

Integrality of L^2 -Betti numbers

THOMAS SCHICK (GÖTTINGEN)

Inside the C^* -algebra of all continuous functions on the circle, the (Laurent) polynomials have a particular property: if the polynomial is not zero, then the operator given on L^2 -functions by multiplication with this function has no non-trivial kernel. In other words: the von Neumann dimension of this kernel is either zero (if the kernel is zero) or 1 (if the polynomial is the zero polynomial).

An operator theoretic version of Atiyah's conjecture about integrality of L^2 -Betti numbers generalizes this from the integers (the dual of the circle groups) to arbitrary discrete groups:

Given a matrix A over the integral group ring of a discrete group G , A can be considered as a bounded operator on $L^2(G)^n$. If G is torsion free, then the von Neumann dimension of this kernel is conjectured to be an integer.

In the talk, we will discuss special cases of this conjecture, as well as generalizations to groups with torsion, and counterexamples to these generalizations. We will also give further information about the possible eigenvalues of such operators.