

The Keynesian Multiplier, Credit-Money and Time*

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Abstract. We analyze whether the standard multiplier principle is consistent with an endogenous credit-money framework and whether it is applicable to time series data. In order to take care of a credit-money framework, we reformulate the sources and uses of funds in the standard multiplier process. In order to tackle the issue of time, we introduce a time component, which measures the length of a multiplier round. We argue that both attempts are incomplete on their own, but can benefit from each other. Together they form a new version of the multiplier depending on the time duration of a multiplier period and a distinction between saving in terms of debt settlement and saving in terms of accumulating receivables. While the comparative-static stability condition of the multiplier can be dropped in this framework, our integrated multiplier reveals a dynamic stability condition for the multiplier process.

Keywords. Keynesian multiplier; endogenous money; velocity of money; credit cycles

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1 Introduction

The discussion of fiscal multipliers has lasted for decades and still economists struggle on the value of the multiplier. Stimulus packages facing the great recession have brought the matter back on the agenda. The literature on multiplier evaluations is growing fast, while at the same time the range of results is increasing.

One reason for the large range of results is simply the method of measuring the multiplier. Spilimbergo et al. (2009: 2) report several definitions of the multiplier in empirical studies and complain about the lack of comparability. For the single case of VAR models the multiplier effect is measured either as *peak response*, *initial response*, or *cumulative response* to either a permanent or one-shot increase in spending. There is no standardized time period, within which the effects are measured.

This heterogeneity stems from the theoretical basis. Multiplier theory does not provide an answer to the question how long the process lasts. As the multiplier is a dimensionless term, the length of a round and the length of the whole adjustment process are undefined *a priori*; they have to be set from outside the model. It turns out that more emphasize should be given to the time dimension of the multiplier process. If empirical evidence should serve as proof or disproof of the multiplier principle, empirical and theoretical work should run on comparable methodical grounds in this regard. However, this is not yet the case, since the multiplier principle is a theoretical construct in a logical-time comparative static framework, while measurement of multiplier effects draws on historical time series data.

This makes the case for transferring the multiplier principle from logical to historical time, i. e. from hypothetical rounds to concrete time periods. For it is of no use to compare the theoretical value of the multiplier after an indefinite number of hypothetical rounds with the empirical value of $\Delta Y/\Delta G$ after a concrete time span. Hence, we will introduce a time component to the multiplier formula, which captures the time duration of a multiplier round, or, in other words, displays how many multiplier rounds take place in a certain period.

A second issue concerns the relation between the multiplier and endogenous money. While both principles are now widely accepted in the literature, the multiplier does not fully account for the nature of a credit-money economy and should therefore be upgraded. We account for a credit-money framework by detailing the leak-ins and leak-outs of the multiplier process, developing a fully-fledged budget constraint, which takes into account all the sources and uses of funds in the process.

The aim of this paper is to tackle both the issues of time and credit-money, and to show that they are interrelated. The paper argues that both attempts are incomplete on their own, but can benefit from each other. Taken together they form a more general multiplier formula. This *integrated multiplier* is consistent

with a credit-money framework and is applicable to historical time.

The paper is organized as follows. The next section reviews the standard multiplier approach and provides a more detailed explanation of the issues mentioned above. The third section reviews the so-called velocity multiplier as a way to introduce time. Section four combines these findings to an integrated multiplier. In section five we deal with the dynamic stability conditions of the multiplier in a credit-money economy. The final section concludes.

A short note on assumptions should be given beforehand: (1) Throughout this paper we consider a closed economy with underemployed resources. (2) Commodity and service prices are mainly determined by unit costs outside the model; consumer price inflation is not a result of an increasing money stock. Thus, we do not distinguish between nominal and real terms. (3) There are no central bank reactions. (4) Changes in distribution of income or assets are not considered.

2 The serial multiplier

2.1 Explanation of the multiplier process

The usual multiplier formula can be derived from the following identities:

$$Y \equiv C + I \tag{1}$$

$$Y \equiv C + S \tag{2}$$

$$S \equiv sY \tag{3}$$

Substituting (3) into (2) yields

$$C \equiv (1 - s)Y. \tag{4}$$

Substituting (4) into (1) yields

$$Y \equiv \frac{1}{s}I \tag{5}$$

$$\Delta Y \equiv \frac{1}{s}\Delta I \tag{6}$$

Considering ΔI to be exogenously determined and s to be a stable behavioral parameter makes (6) a behavioral equation

$$\Delta Y = \frac{1}{s}\Delta I \tag{7}$$

where $1/s$ is the investment multiplier.

The multiplier principle has been viewed by most authors as a way to predict the impact of an exogenous expenditure on overall income or employment.¹ It is this major interpretation of the multiplier—the *serial multiplier*—that empirical investigations of stimulus packages draw upon; and it is this interpretation that we will deal with here. In order to account for the nature of stimulus packages, we will focus on one-shot stimuli.

