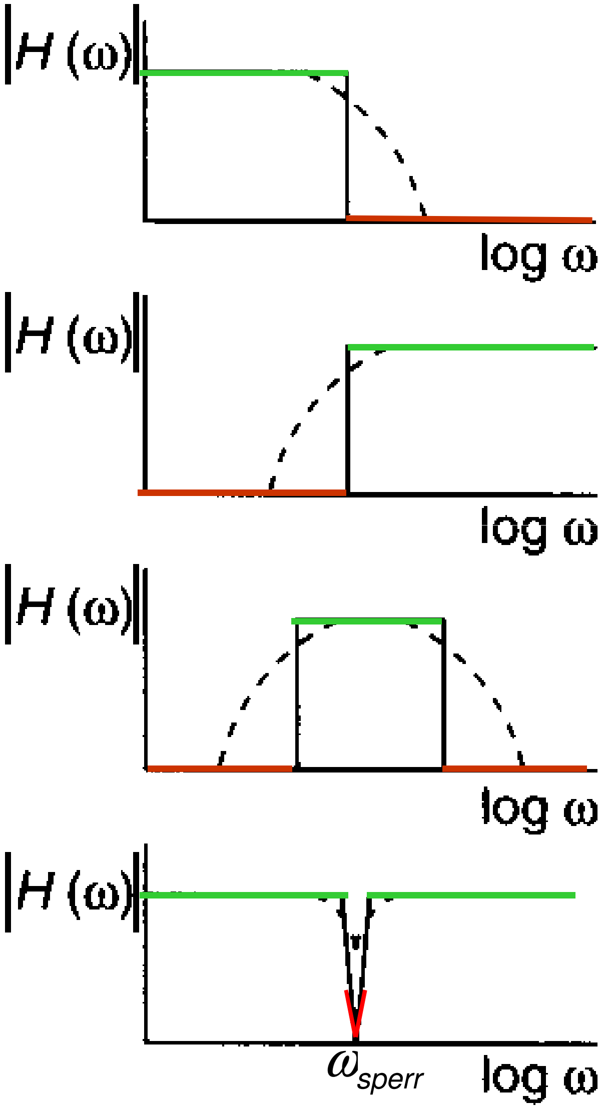
	Schaltung	Übertragungsfunktion G(s)*)	Übergangsfunktion h(t) · (– U)**)	Ausgangsspannung $e_o = f(t)$ $e_i = -U$ für $t > o$ $e_i = o$ " $t < o$
1		$-\frac{R_F}{R_1}$	$U \cdot \frac{R_F}{R_1}$	
2		$-\frac{1}{RC} \cdot \frac{1}{s}$	$U \frac{1}{RC} \cdot t$	
3		$-\frac{1}{R_1 C} \cdot \frac{1}{\left(s + \frac{1}{R_2 C}\right)}$	$U \cdot \frac{R_2}{R_1} \left(1 - e^{-\frac{1}{R_2 C} t}\right)$	
4		$- RC \cdot s$	$U \cdot RC \cdot \delta(t)$	
5		$-\frac{C_1}{C_2} \frac{s}{\left(s + \frac{1}{RC_2}\right)}$	$U \cdot \frac{C_1}{C_2} e^{-\frac{1}{RC_2} t}$	
6		$- R_2 C \cdot \left(s + \frac{1}{R_1 C}\right)$	$U \cdot R_2 C \cdot \delta(t) + U \frac{R_2}{R_1}$	
7		$-\frac{C_1}{C_2} \frac{s + \frac{1}{RC_1}}{s}$	$U \frac{C_1}{C_2} + \frac{1}{RC_2} U \cdot t$	
8		$-\frac{C_2}{C_1} \frac{\left(s + \frac{1}{R_2 C_2}\right)}{\left(s + \frac{1}{R_1 C_1}\right)}$	$U \frac{R_1}{R_2} \left[1 + e^{-\frac{1}{R_1 C} t} \left(\frac{R_2 C_2}{R_1 C_1} - 1\right)\right]$	

