A New Mixed FE-Formulation for Liquid Crystal Elastomer Films

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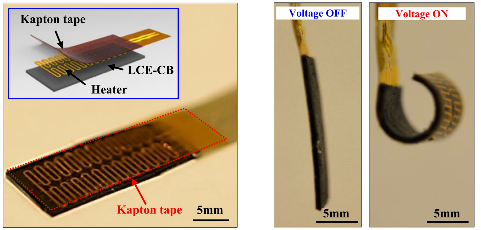
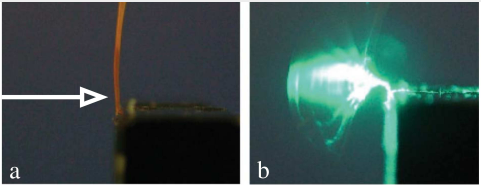
Motivation and goals

Goal 1: FE-simulation of thermal actuation of motion of LCE films



Introduction of Joule heat energy by heating pads, see Cui Y. et al. [2018]

Goal 2: FE-simulation of UV light acuation of motion of LCE films



Using of UV light for inducing vibration, see Corbett & Warner [2009]

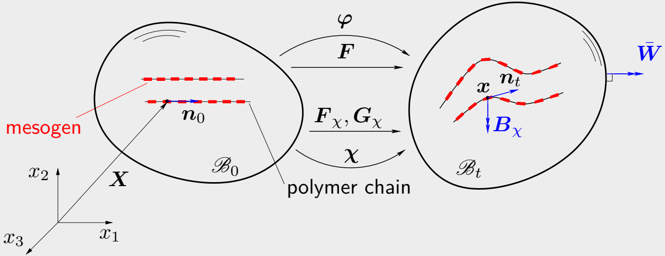
Step 1: FE formulation for actuation of continuum motions by boundary or volume loads

We design a dynamic mixed FE method for continuum motions with internal reorientation

Continuum formulation with reorientation effects

(see e.g. Frank [1958], Leslie [1968], Anderson et al. [1999], Himpel et al. [2008], De Luca et al. [2013])

Continuum configurations of a LCE with orientational loads



1. Orientation mapping

$$\chi : \mathscr{B}_0 \times \mathscr{T} \to \mathbb{R}^{n_{\dim}} \text{ with}$$
$$\chi(X, 0) = n_0(X) \text{ and } n_0 \cdot n_0 = 1$$

2. Orientation tensor

$$F_\chi := \chi \otimes n_0 \qquad n_t = F_\chi \, n_0$$

3. Orient. deformation tensor

$$C_\chi := F^t \, g \, F_\chi = F^t \, g_\chi \, F$$

4. Distorsion tensor

$$K_\chi := F^t \, g \, G_\chi = F^t \, g_K \, F$$

5. Orient. velocity vector

$$v_\chi(X, t) := \dot{\chi}(X, t) = \dot{n}_t$$

6. Orient. momentum vector

$$p_\chi := \rho_0 \left[(l_\chi^2 - l_0^2) A_0 + l_0^2 \, I \right] v_\chi$$
$$A_0 := n_0 \otimes n_0$$

Free energy functions as stress potentials

(cp. Frank [1958], Leslie [1968], Warner et al. [1993], Anderson et al. [1999], Himpel et al. [2008], De Luca et al. [2013])

Free energy associated with orientational deformations

1. Interactive free energy (motivated by Anderson et al. [1999], Himpel et al. [2008])

$$\Psi_i(\boldsymbol{F}^t\,\boldsymbol{g}\,\boldsymbol{\chi}) \equiv \Psi^{\text{ori}}(\boldsymbol{C}_\chi) := \hat{\Psi}^{\text{ori}}(I_1^{\text{ori}}, J_2^{\text{ori}})$$

2. Orientational invariants

$$I_1^{\text{ori}} := \boldsymbol{C}_\chi \boldsymbol{A}_0 : \boldsymbol{G}^{-1} \qquad J_2^{\text{ori}} := \boldsymbol{C}_\chi \boldsymbol{A}_0 : \boldsymbol{C}_\chi \boldsymbol{A}_0$$

Free energy associated with distorsions of the orientation field

1. Frank free energy (motivated by Frank [1958], Leslie [1968], Anderson et al. [1999])

$$\Psi^{\text{dis}}(\boldsymbol{K}_\chi) := \hat{\Psi}^{\text{dis}}(I_1^{\text{dis}}, J_2^{\text{dis}})$$

