Exponents of the localization lengths in the bipartite Anderson model with off-diagonal disorder

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Abstract

We investigate the scaling properties of the two-dimensional (2D) Anderson model of localization with purely off-diagonal disorder (random hopping). In particular, we show that for small energies the infinite-size localization lengths as computed from transfer-matrix methods together with finite-size scaling diverge with a power-law behavior. The corresponding exponents seem to depend on the strength and the type of disorder chosen.

Key words: Localization, off-diagonal disorder, critical exponents, bipartiteness

1. Introduction

Of paramount importance for the theory of disordered systems and the concept of Anderson localization [1–5] is the scaling theory of localization as proposed in 1979 [6]. Especially in 2D, this theory predicts the absence of a disorder-driven MIT for generic situations such that all states remain localized and the system is an insulator [7–9]. However, already early [10,11] it was suggested that an Anderson model of localization with purely off-diagonal disorder might violate this general statement since non-localized states were found at the band center [12–14]. Further numerical investigations in recent years [15–19] have uncovered additional evidence that the localization properties at $E = 0$ are special. In particular, it was found that the divergence in the density of states DOS is accompanied by a divergence of the localization lengths λ [15,16]. This divergence does not violate the scaling arguments [20], since it can be shown that its scaling properties are compatible with critical states only [16], i.e., there are no truly extended states at $E = 0$. Of importance for the model is a very special symmetry around $E = 0$ which holds in the bipartite case of an even number of sites [20,21]. Then the spectrum is symmetric such that for every eigenenergy $E_i < 0$ there is also a state with energy $E_i > 0$. This situation is connected with a so-called chiral universality class. Furthermore, the model is closely connected to the random flux model studied in the quantum-Hall situation

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where the off-diagonal disorder is due to a random magnetic flux through the 2D plaquettes.

Thus although we do not have a true MIT, we nevertheless have a transition from localized via delocalized to localized behavior as we sweep the energy through $E = 0$. We consider a single electron on the 2D lattice with N sites described by the Anderson Hamiltonian

$$H = \sum_{i \neq j}^{N} t_{ij} \left| i \right\rangle \left\langle j \right| + \sum_{i}^{N} \epsilon_i \left| i \right\rangle \left\langle i \right| \qquad (1)$$

where $|i\rangle$ denotes the electron at site i. The onsite energies ϵ_i are set to 0 and the off-diagonal disorder is introduced by choosing random hopping elements t_{ij} between nearest neighbor sites.

We test three different distributions of t_{ij}: (i) a rectangular distribution $t_{ij} \in [c - w/2, c + w/2]$ [15], (ii) a Gaussian distribution $P(t_{ij}) = \exp\left[-(t_{ij} - c)^2/2\sigma^2\right]/\sqrt{2\pi\sigma^2}$, and (iii) a rectangular distribution of the logarithm of t_{ij} where $P(\ln t_{ij}/t_0) = 1/w$ if $|\ln t_{ij}/t_0| \leq w/2$ or $P(\ln t_{ij}/t_0) = 0$ otherwise [14]. The logarithmic distribution appears more suited to model actual physical systems [14]. We also note that the logarithmic distribution avoids problems with zero t elements and thus there is no need to introduce an artificial lower cutoff as for the box and Gaussian distributions [15]. Furthermore, the box and Gaussian distributions will usually have negative t values which correspond to a rather artificial phase shift.

In the case of rectangular and normal distributions we set the width w and the standard deviation σ to 1 and change the center c of the distribution. In the case of the logarithmic t distribution $t_0 = 1$ sets the energy scale and we change the disorder width w. Values of the parameters were $c = 0$, 0.25, 0.5, and 1.0 for the rectangular distribution; $c = 0$ and $c = 0.25$ for the Gaussian distribution and $w = 2$, 6, and 10 for the logarithmic t distribution.