6.7 Active Filters

Filter: Circuit with a frequency dependent frequency response

Pass Band

Attenuation Band



Low pass

High pass

Band pass

Notch

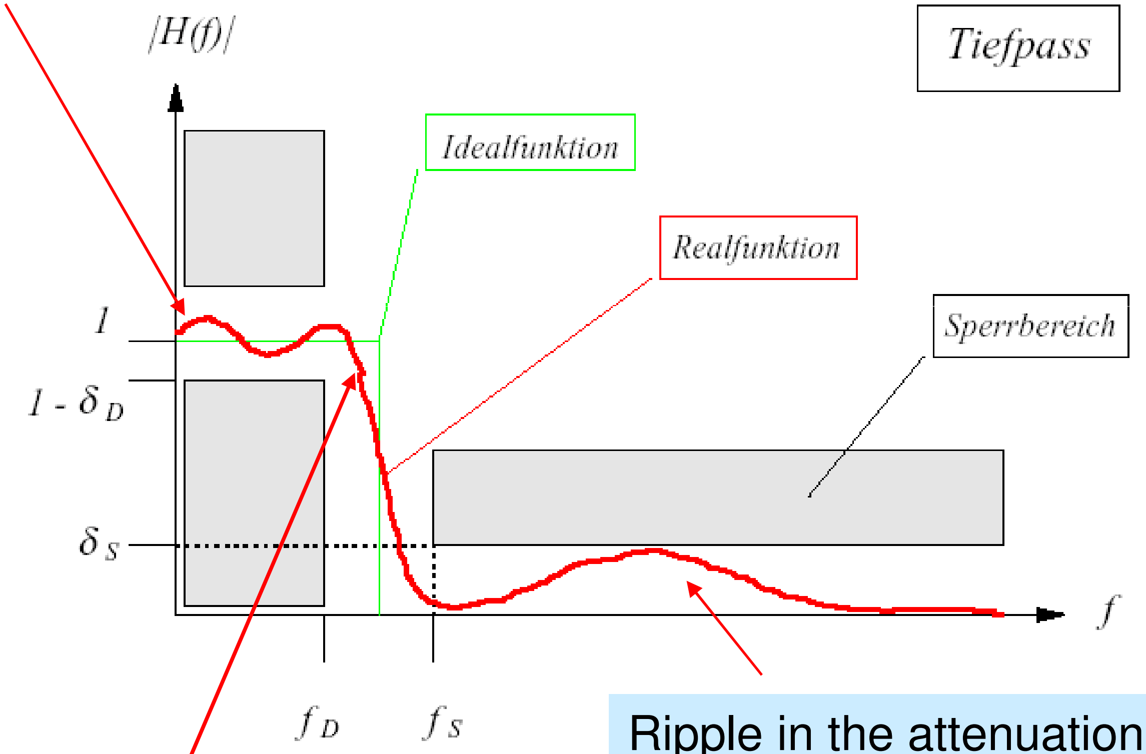
Passive Filter: R, L, C - Filter

Active Filter: Op-Amps, No Inductivities

6.7 Active Filters

Properties of real filters

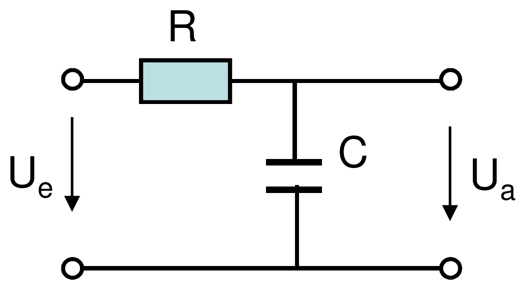
Ripple in the pass band



Ripple in the attenuation band

Cut-off frequency: decay of the modulus by $\frac{1}{\sqrt{2}} \quad (-3dB)$

Low pass- Filter 1st order



$$G(p) = \frac{1}{1 + RCp}$$

One negative Pol

Transfer Function of a Filter of order n

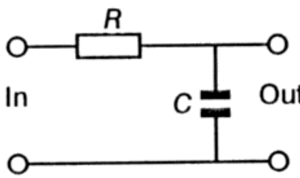
$$G(p) = \frac{A_0}{1 + \sum_{i=1}^{n} c_i p^i}$$

c_i real Filter-Coefficients
n Order of the filter

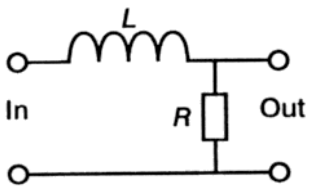
→ n negative pols

6.7 Active Filters

Low pass filter

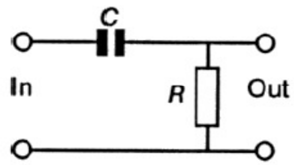


(a)

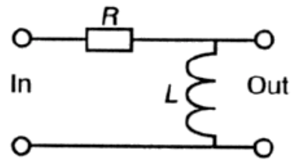


(b)

High pass filter

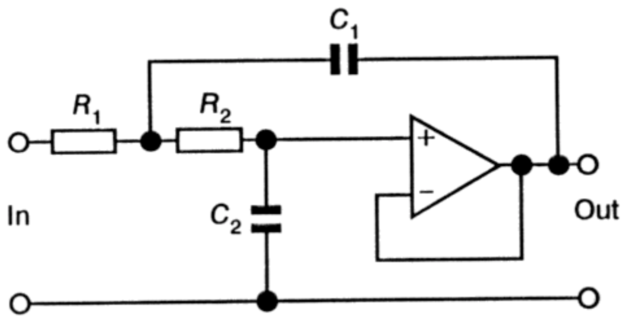


(a)



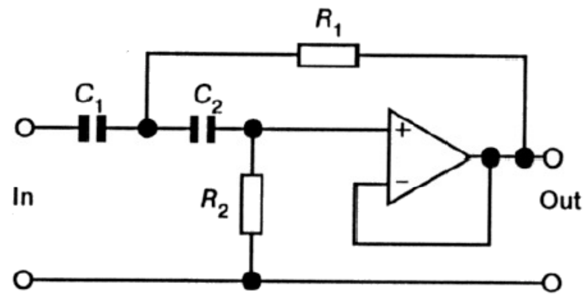
(b)

Active filter



(c)

$R_1=R_2$



(c)

$C_1=C_2$

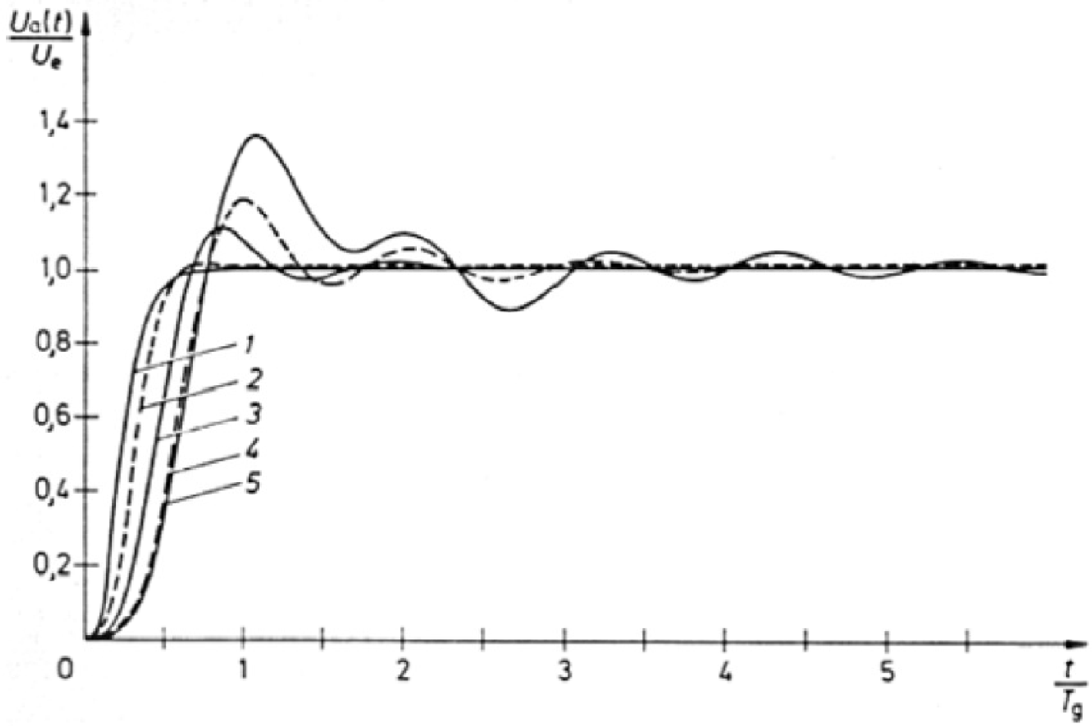
6.7 Active Filters
Properties of different filter types

Gauß (1): flat amplitude characteristic

Bessel (2): Optimal Transfer of square pulses for $f < f_g$
group delay time indipendent on ω, low ripple.

Butterworth (3): Amplitude characteristic optimized for $f < f_g$, constant.