The serial multiplier, which Keynes owes to Kahn (1931), is supposed to resemble a real process of spending and receipts through time, by drawing on comparative-static logical-time analysis. It describes a sequence of events: an initial increase in investment in the *first round* generates additional income which is partly spent for consumption and partly saved in the *second round*; what has been spent for consumption, induces additional income that, again, is spent for consumption and saved in a certain proportion in *round number three*, and so on. Given that $0 < s < 1$, this ‘converging series of ever diminishing waves of expenditures’ (Meade 1975: 84) yields the same formula as (7), yet, this time as the outcome of an equilibrating process in logical time:

$$\Delta Y = (1 + c + c^2 + c^3 + \dots)\Delta I = \frac{1}{s}\Delta I \quad \text{with} \quad 0 < (c, s) < 1 \quad (8)$$

The process stops as soon as additional planned saving equals additional investment again. It is the increasing income that adjusts saving step by step to investment. In other words, the lower the propensity to save s , the more income is generated before saving is equal to investment again.

At a first glance, the multiplier process looks like an inescapable mechanism. There has, however, been a long discussion why individual voluntary decisions of savers and investors generate that outcome.² The prevalent answer states that the generated saving is necessary to finance investment, which can also be found in the General Theory:

‘An increment of investment in terms of wage-units cannot occur unless the public *are prepared to* increase their savings in terms of wage-units. Ordinarily speaking, the public will not do this unless their aggregate income in terms of wage-units is increasing. Thus their effort to consume a part of their increased incomes will stimulate output until the new level [...] of incomes provides a margin of saving *sufficient to correspond* to the increased investment. The multiplier tells us by how

¹Besides, there are other interpretations of the multiplier as a logical relation (Gnos 2008) or as a sectoral equilibrium condition (Hartwig 2004, 2008). Cf. Chick (1983: 253-4) for a more detailed discussion of the methodical difference.

²Cf. for example Dalziel (1996); Moore (1994); Cottrell (1994); Chick (1983); Warming (1932) and more recently Gechert (2011); Bailly (2008); Rochon (2008)

much their employment has to be increased to yield an increase in real income sufficient to induce them *to do the necessary extra saving ...*? (Keynes 1936: 117) (emphases added; S.G.)

Following this rationale, voluntary savings must fund the initial investment which forces them to become equal. According to Shackle (1951: 243), Keynes owes this to Kahn:

‘For it will be demonstrated [...] that, *pari passu* with the building of roads, funds are released from various sources at precisely the rate that is *required to pay the cost of the roads*.’ Kahn (1931: 174) (emphases added; S.G.)

These explanations are—curiously enough—a recurrence to Say’s Law: the multiplier is considered to explain how investment is made possible by savings while the core of the General Theory was meant to explain that investment governs savings.³

Keynes himself overcame this contradiction in his post-General Theory writings (Keynes 1937a: 246-7), (Keynes 1937b: 664-6). These articles laid the foundation for the now widely accepted endogenous money approach which states that finance requires no saving. Financial resources for investment are provided by private banks creating credit *ex nihilo*. When a loan is granted, the borrower holds a debt and a deposit; nobody has saved beforehand. Once the borrower spends the money on newly produced capital goods, the producer receives deposits that can be considered transitory saving. If they are spent later on, someone else earns and transitionally saves them. The overall amount of financial assets is zero at any time because there is still a liability to the bank. Only the investment has created wealth. Thus, finance creates saving via investment and not the other way round. Accepting the endogenous money approach means rejecting the notion that saving finances investment. If investment needs finance but finance does not require saving, there is no market constraint for voluntary saving to be on par with investment. It provides no economic explanation for the outcome of the multiplier process.⁴

However, there is a more pragmatic answer to the question why saving should adjust to investment. Kahn and Keynes simply modeled the process this way to arrive at a finite multiplier value in a comparative-static framework (Hegeland 1966: 61).⁵ From that point of view, savings are a mere residual, a leakage allowing the

³Cf. Trigg (2003) for a further discussion.

⁴There is only the ever-valid *ex post* identity of actual saving and investment. Cf. Gechert (2011) for a more detailed discussion.

⁵Hegeland also considers a second effect. The notion that a governmental expenditure would eventually create adequate savings may have been convincing to politicians of the non-inflationary effects of expansionary fiscal policy.

process to find a position of rest after the initial demand shock. However, to yield that outcome, the serial multiplier process depends on two critical assumptions, namely, (1) Keynes' division of the multiplier and the multiplicand, and (2) his simple consumption function. He considered consumption and investment to be of a completely different nature:

'The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it is usual in a complex system to regard as the *causa causans* that factor which is most prone to sudden and wide fluctuation. More comprehensively, aggregate output depends on the propensity to hoard, on the policy of the monetary authority as it affects the quantity of money, on the state of confidence concerning the prospective yield of capital-assets, on the propensity to spend and on the social factors which influence the level of the money-wage. But of these several factors it is those which determine the rate of investment which are most unreliable, since it is they which are influenced by our views of the future about which we know so little.' (Keynes 1937c: 221)

Keynes divides the multiplier ($1/s$) and the multiplicand (ΔI) in order to separate the more stable from the more fluctuating expenditures. As the citation above shows, he also regards this as the right way to separate cause and effect. Investment demand is very volatile and thus the ultimate *cause* of economic fluctuations. In contrast, consumption is a mere *effect*, and is determined by the simple Keynesian consumption function as a stable share of disposable income. Consequently, the leakage in each round of the process is given by *current income not consumed*.