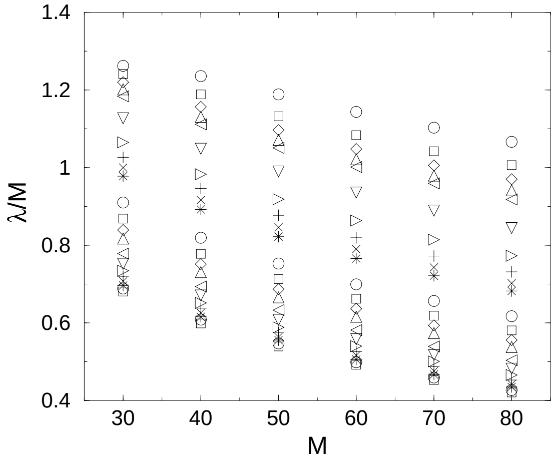

Fig. 1. Reduced localization length λ/M for various system sizes M of a box t distribution with $c = 0$. Symbols indicate different energies ranging from 0.025 (○), 0.0225 (□) to 2×10^{-5} (□).

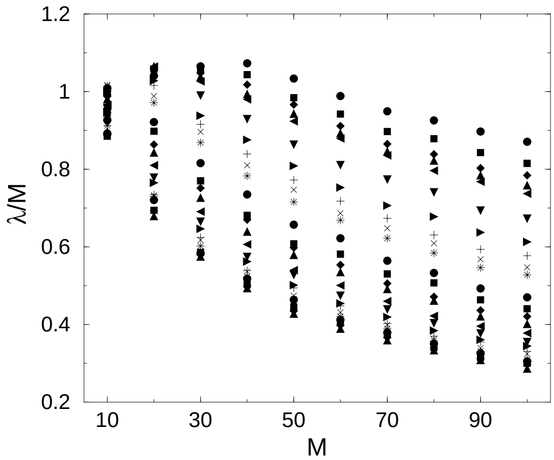

Fig. 2. Reduced localization length λ/M for various system sizes M of a Gaussian t distribution with $c = 0.25$. Symbols indicate different energies ranging from 0.03 (●), 0.0275 (■) to 2×10^{-5} (▲).

2. Computation of the localization lengths at $E \neq 0$

The transfer-matrix method [22,23] was used to compute the localization lengths for strips of various widths M up to $M = 100$ in the energy interval $2 \times 10^{-5} \leq E \leq 0.2048$. In Figs. 1, 2, and 3 we

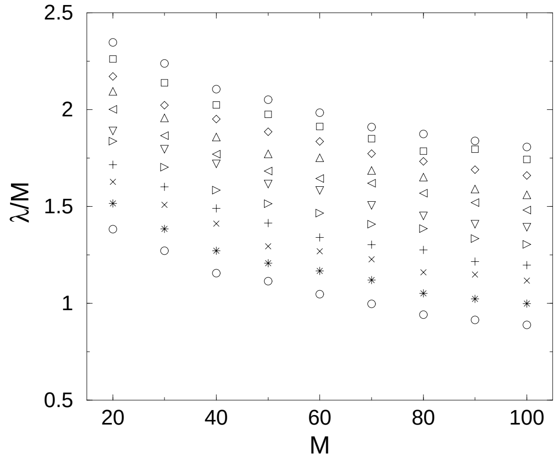

Fig. 3. Reduced localization length λ/M for various system sizes M of a logarithmic t distribution with $w = 2$. Symbols indicate different energies ranging from 0.2048 (○), 0.1024 (□) to 2×10^{-4} (□).