Tschebyscheff (4): Filter with riple ε = 0,5 dB



Group delay time

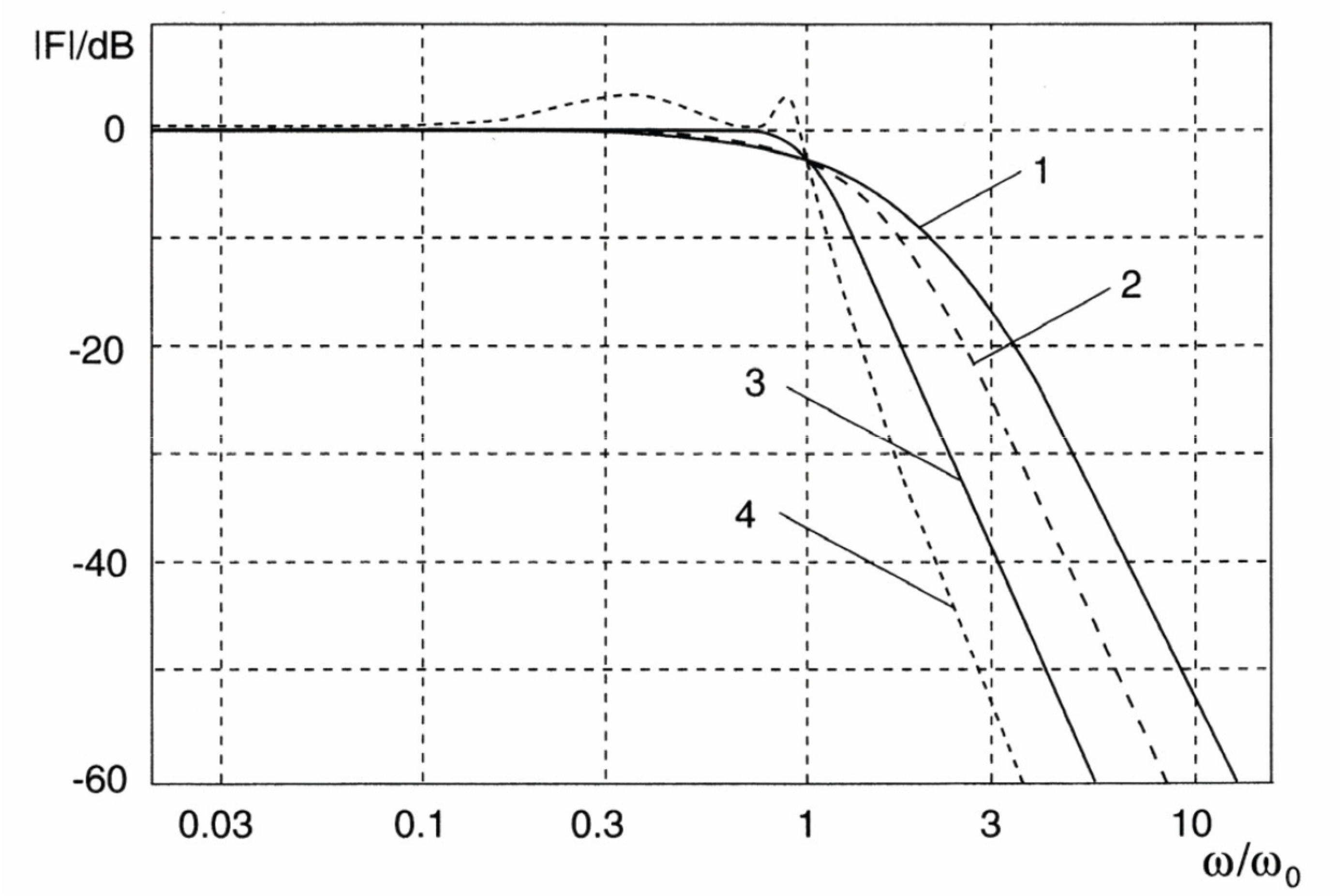
$$t_{gr} = -\frac{\partial \omega}{\partial t}$$

Normed group delay time

$$T_{gr} = -\frac{\omega_g}{2\pi} t_{gr}$$

6.7 Active Filters

Amplitude characteristic of active Filter of 4^{th} Order



Amplitudengänge von Tiefpässen 4. Ordnung
1 Gauß (krit. Dämpfg.), 2 Bessel, 3 Butterworth, 4 Tschebyscheff