2.2 Criticism of the multiplier process

The method may be brilliant from an analytical point of view, but it may not suffice to separate cause and effect. Explaining causality with volatility has been questioned by Villard (1941: 229-33), Lutz (1955: 40-2), and Machlup (1965: 10), among others. Additionally, the Keynesian consumption function has come under criticism from various strands (Modigliani and Brumberg 1954), (Godley and Lavoie 2007: 70), (D'Orlando and Sanfilippo 2010: 1044). Taken together, it is questionable whether the causality intended by the multiplier principle resembles the actual series of spending and receipts in an economy and whether it is valid in a credit-money framework. The following points may specify the concerns:

(1) It is illogical that consumption triggers further expenditures in any round of the multiplier process but the first round. According to the multiplier formula,

consumption can only continue the process, yet not initiate it. This assumption may be justified from a financing point of view: assuming that any initial spending requires new credit (Wray 2011: 8), deficit spending is more common for investment and public expenditures than for consumption. However, debt financed consumption has become increasingly important over the last decades and should not be ruled out (Fontana 2009: 100), (Dutt 2006: 341-3), (Brown 2008: 20), (Cynamon and Fazzari 2008: 8), (Akerlof 2008: 1). We therefore implement initial consumption spending by referring to additional autonomous expenditures (ΔA) starting the process; ΔA contains initial investment, consumption and governmental expenditures.

(2) Related to the first point, it is worthwhile to take a credit-money perspective on the process of spending: any autonomous expenditure comes with new credit creation. The credit view is based on the financing process of investment as described by Davidson (1986: 102) and Chick (1983: 262-3). Initial investment is financed by new loans or drawing on overdraft facilities provided by the banking system. Internal finance (retained earnings, depreciation) plays a subordinate role for financing net-investment. We may easily extend this assumption to additional autonomous expenditures ΔA in general. Under these conditions, the multiplier process starts with credit expansion:

‘Without a rise in overall debt, private or public, the growth process would essentially come to a halt in a monetary economy. Indeed, whether it is business enterprises, households, or governments, in a dynamic growth process someone must first borrow in excess of any “pre-existing” amount of financial resources.’ Seccareccia (2011: 12-3)⁶

The reason why it is feasible to equalize effective demand and a credit expansion comes with the so-called *reflux principle* (Rochon 2008; Lavoie 1999; Kaldor and Trevithick 1981). With respect to the initial borrower it basically says that a loan is supposed to be spent. Debtors do not hold idle money because debit interest rates usually exceed credit interest rates. If there was no usage for excess credit-money, it would be repaid (Lavoie 1999: 106). By aggregate demand, the additional credit-money flows from hand to hand, accompanying the creation of income through the multiplier. This is not to say that money is causal for aggregate demand; but it sheds new light on the multiplier process, as will be argued in more detail in section 4.

(3) Again, related to the first point, it is illogical to regard consumption as the only induced expenditure. Clearly, excluding induced investment controls for a finite multiplier value in a comparative static framework, but this is a mere method-based reason. As regards content, Keynes may have left induced investment out

⁶Cf. Polak (2001: 7) for another application of the credit view.

because he argued in a situation far from full capacity utilization where there is no incentive for the private sector to increase capacity; the marginal propensity to invest may be close to zero under these conditions. However, given that initial spending is for consumption purposes—be it public or private—it increases capacity utilization, and therefore may induce investment. Moreover, the majority of the empirical literature applies the multiplier principle regardless of the phase of the business cycle. Thus, their estimations comprise phases of high capacity utilization, too. As many empirical studies find crowding-in of private investment, the theoretical model should not exclude these effects *a priori*. Of course, there are multiplier-accelerator models that employ an investment function as well. See Hicks (1959) for an early model. However, the present paper combines the marginal propensity to consume and the marginal propensity to invest which yields the *marginal propensity to spend* (ε). So far, the modified multiplier formula reads

$$\Delta Y = \frac{1}{1 - \varepsilon} \Delta A. \quad (9)$$

Assuming that induced expenditures solely stem from income generated in the previous round, controls for $\varepsilon \leq 1$, whereby the series is most likely to converge.

(4) However, in a credit-money economy expenditures are not limited by current income. The factual budget constraint of households is based on cash flows (Brown 2008: 3), and thus includes wealth (Godley and Lavoie 2007: 66), (Zezza 2008: 376), and credit (van Treeck 2009: 475-6), (Zezza 2008: 379), (Bhaduri 2011: 10) as additional sources to spend from. Consequently, the stability condition of the multiplier ($0 < e < 1$) may not hold.