2. Distorsional invariants

$$I_1^{\text{dis}} := (\boldsymbol{K}_\chi - \text{Grad}[\boldsymbol{n}_0]) : \boldsymbol{G}^{-1} \qquad J_2^{\text{dis}} := (\boldsymbol{K}_\chi - \text{Grad}[\boldsymbol{n}_0]) : (\boldsymbol{K}_\chi - \text{Grad}[\boldsymbol{n}_0])$$

Free energy associated with non-isothermal elastic deformations

1. Isotropic compressible free energy (see e.g. Warner et al. [1993], Anderson et al. [1999])

$$\Psi^{\text{ela}}(\boldsymbol{C}, \Theta) := \hat{\Psi}^{\text{ela}}(I_1^{\text{ela}}, J_2^{\text{ela}}, I_3^{\text{ela}}, \Theta)$$

2. Deformation invariants

$$I_1^{\text{ela}} := \boldsymbol{C} : \boldsymbol{G}^{-1} \qquad J_2^{\text{ela}} := \boldsymbol{C} : \boldsymbol{C} \qquad I_3^{\text{ela}} := \det[\boldsymbol{C}]$$

Reorientation with drilling degrees of freedom

Reorientation modelled as dissipative process (cp. Garikipati et al. [2006])

1. Clausius-Planck inequality

$$\mathcal{D}_{\chi}^{\text{int}} := \boldsymbol{N}_{\chi} : \boldsymbol{g}\,\dot{\boldsymbol{F}} - \dot{\Psi}^{\text{ori}}(\boldsymbol{C}_{\chi}) \equiv \left[\boldsymbol{N}_{\chi} - \boldsymbol{F}_{\chi}\,\boldsymbol{S}_{\chi}^{t}\right] : \boldsymbol{g}\,\dot{\boldsymbol{F}} - \boldsymbol{F}\,\boldsymbol{S}_{\chi} : \boldsymbol{g}\,\dot{\boldsymbol{F}}_{\chi} \geq 0$$

2. Normalized orientation vectors guaranteed by drilling degrees of freedom

$$\mathbb{I}^{\text{skw}} : \boldsymbol{g}\,\dot{\boldsymbol{F}}_{\chi}\,\boldsymbol{F}_{\chi}^{-1} = \boldsymbol{\epsilon}\cdot\dot{\boldsymbol{\alpha}} \qquad \dot{\boldsymbol{\alpha}} := \dot{\alpha}^{k}\,\boldsymbol{g}_{k} \circ \boldsymbol{\varphi}(\boldsymbol{X}, t)$$

3. Reorientation dissipation

$$\mathcal{D}_{\chi}^{\text{int}} := \left[\boldsymbol{N}_{\chi} - \boldsymbol{F}_{\chi}\,\boldsymbol{S}_{\chi}^{t}\right] : \boldsymbol{g}\,\dot{\boldsymbol{F}} - \boldsymbol{\tau}_{\chi} : \boldsymbol{\epsilon}\cdot\dot{\boldsymbol{\alpha}} \geq 0$$

4. Coleman-Noll procedure

$$\boldsymbol{N}_{\chi} := \boldsymbol{F}_{\chi}\,\boldsymbol{S}_{\chi}^{t} \qquad \boldsymbol{\tau}_{\chi} := \boldsymbol{F}\,\boldsymbol{S}_{\chi}\,\boldsymbol{F}_{\chi}^{t}$$

Reorientation equations

1. Orientational non-equilibrium stress equation (solved on element level)

$$\boxed{-\frac{1}{2}\,\boldsymbol{\epsilon} : \boldsymbol{\tau}_{\chi} = \boldsymbol{\Sigma}_{\chi}} \qquad \boldsymbol{\Sigma}_{\chi} = V_{\chi}\,\dot{\boldsymbol{\alpha}} \qquad \mathcal{D}_{\chi}^{\text{int}} := 2\,\boldsymbol{\Sigma}_{\chi}\cdot\dot{\boldsymbol{\alpha}} \geq 0$$

2. Global orientation equation with Dirichlet boundary conditions in general

$$\boxed{\dot{\boldsymbol{\chi}} = -\boldsymbol{\epsilon}\cdot\dot{\boldsymbol{\alpha}}\cdot\boldsymbol{\chi}}$$