6.7 Active Filters

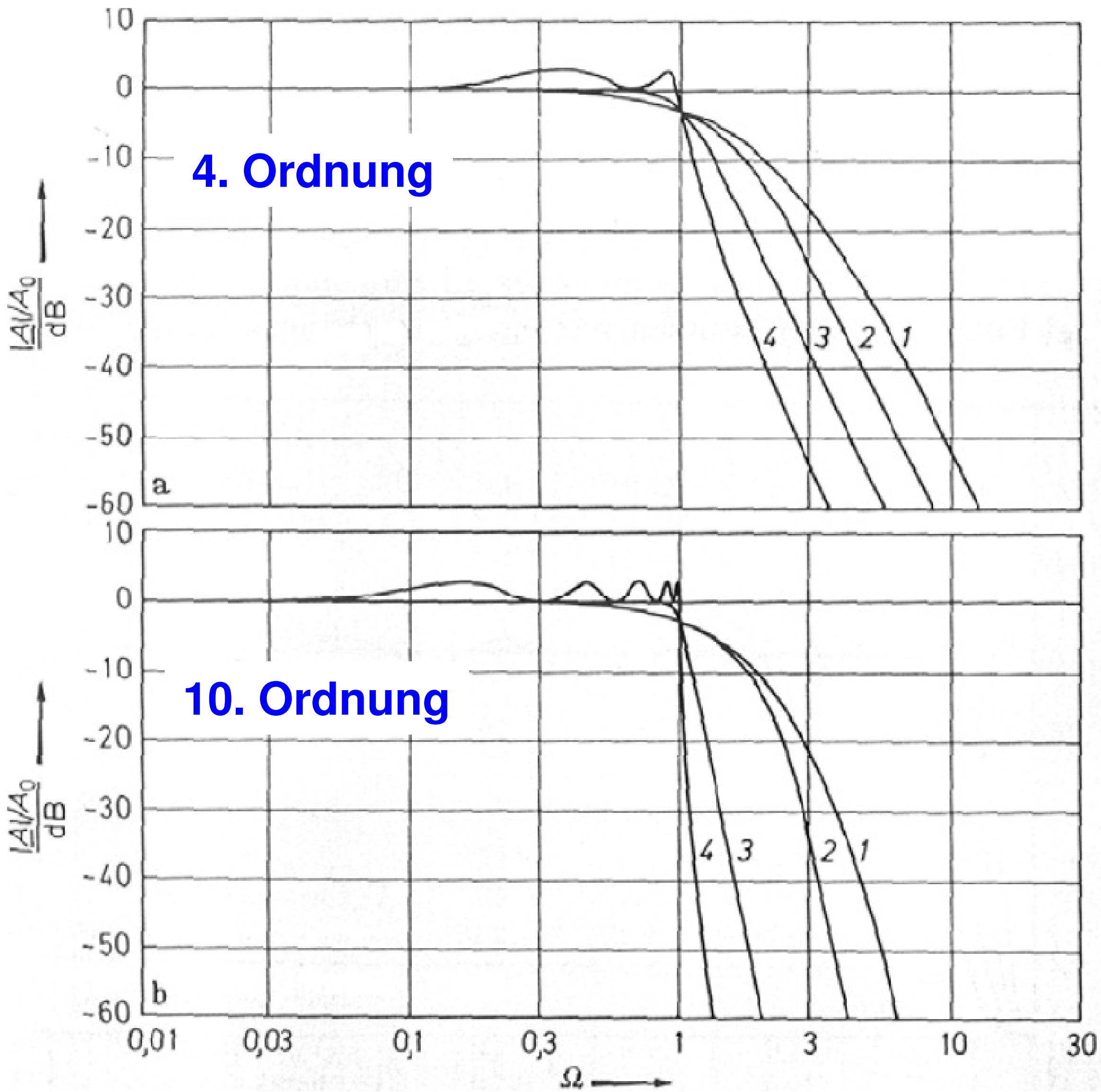
1 RC with critical damping

2 Bessel

3 Butterworth

4 Tschebyscheff with 3 dB ripples

Butterworth
→ maximal constant
frequency response



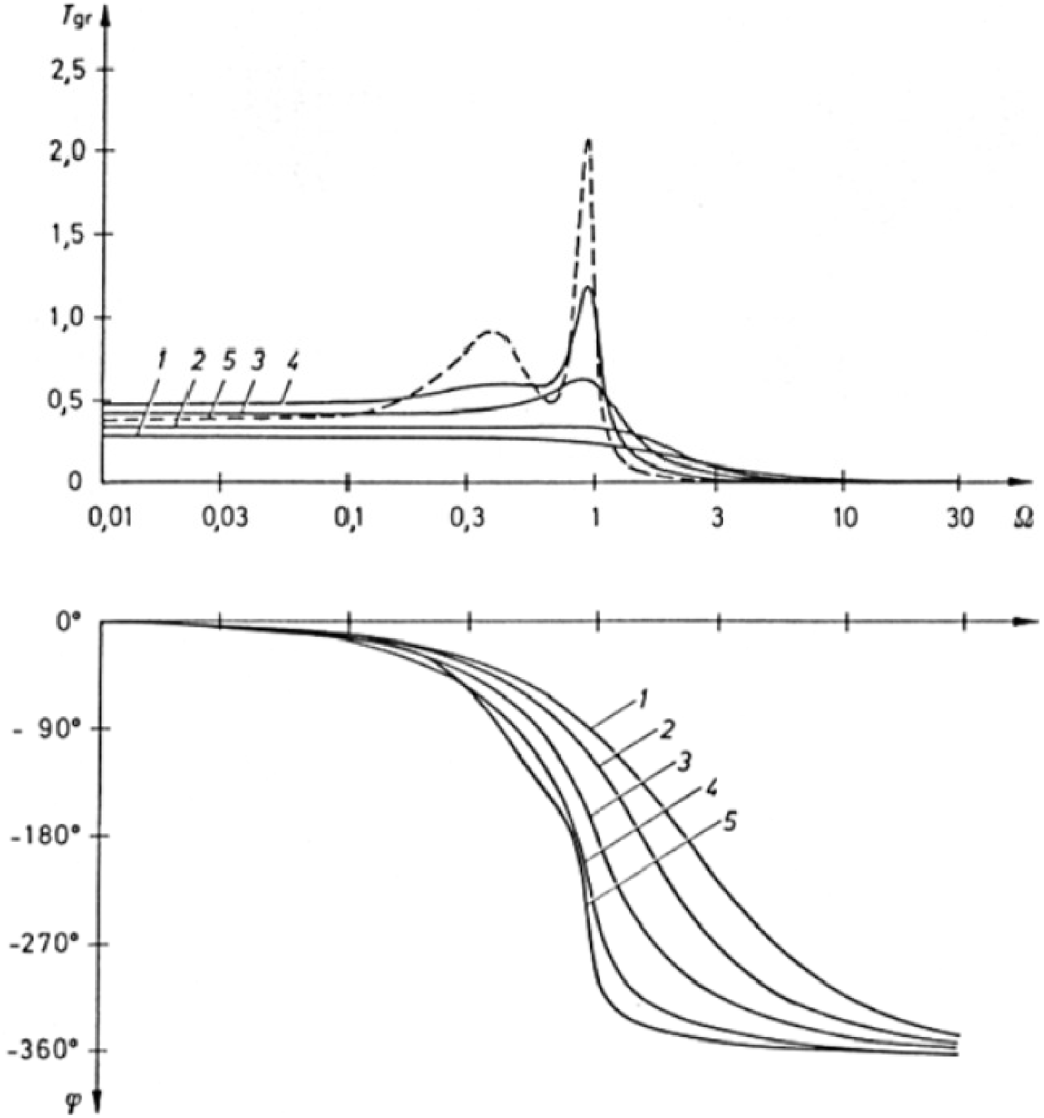
6.7 Active Filters

1 RC with critical damping

2 Bessel

3 Butterworth

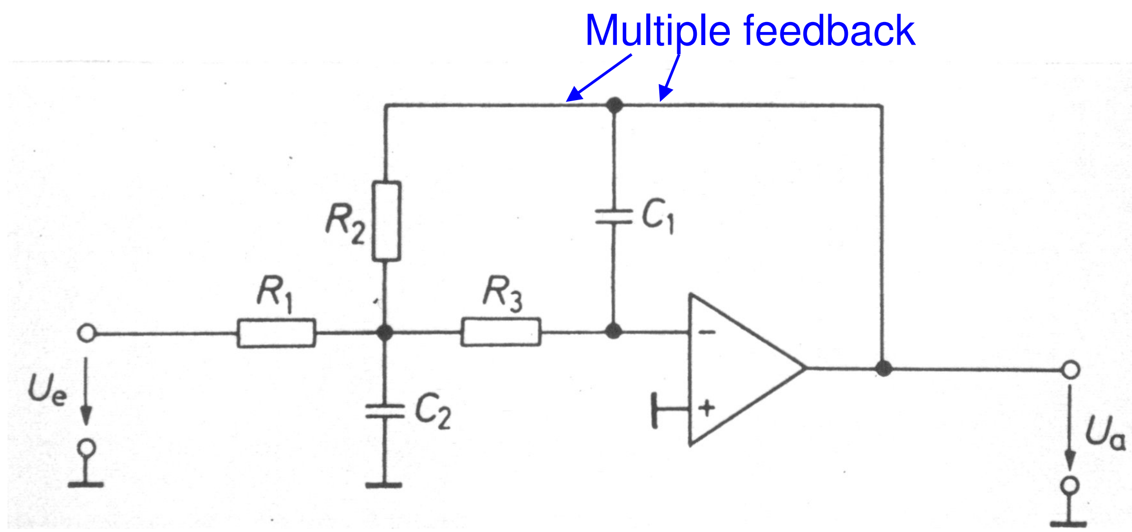
4 Tschebyscheff with 3 dB ripples



Bessel
→ Group delay time
independent on frequency
in the pass Band

6.7 Active Filters

Active Filter 2nd Order



$$G(S) = \frac{A_0}{1 + a_1 S + a_2 S^2}$$

$$S = \frac{p}{\omega_g} = \frac{j\omega}{\omega_g}$$

$$G(S) = -\frac{R_2/R_1}{1 + \omega_g C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)S + \omega_g^2 C_1 C_2 R_2 R_3 S^2}$$