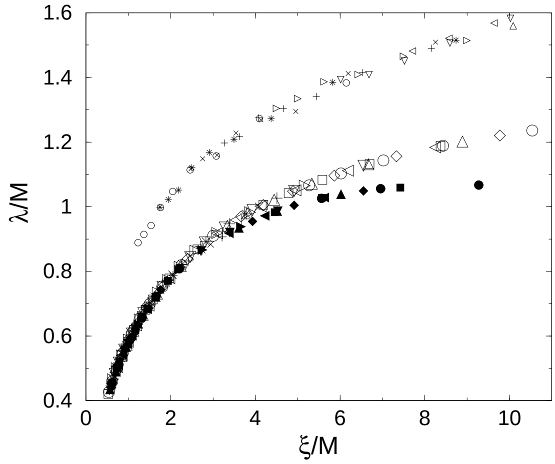

Fig. 4. Finite-size-scaling plots for box ($c = 0$, $M \in [50, 80]$, large open symbols), Gaussian ($c = 0$, $M \in [20, 60]$, filled symbols), and logarithmic ($w = 2$, $M \in [30, 100]$, small open symbols) t distributions.

show the system size dependence for, e.g., special values of c and w and all three disorder distributions. The accuracy of our results was $0.1-0.3\%$ or 1% depending on the disorder distribution and the values of parameters, see Table 1 for actual parameter values. Next, the finite-size-scaling analysis of Ref. [23] was applied to the data. The calculated localization lengths usually increase as the energy approaches 0. Only, for small even width values $(10, 20)$ it decreases significantly close to $E = 0$ [19] which makes finite-size scaling impossible. Therefore the smallest system sizes were dropped during the finite-size scaling procedure. Results for the finite-size scaling curves are shown in Fig. 4 for the three different distributions.

3. Critical exponents

One expects that the scaling parameters ξ obtained from finite-size scaling diverge close to $E = 0$ [24]. However, the precise functional form of this divergence is not yet know. In Ref. [24] it has been suggested that for energies $E > E^*$ the divergence can be described by a power law as

$$\xi(E) \propto \left|\frac{E_0}{E}\right|^{\nu} \tag{2}$$

with the critical exponent ν. For even smaller $|E| \ll E^*$, this behavior should then change to

$$\xi(E) \propto \exp\sqrt{\frac{\ln E_0/E}{A}} \tag{3}$$

with constants E_0 and A given by the renormalization group flow [24]. Double-logarithmic plots of ξ vs. E in Figs. 5, 6 and 7 confirm the power-law behavior with reasonable accuracy down to $E \approx 10^{-4}$. For smaller values it has been shown already in Ref. [19] that a new behavior is to be expected.

Table 1 collects the values of the critical exponent obtained for different disorders. In the case of the logarithmic t distribution and $w = 10$ the power-law divergence fails, therefore the exponent was not calculated. From Table 1, it can be easily seen that all calculated values are in the range $0.2 \le \nu \le 0.5$. The exponent is apparently not universal but seems to depend on the kind of disorder and the actual value of parameters; for stronger disorders ν becomes smaller (for the logarithmic t distribution the disorder strength increases with w [14], for the rectangular distribution the strongest disorder appears at $c = 0.25$

Table 1
Estimated values of the exponents of the localization lengths for various disorder strengths and distributions. The error bars represent the standard deviations from the power-law fit and should be increased by at least one order of magnitude for a reliable representation of the actual errors.

disorder distribution	parameters	accuracy in %	sizes used in finite-size scaling	estimated exponent
box	$c = 0$	0.1-0.2	30-80	0.326 ± 0.002
box	$c = 0$	0.1-0.2	25-65	0.325 ± 0.002
box	$c = 0.25$	0.1-0.2	30-70	0.319 ± 0.001
box	$c = 0.5$	0.1-0.2	30-70	0.361 ± 0.001
box	$c = 1.0$	0.1-0.3	30-70	0.444 ± 0.002
Gaussian	$c = 0$	0.1-0.2	30-60	0.314 ± 0.001
Gaussian	$c = 0.25$	1	30-100	0.310 ± 0.001
Gaussian	$c = 0.25$	1	35-95	0.308 ± 0.001
logarithmic	$w = 2$	1	20-100	0.412 ± 0.007
logarithmic	$w = 6$	1	20-100	0.251 ± 0.004
logarithmic	$w = 10$	1	20-100	—