(5) Besides the sources, it is also necessary to specify the uses of funds. In the simple multiplier model there is only the distinction between consumption and saving. With point (3) we add investment, whereby we distinguish consumption, investment and the residual that is not spent. From a demand perspective, this residual is considered a definite flow out of the circuit and does not re-enter (Palley 1998: 96); from a monetary perspective it is net wealth accumulation, either by accumulating receivables (deposits, bonds, ...) or by reducing liabilities.

We may draw on an example here. Consider a closed economy consisting of a bank (B), a producer of consumption goods (P_C) and a producer of investment goods (P_I), the latter being also the consumer. P_C takes out a loan ΔL in order to buy a newly produced investment good from P_I . First of all, taking out the loan increases both liabilities and receivables of P_C . After production, delivery and payment of the investment good, P_C holds a tangible asset ΔI and the liability to the bank, while P_I holds the receivable to the bank.

Let us consider the simple case where P_I has a marginal propensity to save $s_I = 1$. Then, the equilibrium condition $S = I$ comes true right after the first round,

while liabilities and receivables still exist. If this is a repeated scenario (permanent flow of additional credit financed investment), there will be a converging flow of additional income while the stock of liabilities and receivables grows infinitely. If this is a one-shot scenario, additional income will fall back to zero while the stock of liabilities and receivables persists. In both cases, the ratio of liabilities to income (and as a mirror image the ratio of receivables to income) would raise to infinity. Thus, the comparative-static equilibrium of the simple multiplier model may be stock-flow incoherent (Godley and Lavoie 2007: 70-1).

Let us consider the other case where P_I has a marginal propensity to save $s_I = 0$ and buys consumption goods from P_C . Right after production, delivery and payment of the consumption goods, P_C holds the liabilities and the receivables again. If P_C fully uses the receivables to settle his debt to the bank, i.e. $s_C = 1$, then after the second round $S = I$ holds. Both in a repeated and in a one-shot scenario the ratio of liabilities (and receivables) to income converges, i.e. there is stock-flow coherence. However, this is the only stock-flow coherent example. Any scenario with positive saving of net creditors (such as P_I in our case) yields stock-flow incoherence. The kind of saving is important for the dynamic stability of the system, but it does not appear in the stability condition of the multiplier. Below we will distinguish between saving as a reduction of liabilities and saving as an accumulation of receivables.

All these caveats are clearly interconnected. They augment the possible sources and uses of funds at the beginning and during the multiplier process. The standard multiplier allows for credit financed expenditures in the initial round only, and in any further round current income is the only available source. Consumption and unspecified saving are the only uses. In contrast, in a credit-money economy the following budget constraint holds in any round of the multiplier process:

$$\Delta Y_{r-1} + \Delta D_r + \Delta L_r \equiv \Delta C_r + \Delta I_r + \Delta H_r + \Delta R_r \quad (10)$$

ΔY_{r-1} is the additional disposable income (generated in the previous round), ΔD_r is additional funding out of wealth (reduction of receivables), ΔL_r is additional credit (accumulation of liabilities), ΔC_r is additional consumption, ΔI_r is additional investment, ΔH_r is additional accumulation of wealth (accumulation of receivables), and ΔR_r is additional debt settlement (reduction of liabilities) in round r .⁷ The marginal propensity to spend ε_r is defined as

$$\varepsilon_r \equiv \frac{\Delta C_r + \Delta I_r}{\Delta Y_{r-1}} = \frac{\Delta Y_r}{\Delta Y_{r-1}} \quad (11)$$

From (10) and (11) follows

$$1 - \varepsilon_r \equiv \frac{\Delta H_r - \Delta D_r + \Delta R_r - \Delta L_r}{\Delta Y_{r-1}} \equiv \lambda_r \quad (12)$$

⁷Note: Δ depicts differences to the preceding round.

where λ_r depicts the net outflow from the circuit by net debt settlement ($\Delta R_r - \Delta L_r$) and net hoarding ($\Delta H_r - \Delta D_r$).⁸ It is now obvious that $\varepsilon_r > 1$ can occur whenever the numerator in (12) becomes negative. That means, more funds are floated into the circuit than withdrawn from the circuit ($\lambda_r < 0$), which corresponds to a negative propensity to save for the standard multiplier. However, this infringes the stability condition of the comparative-static model (now: $0 < \lambda_r < 1$); the multiplier wouldn't converge to a finite value. The usual solution is to restrict the analysis to stable cases.

As we are concerned with making the multiplier principle empirically applicable, it is not viable to look at stable cases only. Thus, in the present paper we develop a method to determine the multiplier without such a restriction, by introducing a time component to the multiplier formula.

This requires some further considerations that are closely related to our second issue of transferring the multiplier from logical to historical time. When applying the multiplier principle to empirical data, one has to switch from rounds to periods. As the multiplier is a dimensionless term, the length of a round and the length of the whole adjustment process are undefined *a priori*; they have to be set from outside the model. This is usually done by either setting one round as one period, as in Dalziel (1996), or by assigning the whole adjustment process to one period, as in Godley and Lavoie (2007); Pusch and Rannenberg (2011). In order to refrain from an arbitrary choice, the multiplier model will be augmented by a time-component, which captures the time duration of a multiplier round, or, in other words, displays how many multiplier rounds take place in a concrete period. The so-called velocity multiplier will serve as a starting point.