Comparison of Coefficients→ $A_0 = -\frac{R_2}{R_1}$

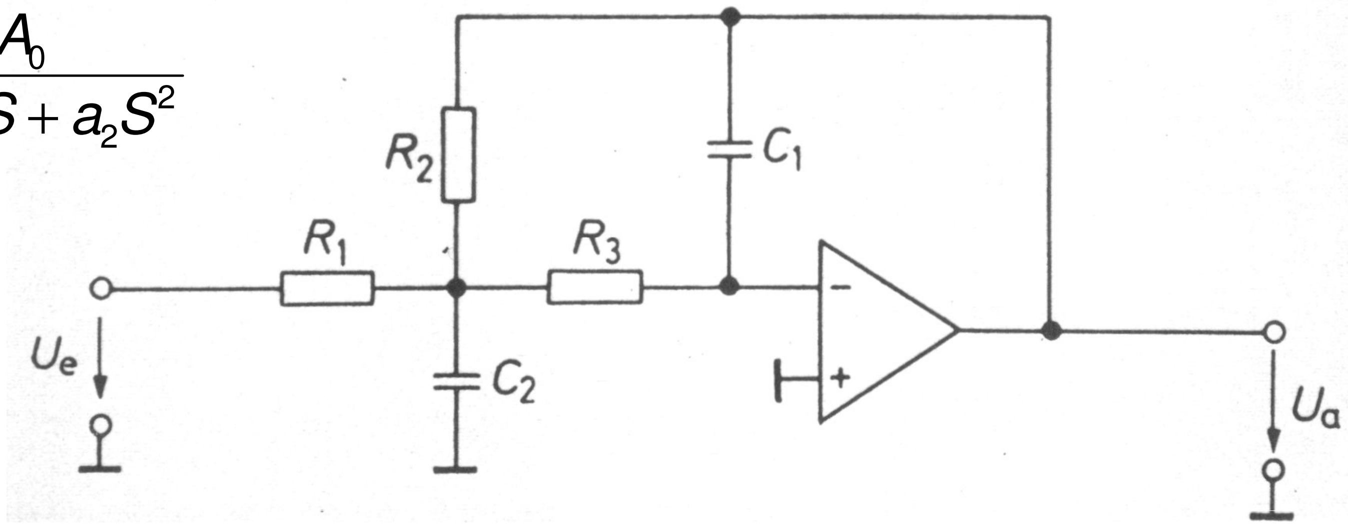
$$a_1 = \omega_g C_1\left(R_2 + R_3 + \frac{R_2 R_3}{R_1}\right)$$

$$a_2 = \omega_g^2 C_1 C_2 R_2 R_3$$

6.7 Active Filters

Active Filter 2nd Order

$$G(S) = \frac{A_0}{1 + a_1 S + a_2 S^2}$$



$$A_0 = -\frac{R_2}{R_1}$$

$$a_1 = \omega_g C_1 \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right)$$

$$a_2 = \omega_g^2 C_1 C_2 R_2 R_3$$

Capacitance values are given

$$\frac{C_2}{C_1} \geq \frac{4a_2(1 - A_0)}{a_1^2}$$

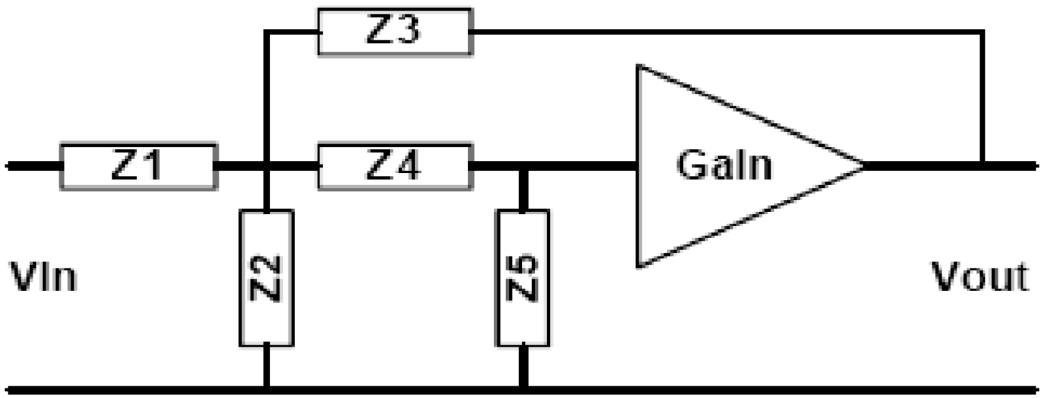
$$R_1 = -\frac{R_2}{A_0}$$

$$R_2 = \frac{a_1 C_2 - \sqrt{a_1^2 C_2^2 - 4C_1 C_2 a_2 (1 - A_0)}}{4\pi f_g C_1 C_2}$$

$$R_3 = \frac{a_2}{4\pi^2 f_g^2 C_1 C_2 R_2}$$

6.7 Active Filters

General



$$\frac{Vout}{Vin} = \frac{Gain}{1 + \left(\frac{Z_4 + Z_1}{Z_5}\right) + \frac{Z_1}{Z_2}\left(1 + \frac{Z_4}{Z_5}\right) + \frac{Z_1 Z_4}{Z_3 Z_5} + \frac{Z_1}{Z_3}(1\text{-}Gain)}$$

Low pass	High pass	Band pass
$Z_1 = R_1$	$Z_1 = 1/s\ C_1$	$Z_1 = R_1$
$Z_2 =$ open	$Z_2 =$ open	$Z_2 =$ open
$Z_3 = 1/s\ C_1$	$Z_3 = R_1$	$Z_3 = 1/s\ C_1$
$Z_4 = R_2$	$Z_4 = 1/s\ C_2$	$Z_4 = 1/s\ C_2$
$Z_5 = 1/s\ C_2$	$Z_5 = R_2$	$Z_5 = R_3 // C_3$