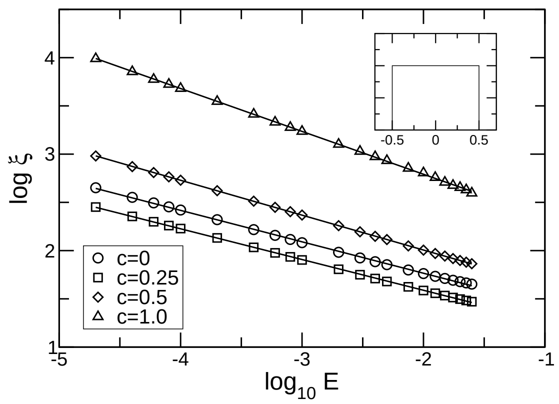

Fig. 5. Variation of the infinite-size localization length ξ with E for box distributions. The inset shows the t distribution for $c = 0$.

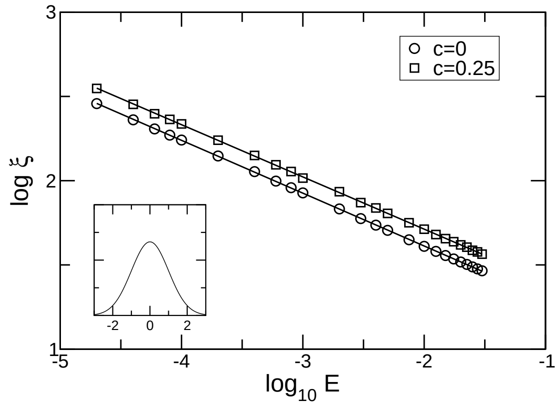

Fig. 6. Variation of the infinite-size localization length ξ with E for Gaussian distributions. The inset shows the t distribution for $c = 0$.

[15]). This non-universality is in agreement with the results of Ref. [24].

As the localization lengths calculated for odd and even strip widths may exhibit different behavior [14,19] we repeated the procedure also for some odd-width systems for rectangular and Gaussian distributions. The difference is within error bars, thus for these disorder strengths the effect is negligible.

4. Conclusions

Our results suggest that the localization-delocalization-localization present in the off-diagonal Anderson model of localization in 2D can be described by a set of exponents that model the divergence of the localization lengths ξ at $E = 0$. Note that these exponents are in reasonable agreement with the exponent 0.5 first estimated for the scaling of the participation numbers in Ref. [15]. Down to $E \approx 10^{-4}$ the power-law behavior can model the data reasonably well. Thus we ex-

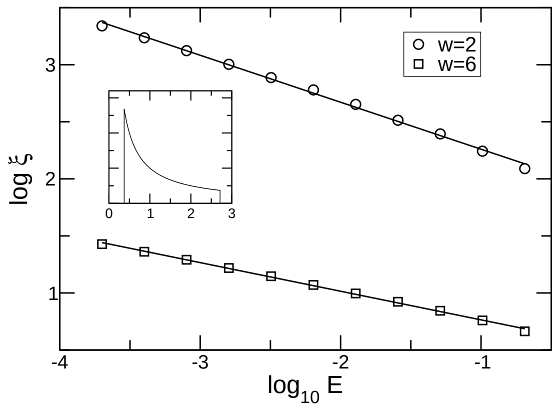

Fig. 7. Variation of the infinite-size localization length ξ with E for logarithmic distributions. The inset shows the t distribution for $w = 2$.

pect the crossover predicted in Ref. [24] to appear at smaller energies. We find that the exponents depend on the strength and distribution of the off-diagonal disorder also in agreement with Ref. [24]. Currently, we are extending these calculations to smaller energies.

We note that it might be interesting to also investigate the situation in honeycomb lattices [25], where the van Hove singularity of the square lattice at $E = 0$ does not interfere with the divergence due to the bipartiteness which is of interest here.

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