3 The velocity multiplier

In section 2 we emphasized the need for a time-dependent multiplier due to an arbitrary conversion from rounds to periods when applying the standard multiplier to time series data. The duration of the multiplier process was intensively debated in the aftermath of Kahn (1931). The discussion led to the development of the so-called velocity multiplier, where the marginal propensity to consume is replaced by the velocity of money.⁹ A recent contribution to this approach was made by Moore (2008), who derives his multiplier formula as follows: the whole volume of trading T_t in a certain period of time equals the amount of money that flows from hand to hand. This amount is decomposed by the stock of money M_t^* and its

⁸“Hoarding” includes deposits, bonds, cash, ...

⁹Among the early proponents of the velocity approach were Lutz (1955); Machlup (1939); Anderson (1945); Clark (1935); Angell (1941). Early opponents were Samuelson (1942); Turvey (1948); Goodwin (2005); Ackley (1951); Hegeland (1966)

transactions velocity V_t^* . Then the truism holds that

$$T_t \equiv M_t^* V_t^* \quad (13)$$

which resembles the Quantity Equation for the whole volume of trading. By focussing on the amount of money that is used to buy currently produced commodities and services, the standard Quantity Equation comes into being:

$$Y_t \equiv M_t V_t \quad (14)$$

A change in aggregate demand (ΔY_t) comes along with changes in the stock and/or in the velocity of money. Totally differentiating (14) yields

$$\Delta Y_t \equiv \Delta M_t V_t + \Delta V_t M_t + \Delta M_t \Delta V_t \quad (15)$$

which is the additional income compared to the baseline.¹⁰ To determine ΔY_t , Moore (2008: 125) assumes the following. (1) The change of velocity of money is expected to be zero ($\Delta V_t = 0$). (2) New investment is totally financed by bank lending and nothing else but investment expenditure demands new money ($\Delta I_t = \Delta M_t$). This way, (15) becomes

$$\Delta Y_t = V_t \Delta I_t \quad (16)$$

where the income velocity of money for a certain period resembles the Keynesian multiplier. ‘In each period the Keynesian multiplier is the income velocity of money!’ (Moore 2008: 126).

The idea breaks away from Keynes’ original reasoning whereby the amount of investment is the *causa causans* of aggregate demand just because it is its most volatile component. For Moore, volatility is not the decisive factor; rather it is the kind of funding that makes the difference between I and C . While consumption is usually paid out of current or past income, net investment is financed by newly created bank loans or by drawing on overdraft facilities. Thus, investment drives aggregate demand precisely because someone is ready to go into debt. It is the additional creditworthy demand that creates credit-money and thus initiates the multiplier effect; and it is the velocity of the additional credit-money that accounts for the value of the multiplier.¹¹

¹⁰Note: In this historical time approach Δ in general indicates deviations from the baseline scenario (without additional demand); it does not indicate differences to the previous period.

¹¹Rochon (2008: 168) notes that Moore’s recourse to the Quantity Equation has been criticized by some authors as ‘smacking of monetarism’. However, Moore does nothing more than referring to the Quantity *Equation*, which is by itself a mere identity. *Neither* does he refer to Quantity *Theory*, which tries to explain inflation with monetary growth, *nor* to Neo-Quantity Theory, where M is determined by the central bank. To him, M is fully determined by creditworthy demand. Still, aggregate demand drives growth of income.

An additional amount of money—created by someone who is willing to borrow and by a bank that is willing to lend—induces a succession of expenditures and receipts, just like the process of the serial multiplier suggests. The crucial difference is: nothing leaks out of the circuit in Moore’s version ($\lambda_t = 0$). Thus, there is no resting point the system is gravitating towards. Instead of that, the process happens in real time, which allows to measure the laps of the amount of money in the circuit. Whenever someone receives an inflow, it will be spent on newly produced goods sooner or later. The sooner this is done, the higher is the income velocity of money, and thus, the multiplier.¹²

The question: “How large is the multiplier in a certain period of time?” boils down to: “How often will the additional money be spent on newly produced goods during that certain period?” The velocity multiplier has an infinite value for an infinite period, but a definite value for a definite period of, say, 4 per year or 1 per quarter. The serial multiplier is the antipode: it yields a finite value for an infinite succession of rounds while it does not tell how long one round actually takes.

This entails a considerable advantage of the velocity multiplier. As the income velocity of money has a concrete time dimension, it is applicable to time series data without arbitrarily converting rounds into periods. The velocity approach makes an attempt to determine the *income propagation period* (Machlup 1939: 5), i. e. the duration of a multiplier round.

The introduction of a time component yields a second advantage: even without the usual stability condition of the serial multiplier ($0 < \lambda_t < 1$), a concrete multiplier value is calculable for a certain period.

On the other hand, Moore’s multiplier has at least three distinct weaknesses.