6.7 Active Filters

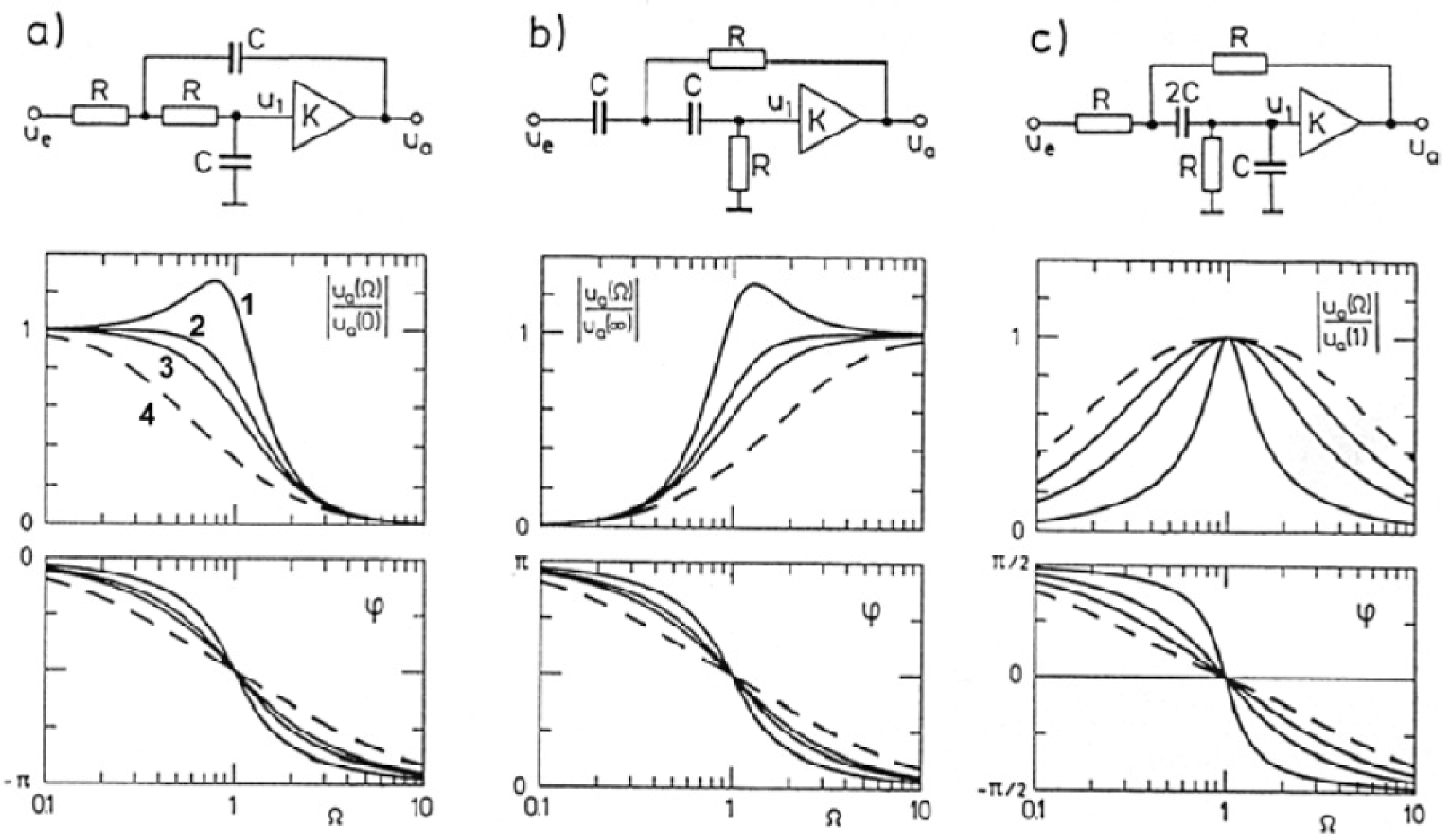
General

1. k=2.144 Tschebyscheff-Filter (2dB)