Variational-based weak formulation (I)

Principle of virtual power extended to mixed fields

1. Incremental principle of virtual power

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}(\dot{\tilde{\boldsymbol{U}}}_1, \ldots \dot{\tilde{\boldsymbol{U}}}_s, \tilde{\boldsymbol{V}}_1, \ldots \tilde{\boldsymbol{V}}_p)\, \mathrm{d}t = 0$$

2. Total virtual power of deformation $\boldsymbol{\varphi}$, temperature Θ and orientation $\boldsymbol{\chi}$

$$\delta_* \dot{\mathcal{H}} := \delta_* \mathcal{P}_\varphi + \delta_* \mathcal{P}_\Theta + \delta_* \mathcal{P}_\chi \qquad \mathcal{H} := \mathcal{T} + \Pi^{\mathrm{int}} + \Pi^{\mathrm{ext}}$$

Virtual power associated with the motion (I)

1. Virtual power of motion

$$\delta_* \mathcal{P}_\varphi := \delta_* \dot{\mathcal{T}}_\varphi(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) + \delta_* \dot{\Pi}_\varphi^{\mathrm{ext}}(\dot{\boldsymbol{\varphi}}, \tilde{\boldsymbol{R}}) + \delta_* \dot{\Pi}_\varphi^{\mathrm{int}}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{C}}}, \tilde{\boldsymbol{P}}, \tilde{\boldsymbol{S}})$$

2. Path-independent virtual kinetic power

$$\delta_* \dot{\mathcal{T}}_\varphi(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) := \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{v}} \cdot [\rho_0\, \boldsymbol{v} - \boldsymbol{p}]\, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{p}} \cdot [\dot{\boldsymbol{\varphi}} - \boldsymbol{v}]\, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \dot{\boldsymbol{p}}\, \mathrm{d}V$$

3. Path-(in)dependent virtual external power

$$\begin{aligned} \delta_* \dot{\Pi}_\varphi^{\mathrm{ext}}(\dot{\boldsymbol{\varphi}}, \tilde{\boldsymbol{R}}) := & - \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \boldsymbol{B}\, \mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \bar{\boldsymbol{T}}\, \mathrm{d}A \\ & - \int_{\partial_\varphi \mathscr{B}_0} \delta_* \tilde{\boldsymbol{R}} \cdot [\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}]\, \mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \tilde{\boldsymbol{R}}\, \mathrm{d}A \end{aligned}$$

Variational-based weak formulation (II)

Virtual power associated with the motion (II)

Path-independent virtual internal power $\delta_* \dot{\Pi}^{\rm int}_{\varphi}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{C}}}, \tilde{\boldsymbol{P}}, \tilde{\boldsymbol{S}}) := \delta_* \mathcal{P}^{\rm int}_{\varphi}$

$$\delta_* \mathcal{P}^{\rm int}_{\varphi} := \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{P}} : \left[\mathrm{Grad}[\dot{\boldsymbol{\varphi}}] - \dot{\tilde{\boldsymbol{F}}} \right] \mathrm{d}V + \frac{1}{2} \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{S}} : \left[\frac{\partial}{\partial t} \left(\tilde{\boldsymbol{F}}^t \tilde{\boldsymbol{F}} \right) - \dot{\tilde{\boldsymbol{C}}} \right] \mathrm{d}V$$
$$+ \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{C}}} : \left[\frac{\partial \Psi}{\partial \tilde{\boldsymbol{C}}} - \frac{1}{2} \tilde{\boldsymbol{S}} \right] \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{F}}} : \left[\tilde{\boldsymbol{F}} \tilde{\boldsymbol{S}} - \tilde{\boldsymbol{P}} \right] \mathrm{d}V + \int_{\mathscr{B}_0} \tilde{\boldsymbol{P}} : \mathrm{Grad}[\delta_* \dot{\boldsymbol{\varphi}}] \, \mathrm{d}V$$

Virtual power associated with the thermal evolution (I)

1. Virtual power of thermal evolution

$$\delta_* \mathcal{P}_{\Theta} := \delta_* \dot{\Pi}^{\rm ext}_{\Theta}(\dot{\Theta}, \tilde{\Theta}, \tilde{\lambda}, \tilde{h}) + \delta_* \dot{\Pi}^{\rm int}_{\Theta}(\dot{\Theta}, \dot{\eta}, \tilde{\Theta})$$