(1) The assumption $\Delta I_t = \Delta M_t$ is not homogeneous in dimensions. ΔI_t measures a change in a flow during a period; ΔM_t measures a change in a stock comparing the beginning and the end of one period. In the next section we overcome this problem by exclusively referring to ΔM_t , which represents the impulse of aggregate demand from a credit-money perspective.

(2) The income velocity is not a fruitful variable. Figures of the income velocity are usually taken *ex post* by dividing a certain money aggregate by income ($V_t = M_t/Y_t$). In this sense, V_t is a mere residual to fulfill the Quantity Equation. Depending on the money aggregate under consideration, V_t is higher or lower when M_t is defined narrower or broader. However, here the velocity is regarded as a behavioral parameter that displays the duration of a multiplier round or, more precisely, the number of multiplier rounds in a period. Therefore it must draw on institutional characteristics of the economy. We will reconsider that problem later.

(3) The exclusion of leak-ins and leak-outs does not fit to the credit-money framework laid out in section 2. Once the additional volume of credit-money

¹²Cf. the dynamic model of Andresen (2006) for an application of the velocity multiplier.

ΔM_t has been created, it is fully kept within the circuit during the whole period under consideration. By this, Moore implicitly sets the leakage to $\lambda_t = 0$ and the propensity to spend to $\varepsilon_t = 1$. Moore assumes that unspent credit-money does not leak out, but becomes deposits on bank accounts where banks use it for further business. Moore (2006: 366) terms this *convenience lending* by depositors. Only the speed of spending (V_t) limits the multiplication. This is the reason why an endless period implies an infinite velocity multiplier, which comes along with dynamic instability as well. We will tackle this issue by combining the velocity multiplier with the extended sources and uses of funds as discussed in section 2.

To sum up, we may state that the velocity multiplier is an incomplete approach. It suffers from the absence of a leakage and the diffuse concept of the velocity of money. It, however, makes an important contribution to the multiplier principle by bringing in the duration of a multiplier round as a determinant of the income effect within a certain period. Clearly, if V_t is to be of use, it must not represent the residual income velocity of money. It must depict the economy's behavioral income propagation period, more precisely: the number of multiplier rounds within a period. This is the way we use V_t in the following.

4 An integrated multiplier

The budget constraint for a credit-money economy is also valid in historical time. Now we consider the case where at the beginning of a period there is a credit-financed demand impulse circulating for a number of rounds in the period ($V_t \Delta M_t$). During the period agents can draw on further funds that circulate as well ($V_t \Delta D_t + V_t \Delta L_t$). Funds are used for aggregate demand ($\Delta C_t + \Delta I_t$), hoarding ($V_t \Delta H_t$) and debt settlement ($V_t \Delta R_t$). For simplicity we assume that all funds circulate with the economy's average frequency V_t :

$$V_t \Delta M_t + V_t \Delta D_t + V_t \Delta L_t \equiv \Delta C_t + \Delta I_t + V_t \Delta H_t + V_t \Delta R_t \quad (17)$$

Net debt settlement equals $V_t(\Delta R_t - \Delta L_t)$ and net hoarding equals $V_t(\Delta H_t - \Delta D_t)$. Let us set $\Delta H_t - \Delta D_t = 0$ for a while, which means that we focus on additional credit creation and repayment; we neglect saving (dissaving) in terms of accumulating (reducing) receivables. The net leakage from the circuit now reads

$$\Delta R_t - \Delta L_t = \lambda_t \Delta M_t. \quad (18)$$

λ_t depicts the share of the credit impulse that is used for net debt settlement in each income propagation period and does not induce further demand. Taking this into account, (17) becomes

$$V_t(1 - \lambda_t)\Delta M_t = \Delta C_t + \Delta I_t. \quad (19)$$

The share of the impulse that is not used for net debt settlement induces aggregate demand, $\Delta C_t + \Delta I_t$, which equals the additional income (ΔY_t) generated in that period. From this follows the integrated multiplier formula:

$$\Delta Y_t = V_t(1 - \lambda_t)\Delta M_t \quad (20)$$

The integrated multiplier is both time dependent and allows for inflows and outflows. There are two important channels, namely the frequency of multiplier rounds (V_t) and the magnitude of the net leakage (λ_t) within a multiplier round. The higher the frequency, the higher is the multiplier. The more intense the leakage, the lower is the multiplier. Both determinants are related to time, and so must be the multiplier.

Suppose the creation of an additional amount of credit-money at the beginning of period t . After the money is spent, it will induce a succession of receipts and expenditures. The economy's average frequency V_t determines, how often the additional money circulates for newly produced goods and assets during a given period. The leakage λ_t determines on average, how much of a receipt is spent again and how much of it is used for net debt settlement in each multiplier round. Together, they determine, how much additional income is generated out of the initial loan within a certain period.