2. k = 1,586 Butterworth-Filter

3. k=1.286 Bessel-Filter

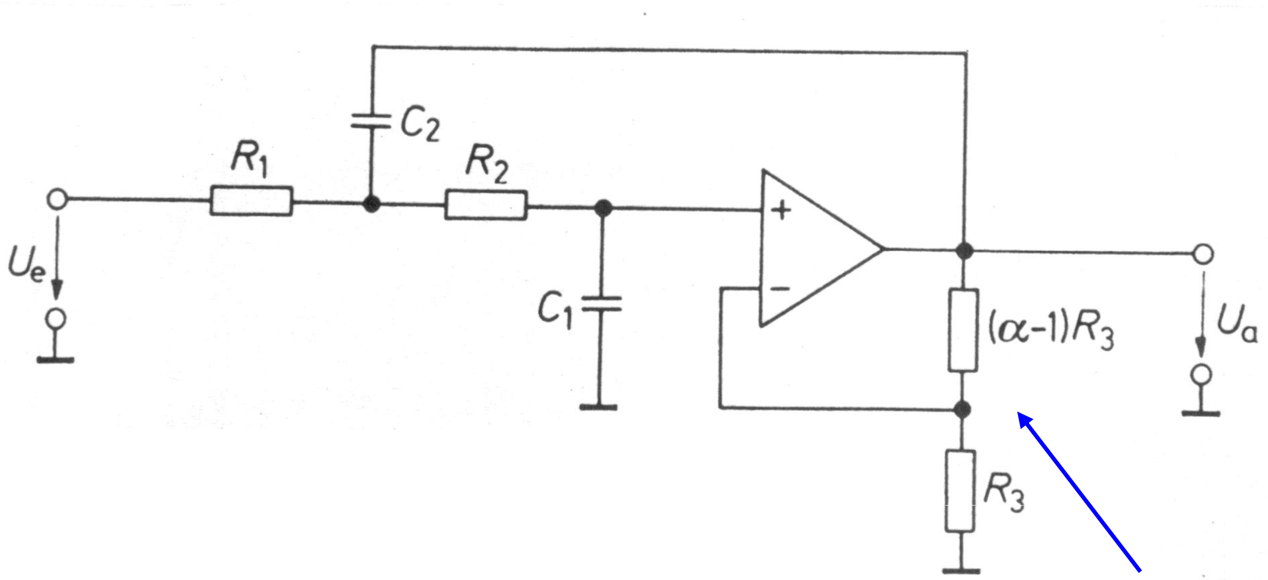
4. k=1 kritische Dämpfung - - -



Low pass High pass Band pass

6.7 Active Filters

Positive Feed Back Low Pass Amplifier 2nd Order



Amplification is hold on a specific value

Transfer function

$$G(S) = \frac{A_0}{1 + a_1 S + a_2 S^2}$$

$$A_0 = \alpha$$

$$A_1 = \omega_g \left[C_1 (R_1 + R_2) + (1-\alpha) R_1 C_2 \right]$$

$$a_2 = \omega_g^2 C_1 C_2 R_2 R_3$$

6.7 Active Filters

Positive Feed Back Low Pass Amplifier 2nd Order

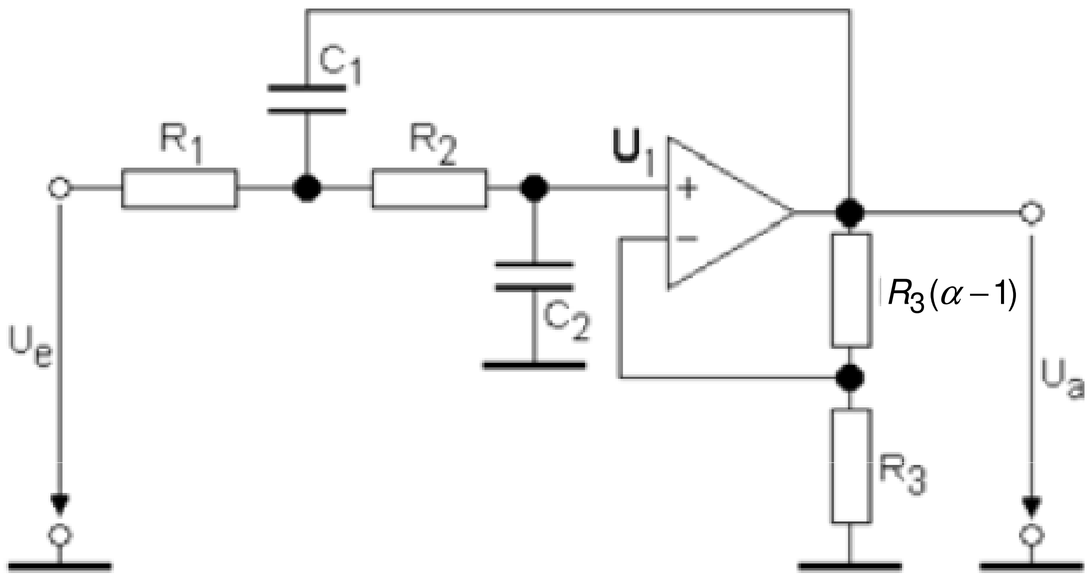
Special case:

Für $R_1=R_2=R_3=R$ und $C_1=C_2=C$

$$A_0 = \alpha$$

$$a_1 = \omega_g RC(1-\alpha)$$

$$a_2 = (\omega_g RC)^2$$

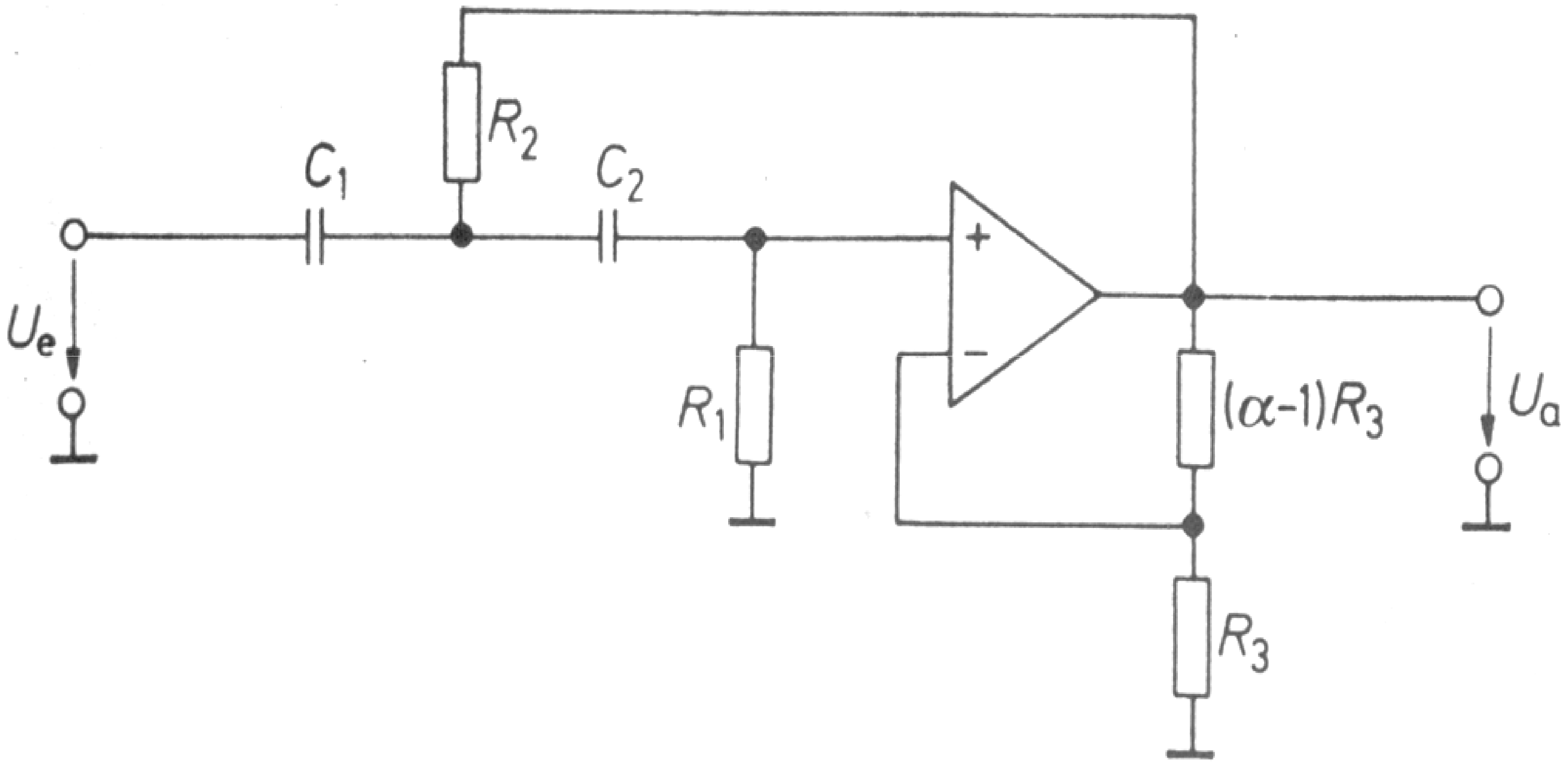


	Critic Damping	Bessel-Filter	Butterworth-Filter	Tschebyscheff-Filter with 1 dB ripple
α	1,0	1,268	1,856	1,955