2. Path-dependent virtual external power

$$\delta_* \dot{\Pi}^{\rm ext}_{\Theta}(\dot{\Theta}, \tilde{\Theta}, \tilde{\lambda}, \tilde{h}) := \int_{\mathscr{B}_0} \delta_* \tilde{\Theta} \, \frac{D^{\rm tot}}{\Theta} \, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{\Theta} \, \mathrm{Grad}[\delta_* \tilde{\Theta}] \cdot \boldsymbol{Q} \, \mathrm{d}V$$
$$+ \int_{\partial_Q \mathscr{B}_0} \delta_* \tilde{\Theta} \, \frac{\bar{Q}}{\Theta} \, \mathrm{d}A + \int_{\partial_{\Theta} \mathscr{B}_0} \delta_* \tilde{\Theta} \, \tilde{\lambda} \, \mathrm{d}A - \int_{\partial_{\dot{\Theta}} \mathscr{B}_0} \delta_* \dot{\Theta} \, \tilde{h} \, \mathrm{d}A$$
$$+ \int_{\partial_{\Theta} \mathscr{B}_0} \delta_* \tilde{\lambda} \left[\tilde{\Theta} - \Theta_{\infty} \right] \mathrm{d}A - \int_{\partial_{\dot{\Theta}} \mathscr{B}_0} \delta_* \tilde{h} \left[\dot{\Theta} - \dot{\bar{\Theta}} \right] \mathrm{d}A$$

with

$$D^{\rm tot} := -\frac{1}{\Theta} \, \mathrm{Grad}[\tilde{\Theta}] \cdot \boldsymbol{Q} + 2 \, \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{\Sigma}_{\chi} \qquad \text{and} \qquad \boldsymbol{Q} := -k_0 \, \det[\tilde{\boldsymbol{F}}] \, \tilde{\boldsymbol{C}}^{-1} \, \mathrm{Grad}[\Theta]$$

Variational-based weak formulation (III)

Virtual power associated with the thermal evolution (II)

Path-independent virtual internal power

$$\delta_* \dot{\Pi}^{\text{int}}_{\Theta}(\dot{\Theta}, \dot{\eta}, \tilde{\Theta}) := \int_{\mathscr{B}_0} \delta_* \dot{\Theta} \left(\frac{\partial \Psi}{\partial \Theta} + \eta \right) \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\eta} \left(\Theta - \tilde{\Theta} \right) \mathrm{d}V - \int_{\mathscr{B}_0} \delta_* \tilde{\Theta} \, \dot{\eta} \, \mathrm{d}V$$

Virtual power associated with the reorientation (I)

1. Virtual power of reorientation

$$\delta_* \mathcal{P}_{\chi} := \delta_* \dot{\mathcal{T}}_{\chi}(\dot{\boldsymbol{\chi}}, \dot{\boldsymbol{v}}_{\chi}, \dot{\boldsymbol{p}}_{\chi}) + \delta_* \mathcal{P}^{\text{ext}}_{\chi} + \delta_* \mathcal{P}^{\text{int}}_{\chi}$$

2. Path-independent virtual kinetic power

$$\delta_* \dot{\mathcal{T}}_{\chi}(\dot{\boldsymbol{\chi}}, \dot{\boldsymbol{v}}_{\chi}, \dot{\boldsymbol{p}}_{\chi}) := \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{v}}_{\chi} \cdot \left(\rho_0 \left[(l_{\chi}^2 - l_0^2) \boldsymbol{A}_0 + l_0^2 \, \boldsymbol{I} \right] \boldsymbol{v}_{\chi} - \boldsymbol{p}_{\chi} \right) \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{p}}_{\chi} \cdot [\dot{\boldsymbol{\chi}} - \boldsymbol{v}_{\chi}] \, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \dot{\boldsymbol{p}}_{\chi} \, \mathrm{d}V$$