So far, we only considered saving in terms of net debt settlement ($\Delta R_t - \Delta L_t$) and disregarded net hoarding ($\Delta H_t - \Delta D_t$). Clearly, both ways of net saving are *non-demand* and thus they have the same short run effects on income. However, from a monetary perspective net debt settlement is a definite leakage because the economy's gross debt level and the amount of money shrinks; net hoarding, on the contrary, is not a leakage in the strict sense. It maintains the debt level, but the hoarded receivables do not circulate in the productive sphere anymore—their velocity slows down to zero.¹³ Therefore, they reduce the economy's average V_t . So, both net debt settlement and net hoarding delimit the multiplier, but through different channels. While net debt settlement reduces the amount of money through the leakage ($\lambda_t\Delta M_t$), net hoarding reduces the average frequency (V_t).

It can be shown that formula (20) is an integrated or general version of the serial and the velocity multiplier. Compared to the velocity multiplier, it is more general as it allows for a leakage through the reflux principle. Compared to the serial multiplier, it is more general as it allows for measurement of the multiplier in historical time and is not constrained to $0 < \lambda_t < 1$, i. e. it allows for additional net inflows to the circuit.

In other words: both the serial multiplier and the velocity multiplier can be derived from (20) by making special constraints. The velocity multiplier is a

¹³This does not mean that the money is idle though. It may well circulate with a high velocity in the financial sphere, but from the perspective of the productive sphere it stands still.

special case of the integrated multiplier by setting the leakage to $\lambda_t = 0$ and by equating additional credit-demand and additional investment ($\Delta M_t = \Delta I_t$). The initial investment is put in place at the very beginning of the period under consideration to ensure dimensional homogeneity. Equation (20) becomes

$$\Delta Y_t = V_t \Delta I_t \tag{21}$$

which resembles equation (16).

Also the serial multiplier can be derived from (20). Moving from logical rounds to concrete time intervals requires setting the length of one round. Common practice defines one round as one period whereby the velocity of money is set to one per period ($V_t = 1/t$). Moreover, the serial multiplier assigns the role of the leakage to the marginal propensity to save ($\lambda_t = s$), and the role of additional autonomous demand to investment of the initial round ($\Delta M_t = \Delta I_0$). This yields

$$\Delta Y_t = (1 - s)\Delta I_0 = c\Delta I_0 \tag{22}$$

which is the additional income generated by consumption expenditures in the round succeeding the initial round. The transformation reveals that the usual multiplier formula is only applicable when the duration of a multiplier round is set to one per period, a point which was already made by Tsiang (1956: 555-6).

Both special cases of the integrated multiplier make *ad hoc* assumptions concerning parameter values that should rather be determined empirically in order to calculate the multiplier effect for a certain time span properly. This is possible with the integrated multiplier because it is not just a theoretical construct, but it can be directly applied to empirical questions. Multiplier effects are not calculated via identification of public spending shocks and GDP effects, but via direct determination of the behavioral parameters V_t and λ_t for a given scenario.

The problem of measurement of these parameters has already been discussed in section 3 and it applies to the integrated multiplier as well. Determining the behavioral income propagation period goes beyond the scope of this paper though.¹⁴ In any event, the integrated multiplier at least lays open the problem of the length of the multiplier period.

The integrated multiplier combines the useful properties of its special versions and avoids their questionable properties at the same time. As V_t has a concrete time dimension, our multiplier applies to processes in historical time. It does not need the comparative-static stability condition of the serial multiplier, just to generate a calculable outcome. Depending on the values and the relation of

¹⁴Early contributions to the velocity approach estimate it to be about a quarter (Machlup 1965: 26); (Villard 1941: 256); (Angell 1941: 145); (Clark 1935: 87).

parameters V_t and λ_t , the integrated multiplier does not necessarily converge to a finite value (as with the serial multiplier), and it does not necessarily end up infinite for an infinite period of time (as with the velocity multiplier). So, even in times of accelerating growth, when demand induces further credit-financed demand ($\lambda_t < 0$), and in times of debt deflation, when the initial impulse is more than offset by net outflows ($\lambda_t > 1$), the multiplicative effect is calculable for a given period.

5 Dynamic stability of the multiplier process

It should be emphasized again that the volume of credit-money itself is not causal to the multiplier process, but comes along with effective demand. We take the credit perspective for two reasons. First, it is plausible that an additional autonomous demand is credit-financed. Second, the reflux principle rules out cases where someone takes out a loan and holds it idle. The integrated multiplier offers an extended view on the financing level and the time dimension of the multiplier; but there is more about it.

In section 2 we addressed the impact of different kinds of saving on dynamic stability. While net debt settlement ($\Delta R_t - \Delta L_t$) reduces the stock of credit-money, net hoarding ($\Delta H_t - \Delta D_t$) keeps it up. In the long run, sustained net hoarding gives rise to dynamic instability. With a one-shot credit-financed demand impulse, the additional income effect will cease and income will return to its baseline value after some time, while receivables (and liabilities) will remain on a higher level; a permanent credit-financed increase of demand yields a higher permanent level of income, while receivables (and liabilities) accumulate on and on. In both cases the ratio of $\Delta M_t / \Delta Y_t$ grows infinitely, making the process stock-flow incoherent.