▷ α defines the filter typ

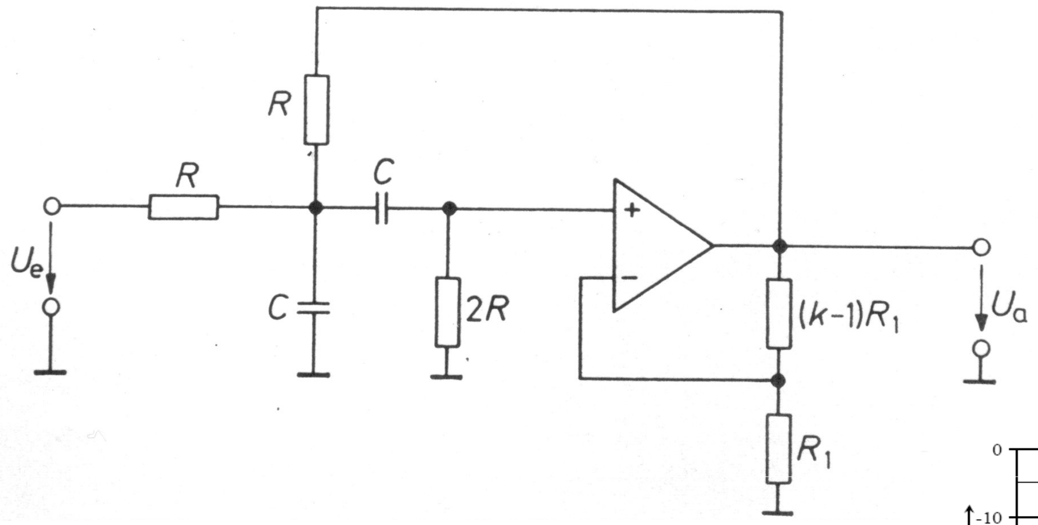
6.7 Active Filters

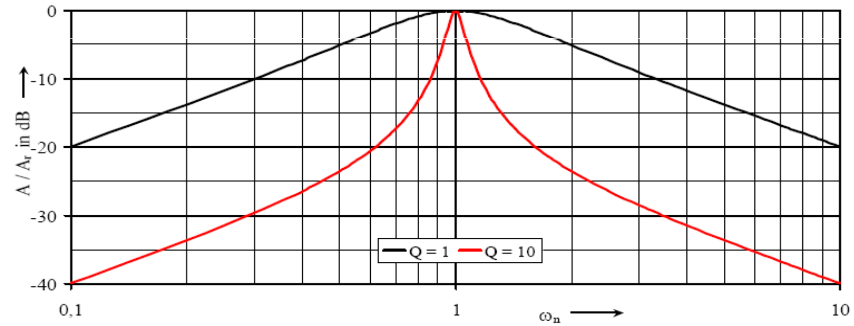
Positive Feedback High Pass Filter



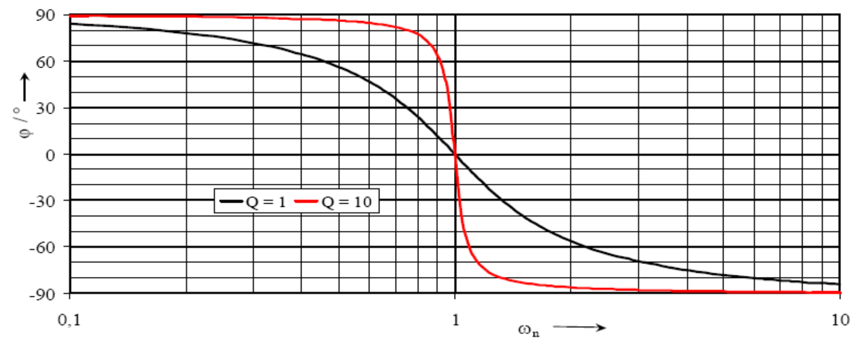
6.7 Active Filters

Band Pass Filter with a Simple Positive Feed Back





Amplitudengang für Bandpassfilter 2. Ordnung mit der Güte Q = 1 und Q = 10



Phasengang für Bandpassfilter 2. Ordnung mit der Güte Q = 1 und Q = 10

Resonance frequency (Extreme low damping)

$$f_r = \frac{1}{2\pi RC}$$

Amplification at f_r

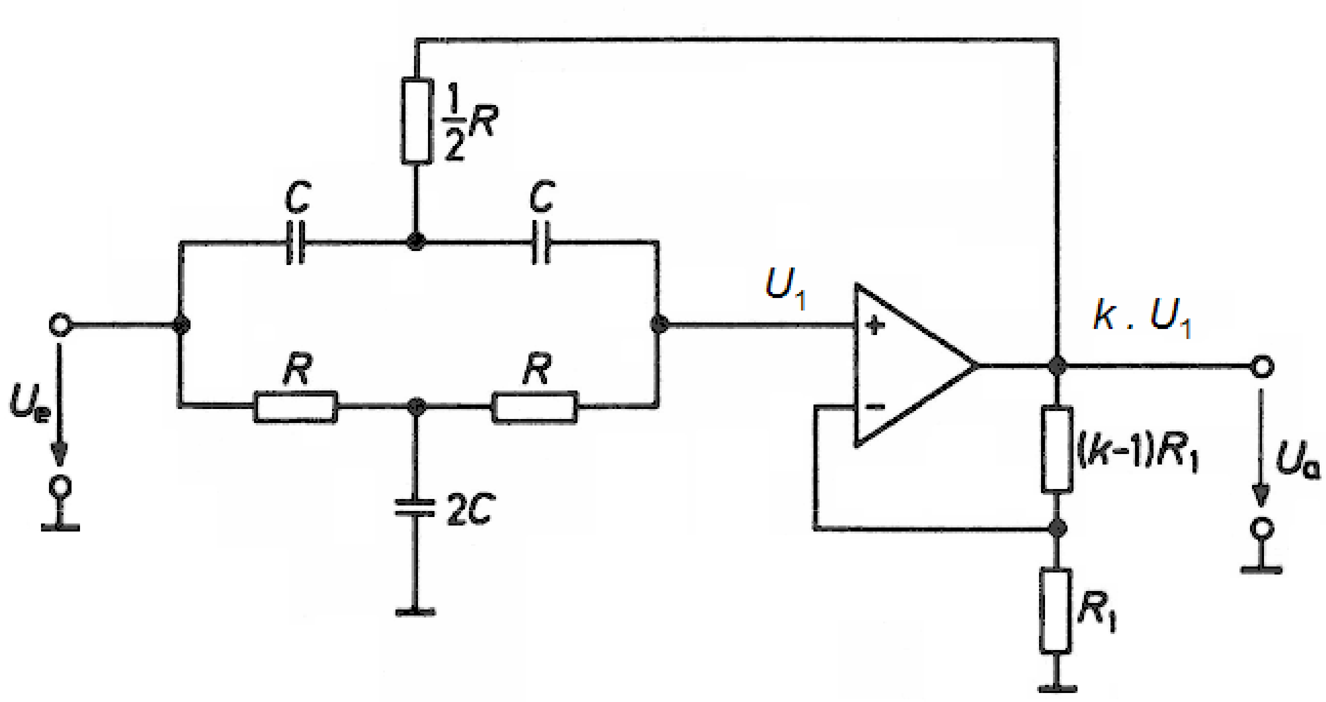
$$A_r = \frac{k}{3-k}$$

Performance of rejection

$$Q = \frac{f_r}{\Delta f} = \frac{1}{3-k}$$

6.7 Active Filters

Aktives Doppel-T-Sperrfilter, Notch-Filter



Resonance frequency
(unfinite barrier effect)

$$f_r = \frac{1}{2\pi RC}$$

Amplification

$$A_0 = k$$

Performance of rejection

$$Q = \frac{f_r}{\Delta f} = \frac{1}{2(2-k)}$$