Solving Poisson's and Maxwell's equations by the Fourier–Singular Complement Method

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The Singular Complement Method (SCM) was developed [1,2] as an alternative to the "usual" edge element and finite volume methods, in order to solve Maxwell's equations in singular domains. It can also serve as an accelerator of convergence for the usual $\mathbb{P}_1$ element method for Poisson's equation, as well as for the Lamé and Stokes problems. The SCM is based on a splitting of the "natural" spaces of solutions into a *regular* part (which has the regularity expected for a smooth or convex domain) and a *singular* part (which needs some special representation). It incorporates the well-known computation of *singularity coefficients* (a.k.a. "stress intensity factors" in mechanics).

However, the SCM had so far been implemented only in two-dimensional situations. This limitation stemmed from the difficulties of practical description of the singular spaces in general three-dimensional geometries.

We show how the SCM can be extended to some simple, but genuinely three-dimensional situations: prismatic or axisymmetric domains with arbitrary data. In this case, the splitting w.r.t. regularity can be combined by a Fourier expansion in the longitudinal or azimuthal coordinate.

This Fourier–Singular Complement Method (FSCM) achieves a good compromise between simplicity and efficiency: it has the optimal convergence rate for $\mathbb{P}_1$ element methods with an L^2 data [3], and the least computational cost among the optimally convergent methods. It can be easily extended to the time-dependent Maxwell equations [4].

References:

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