3. Path-dependent virtual external power

$$\delta_* \dot{\Pi}^{\text{ext}}_{\chi}(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\chi}}, \tilde{\boldsymbol{Z}}, \tilde{\boldsymbol{\tau}}_n, \tilde{\boldsymbol{\nu}}) =: \delta_* \mathcal{P}^{\text{ext}}_{\chi}$$

where

$$\delta_* \mathcal{P}^{\text{ext}}_{\chi} := -\int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \boldsymbol{B}_{\chi} \, \mathrm{d}V - \int_{\partial_W \mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \bar{\boldsymbol{W}} \, \mathrm{d}A - \int_{\partial_{\chi} \mathscr{B}_0} \delta_* \tilde{\boldsymbol{Z}} \cdot [\dot{\boldsymbol{\chi}} - \dot{\bar{\boldsymbol{\chi}}}] \, \mathrm{d}A - \int_{\partial_{\chi} \mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \tilde{\boldsymbol{Z}} \, \mathrm{d}A - \int_{\partial_{\chi} \mathscr{B}_0} 2 \, \delta_* \tilde{\boldsymbol{\tau}}_n \cdot \tilde{\boldsymbol{\nu}} \, \mathrm{d}A - \int_{\partial_{\chi} \mathscr{B}_0} 2 \, \delta_* \tilde{\boldsymbol{\nu}} \cdot \tilde{\boldsymbol{\tau}}_n \, \mathrm{d}A + \int_{\mathscr{B}_0} 2 \, \delta_* \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{\Sigma}_{\chi} \, \mathrm{d}V$$

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Variational-based weak formulation (IV)

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Virtual power associated with the reorientation (II)

Path-independent virtual internal power

$$\delta_* \dot{\Pi}_\chi^{\rm int}(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\chi}}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{F}}}_\chi, \dot{\tilde{\boldsymbol{G}}}_\chi, \dot{\tilde{\boldsymbol{C}}}_\chi, \dot{\tilde{\boldsymbol{K}}}_\chi, \tilde{\boldsymbol{\tau}}_n, \tilde{\boldsymbol{P}}_\chi, \tilde{\boldsymbol{P}}_K, \tilde{\boldsymbol{S}}_\chi, \tilde{\boldsymbol{S}}_K) := \delta_* \mathcal{P}_\chi^{\rm int}$$

where

$$\begin{aligned}
\delta_* \mathcal{P}_\chi^{\rm int} := & \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{F}}} : \left[\tilde{\boldsymbol{F}}_\chi \tilde{\boldsymbol{S}}_\chi^t + \tilde{\boldsymbol{G}}_\chi \tilde{\boldsymbol{S}}_K^t\right] \mathrm{d}V + \int_{\mathscr{B}_0} 2\, \delta_* \tilde{\boldsymbol{\tau}}_n \cdot [\dot{\boldsymbol{\chi}} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{\chi}]\, \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{P}}_\chi : \left[\dot{\boldsymbol{\chi}} \otimes \boldsymbol{n}_0 - \dot{\tilde{\boldsymbol{F}}}_\chi\right] \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{P}}_K : \left[\mathrm{Grad}[\dot{\boldsymbol{\chi}}] - \dot{\tilde{\boldsymbol{G}}}_\chi\right] \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{S}}_\chi : \left[\frac{\partial}{\partial t}\left(\tilde{\boldsymbol{F}}^t \tilde{\boldsymbol{F}}_\chi\right) - \dot{\tilde{\boldsymbol{C}}}_\chi\right] \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{S}}_K : \left[\frac{\partial}{\partial t}\left(\tilde{\boldsymbol{F}}^t \tilde{\boldsymbol{G}}_\chi\right) - \dot{\tilde{\boldsymbol{K}}}_\chi\right] \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{C}}}_\chi : \left[\frac{\partial \Psi}{\partial \tilde{\boldsymbol{C}}_\chi} - \tilde{\boldsymbol{S}}_\chi\right] \mathrm{d}V + \int_{\mathrm{B}_0} \delta_* \dot{\tilde{\boldsymbol{K}}}_\chi : \left[\frac{\partial \Psi}{\partial \tilde{\boldsymbol{K}}_\chi} - \tilde{\boldsymbol{S}}_K\right] \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{F}}}_\chi : [\tilde{\boldsymbol{F}}\, \tilde{\boldsymbol{S}}_\chi - \tilde{\boldsymbol{P}}_\chi]\, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{G}}}_\chi : [\tilde{\boldsymbol{F}}\, \tilde{\boldsymbol{S}}_K - \tilde{\boldsymbol{P}}_K]\, \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \tilde{\boldsymbol{P}}_\chi : [\delta_* \dot{\boldsymbol{\chi}} \otimes \boldsymbol{n}_0]\, \mathrm{d}V + \int_{\mathrm{B}_0} \tilde{\boldsymbol{P}}_K : \mathrm{Grad}[\delta_* \dot{\boldsymbol{\chi}}]\, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \tilde{\boldsymbol{S}}_\chi : \tilde{\boldsymbol{F}}^t (\boldsymbol{\epsilon} \cdot \boldsymbol{\alpha})\, \tilde{\boldsymbol{F}}_\chi\, \mathrm{d}V \\
& + \int_{\mathscr{B}_0} \left[\frac{1}{2} \boldsymbol{\epsilon} : \boldsymbol{\tau}_\chi - \tilde{\boldsymbol{\tau}}_n \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{\chi}\right] \cdot 2\, \delta_* \dot{\boldsymbol{\alpha}}\, \mathrm{d}V + \int_{\mathscr{B}_0} 2\, \tilde{\boldsymbol{\tau}}_n \cdot \delta_* \dot{\boldsymbol{\chi}}\, \mathrm{d}V
\end{aligned}$$