Only if saving is done in terms of net debt settlement, this yields a dynamic equilibrium. Given a one-shot impulse, both the income effect and the amount of circulating credit will pass off; given a repeated impulse, both the income effect and the amount of credit-money will converge to a finite value. This gives rise to a new understanding of the multiplier process in the long run: the multiplier *does not* show the income generating process until an initial investment is financed or paid by savings. What it *does* show, is the income generating process until an additional amount of credit-money is repaid.

Suppose a one-shot credit financed demand impulse (ΔM_t) at the beginning of period t . The remaining impulse at the beginning of period $t + 1$ is

$$\Delta M_{t+1} = \Delta M_t - (\Delta R_t - \Delta L_t) = (1 - \lambda_t) \Delta M_t \quad (23)$$

which still generates additional income in this period.¹⁵ The multiplier principle

¹⁵Again, ΔM_{t+1} depicts the additional amount of credit money compared to the baseline scenario. It is *not* the difference to M_t .

has worked out fully when at some period τ the debt level M_τ is back at the baseline, which signals a steady state.

However, for multiplier effects to be measurable the steady state does not need to come into being because empirical measurement usually refers to a concrete short-term period. The leakage does neither need to be positive nor complete in such a period. There may be times of optimism, when an autonomous demand triggers accelerating credit-financed demand expansion, which comes with a negative λ_t . The effects can still be measured for this time span via the integrated multiplier yielding a relatively large multiplier value. In contrast, there may be times of debt deflation with a low integrated multiplier value due to an intense leakage.

Nevertheless, a stock-flow incoherent setting in the short run may entail repercussions from stocks on flows that influence the parameters of the multiplier in upcoming periods. More precisely, an increasing debt-to-income ratio in the economy may enhance the propensity to settle debt (λ_t) and thus reduce the multiplier effect in the future. To capture these effects, the integrated multiplier can be extended to a dynamic model, where V_t and λ_t are endogenously determined. This is linked to the growing literature on credit cycles and their influence on the business cycle (Fisher 1933; Palley 1994; Biggs et al. 2009; Keen 2010; Raberto et al. 2011).

6 Conclusion

The present paper discussed the shortcomings of the simple Keynesian multiplier model with respect to the characteristics of a credit-money economy and a historical-time framework. Indeed, the well-known serial multiplier is a comprehensible way to model the process of expenditures and receipts stemming from an initial demand for capital goods, but the formula merely looks as though it entails an ever-valid mechanism. Reconsidering the sources and uses of funds in a credit-money economy reveals some degrees of freedom that should not be set *ad hoc*. When induced investment and a more realistic budget constraint are taken into account, the comparative-static stability condition ($0 < s < 1$) may not hold. Additionally, the serial multiplier formula provides no information regarding the length of the process, causing arbitrariness when it comes to empirically determining the multiplier from time series data.

Thus, we develop an alternative approach where the stability condition is not needed to calculate a finite multiplier value. We do this by introducing a time component that captures the number of multiplier rounds proceeding in a concrete historical time period. We combine the extensions concerning credit-money and time to our *integrated multiplier*, which has two channels of influence—the number of multiplier rounds V_t per period and the magnitude of the net leakage λ_t in each of these rounds.

This new multiplier has several advantages. First, it takes into account a time dimension, whereby it allows for a rule-based conversion from hypothetical rounds to concrete periods. Second, with the time component the multiplier value is calculable for a given period, even when the comparative-static stability condition does not hold. Third, the budget constraint that comes with our multiplier version fits to a credit-money economy. Fourth, it allows for induced investment.

With these advantages the integrated multiplier is not just a theoretical construct, but it can be directly applied to empirical questions. Multiplier effects are not calculated via identification of public spending shocks and GDP effects, but via determination of the behavioral parameters V_t and λ_t for a given scenario. As a drawback of this method, it may be a difficult task to empirically determine these parameter values and we do not have a satisfactory solution so far. This is an issue for future research.

As we look at the multiplier from a monetary perspective, the conditions of a dynamic equilibrium are revealed, namely, a stock-flow coherent liabilities-to-income ratio (and, as a mirror image, a stock-flow coherent receivables-to-income ratio). The dynamic stability condition supersedes the comparative-static one. This makes a new understanding of the multiplier process in the long run: the multiplier *does not* show the income creating process until an initial investment is financed or paid by savings. What it *does* show, is the income creating process until an additional amount of credit-money is repaid.

This relates the multiplier analysis to processes of leveraging and deleveraging that may entail repercussions on the parameters of the multiplier, which then depends on the overall level of debt in the economy. In a next step, a dynamic model could tackle this issue by endogenising the parameters V_t and λ_t . Of course, that will make it necessary to identify the determinants of V_t and λ_t . To put it with Machlup's words:

‘The theory of the Multiplier, if it is to be of use to those who wish to know the possible and the probable effects of public works, must renounce the attractive appearance of neatness and preciseness. The two variables which seem to play the main parts in the play of the Multiplier must be decomposed into the all too large number of variables which play the important roles in the real world.’
(Machlup 1939: 27)

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