A lower bound on the lowest spectral gap of Schrödinger operators with Kato class measures

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We consider a Schrödinger operator on $\mathbb{R}^n$ formally given by

$$H_\mu = -\Delta - \mu,$$

with a compactly supported measure $\mu \geqslant 0$ from the Kato class. It is known that then $\sigma_{\mathrm{ess}}(H_\mu) = [0, \infty)$. Assuming that H_μ has two lowest eigenvalues $\lambda_0 < \lambda_1 < 0$ with corresponding eigenfunctions $\varphi_0, \varphi_1 \in L_2(\mathbb{R}^n)$, the lowest spectral gap can be expressed as

$$\lambda_1 - \lambda_0 = \|\varphi_1\|_2^{-2} \int \left|\nabla \tfrac{\varphi_1}{\varphi_0}\right|^2 \varphi_0^2 \,.$$

We demonstrate how this formula can be used to obtain an explicit lower bound on the spectal gap if μ is of the form $\mu = g\sigma_\Gamma$, where Γ is a compact $(n-1)$-dimensional submanifold of $\mathbb{R}^n$, σ_Γ the surface measure on Γ, and $0 \leqslant g \in L_\infty(\Gamma)$. In dimension $n = 2$ this was accomplished by S. Kondej and I. Veselic (2006). The approach we present is also applicable to more general measures satisfying the condition

$$\mu\big(B(x,r)\big) \leqslant c r^{n-\alpha} \qquad (x \in \mathbb{R}^n,\ r > 0)$$

for some $c > 0$, $\alpha \in [0, 2)$ (which implies that μ is in the Kato class). For $\mu = g\sigma_\Gamma$, this condition is satisfied with $\alpha = 1$.