Total virtual power in the incremental principle

$$\delta_* \dot{\mathcal{H}} := \delta_* \mathcal{P}_\varphi + \delta_* \mathcal{P}_\theta + \delta_* \dot{\mathcal{T}}_\chi(\dot{\boldsymbol{\chi}}, \dot{\boldsymbol{v}}_\chi, \dot{\boldsymbol{p}}_\chi) + \delta_* \mathcal{P}_\chi^{\rm ext} + \delta_* \mathcal{P}_\chi^{\rm int}$$

Global weak forms of motion with reorientation

Weak balance of linear momentum

$$\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\varphi}} \cdot [\dot{\boldsymbol{p}} - \boldsymbol{B}]\,\mathrm{d}V\,\mathrm{d}t - \int_{\mathscr{T}_n}\int_{\partial_T\mathscr{B}_0} \delta_*\dot{\boldsymbol{\varphi}} \cdot \bar{\boldsymbol{T}}\,\mathrm{d}A\,\mathrm{d}t$$

$$+ \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \mathrm{Grad}[\delta_*\dot{\boldsymbol{\varphi}}] : \tilde{\boldsymbol{P}}\,\mathrm{d}V\,\mathrm{d}t = \int_{\mathscr{T}_n}\int_{\partial_\varphi\mathscr{B}_0} \delta_*\dot{\boldsymbol{\varphi}} \cdot \tilde{\boldsymbol{R}}\,\mathrm{d}A\,\mathrm{d}t$$

Weak balance of thermal momentum (cf. Romero [2010])

$$\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \delta_*\tilde{\Theta}\left[\dot{\eta} - \frac{D^{\mathrm{tot}}}{\Theta}\right]\mathrm{d}V\,\mathrm{d}t - \int_{\mathscr{T}_n}\int_{\partial_Q\mathscr{B}_0} \delta_*\tilde{\Theta}\,\frac{\bar{Q}}{\Theta}\,\mathrm{d}A\,\mathrm{d}t$$

$$- \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \mathrm{Grad}[\delta_*\tilde{\Theta}] \cdot \frac{1}{\Theta}\,\boldsymbol{Q}\,\mathrm{d}V\,\mathrm{d}t = \int_{\mathscr{T}_n}\int_{\partial_\Theta\mathscr{B}_0} \delta_*\tilde{\Theta}\,\tilde{\lambda}\,\mathrm{d}A\,\mathrm{d}t$$

Weak balance of orientational momentum

$$\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\chi}} \cdot [\dot{\boldsymbol{p}}_\chi + 2\,\tilde{\boldsymbol{\tau}}_n - \boldsymbol{B}_\chi]\,\mathrm{d}V\,\mathrm{d}t - \int_{\mathscr{T}_n}\int_{\partial_W\mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \delta_*\dot{\boldsymbol{\chi}}\,\mathrm{d}A\,\mathrm{d}t$$

$$+ \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \tilde{\boldsymbol{P}}_K : \mathrm{Grad}[\delta_*\dot{\boldsymbol{\chi}}]\,\mathrm{d}V\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \tilde{\boldsymbol{P}}_\chi : [\delta_*\dot{\boldsymbol{\chi}} \otimes \boldsymbol{n}_0]\,\mathrm{d}V\,\mathrm{d}t = \int_{\mathscr{T}_n}\int_{\partial_\chi\mathscr{B}_0} \tilde{\boldsymbol{Z}} \cdot \delta_*\dot{\boldsymbol{\chi}}\,\mathrm{d}A\,\mathrm{d}t$$

Weak balance of orientation rate

$$\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} 2\,\delta_*\tilde{\boldsymbol{\tau}}_n \cdot [\dot{\boldsymbol{\chi}} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{\chi}]\,\mathrm{d}V\,\mathrm{d}t = \int_{\mathscr{T}_n}\int_{\partial_\chi\mathscr{B}_0} 2\,\delta_*\tilde{\boldsymbol{\tau}}_n \cdot \tilde{\boldsymbol{\nu}}\,\mathrm{d}A\,\mathrm{d}t$$

Balance laws of the weak formulation (I)

Energy and momentum functions of the LCE extended continuum

Kinetic energy $\mathcal{T}(t) := \int_{\mathscr{B}_0} \frac{1}{2}\,\boldsymbol{v}\cdot\boldsymbol{p}\,\mathrm{d}V$	Kinetic energy of orientation $\mathcal{T}_\chi(t) := \int_{\mathscr{B}_0} \frac{1}{2}\,\boldsymbol{v}_\chi\cdot\boldsymbol{p}_\chi\,\mathrm{d}V$	Potential energy $\Pi^{\mathrm{int}}(t) := \int_{\mathscr{B}_0} \Psi\,\mathrm{d}V$
Linear momentum $\boldsymbol{L}(t) := \int_{\mathscr{B}_0} \boldsymbol{p}\,\mathrm{d}V$	Angular momentum $\boldsymbol{J}(t) := \int_{\mathscr{B}_0} \boldsymbol{\varphi}\times\boldsymbol{p}\,\mathrm{d}V$	Momentum of orientation $\boldsymbol{L}_\chi(t) := \int_{\mathscr{B}_0} \boldsymbol{p}_\chi\,\mathrm{d}V$
Moment of momentum $\boldsymbol{J}_\chi(t) := \int_{\mathscr{B}_0} \boldsymbol{\chi}\times\boldsymbol{p}_\chi\,\mathrm{d}V$	Reorientation function $\mathcal{C}^{\mathrm{ori}}(t) := \int_{\mathscr{B}_0} [\lVert\boldsymbol{\chi}\rVert^2 - 1]\,\mathrm{d}V$	Thermal energy $\Pi^{\mathrm{the}}(t) := \int_{\mathscr{B}_0} \Theta\,\eta\,\mathrm{d}V$
Entropy $\mathcal{S}(t) := \int_{\mathscr{B}_0} \eta\,\mathrm{d}V$	Total energy $\mathcal{H} := \mathcal{T} + \mathcal{T}_\chi + \Pi^{\mathrm{int}} + \Pi^{\mathrm{the}} + \Pi^{\mathrm{ext}}$	Lyapunov function $\mathcal{F} := \mathcal{H} - \Theta_\infty\,\mathcal{S}$

Linear momentum

(symmetry of virtual translations)

$$\boldsymbol{L}(t_{n+1}) - \boldsymbol{L}(t_n) = \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \boldsymbol{B}\,\mathrm{d}V\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_T\mathscr{B}_0} \bar{\boldsymbol{T}}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi\mathscr{B}_0} \tilde{\boldsymbol{R}}\,\mathrm{d}A\,\mathrm{d}t$$

Orientational momentum

(symmetry of virtual orientations)

$$\boldsymbol{L}_\chi(t_{n+1}) - \boldsymbol{L}_\chi(t_n) = \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} [\boldsymbol{B}_\chi - 2\,\tilde{\tau}_n - \tilde{\boldsymbol{P}}_\chi\,\boldsymbol{n}_0]\,\mathrm{d}V\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_W\mathscr{B}_0} \bar{\boldsymbol{W}}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\chi\mathscr{B}_0} \tilde{\boldsymbol{Z}}\,\mathrm{d}A\,\mathrm{d}t$$

Balance laws of the weak formulation (II)

Thermal momentum (cf. Romero [2010])

$$\mathcal{S}(t_{n+1}) - \mathcal{S}(t_n) = \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \frac{D^{\mathrm{tot}}}{\Theta}\,\mathrm{d}V\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_Q\mathscr{B}_0} \frac{\bar{Q}}{\Theta}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\Theta\mathscr{B}_0} \tilde{\lambda}\,\mathrm{d}A\,\mathrm{d}t$$

Angular momentum (symmetry of virtual rotations)

$$\begin{aligned}\boldsymbol{J}(t_{n+1}) - \boldsymbol{J}(t_n) = &\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \boldsymbol{\varphi}\times\boldsymbol{B}\,\mathrm{d}V\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_T\mathscr{B}_0} \boldsymbol{\varphi}\times\bar{\boldsymbol{T}}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi\mathscr{B}_0} \boldsymbol{\varphi}\times\tilde{\boldsymbol{R}}\,\mathrm{d}A\,\mathrm{d}t \\ &+ \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} [\tilde{\boldsymbol{F}}_\chi\tilde{\boldsymbol{S}}_\chi^t + \tilde{\boldsymbol{G}}_\chi\tilde{\boldsymbol{S}}_K^t]\times\tilde{\boldsymbol{F}}\,\mathrm{d}V\mathrm{d}t\end{aligned}$$

Moment of orientational momentum (symmetry of virtual reorientations)

$$\begin{aligned}\boldsymbol{J}_\chi(t_{n+1}) - \boldsymbol{J}_\chi(t_n) = &\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \boldsymbol{\chi}\times\boldsymbol{B}_\chi\,\mathrm{d}V\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_W\mathscr{B}_0} \boldsymbol{\chi}\times\bar{\boldsymbol{W}}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\chi\mathscr{B}_0} \boldsymbol{\chi}\times\tilde{\boldsymbol{Z}}\,\mathrm{d}A\,\mathrm{d}t \\ &- \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} [\tilde{\boldsymbol{F}}_\chi\tilde{\boldsymbol{S}}_\chi^t + \tilde{\boldsymbol{G}}_\chi\tilde{\boldsymbol{S}}_K^t]\times\tilde{\boldsymbol{F}}\,\mathrm{d}V\mathrm{d}t - \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \boldsymbol{\chi}\times 2\,\tilde{\boldsymbol{\tau}}_n\,\mathrm{d}V\mathrm{d}t\end{aligned}$$

Kinetic energy of motion (symmetry of virtual time shifts)

$$\begin{aligned}\mathcal{T}(t_{n+1}) - \mathcal{T}(t_n) = &\int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \boldsymbol{B}\cdot\dot{\boldsymbol{\varphi}}\,\mathrm{d}V\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_T\mathscr{B}_0} \bar{\boldsymbol{T}}\cdot\dot{\boldsymbol{\varphi}}\,\mathrm{d}A\,\mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi\mathscr{B}_0} \tilde{\boldsymbol{R}}\cdot\dot{\boldsymbol{\varphi}}\,\mathrm{d}A\,\mathrm{d}t \\ &- \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} [\tilde{\boldsymbol{S}} : \dot{\boldsymbol{F}}^t\tilde{\boldsymbol{F}} + \tilde{\boldsymbol{S}}_\chi : \dot{\boldsymbol{F}}^t\tilde{\boldsymbol{F}}_\chi + \tilde{\boldsymbol{S}}_K : \dot{\boldsymbol{F}}^t\tilde{\boldsymbol{G}}_\chi]\,\mathrm{d}V\mathrm{d}t\end{aligned}$$

Balance laws of the weak formulation (III)

Kinetic energy of orientation

(symmetry of virtual time shifts)

$$\mathcal{T}_\chi(t_{n+1}) - \mathcal{T}_\chi(t_n) = \int_{\mathcal{T}_n}\int_{\mathcal{B}_0} \boldsymbol{B}_\chi \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}V\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\partial_\varphi \mathcal{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\partial_\varphi \mathcal{B}_0} \tilde{\boldsymbol{Z}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t - \int_{\mathcal{T}_n}\int_{\mathcal{B}_0} [\tilde{\boldsymbol{S}}_\chi : \tilde{\boldsymbol{F}}^t \left(\dot{\tilde{\boldsymbol{F}}}_\chi + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \cdot \tilde{\boldsymbol{F}}_\chi\right) + \tilde{\boldsymbol{S}}_K : \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}}_\chi + D_\chi^{\mathrm{int}}]\, \mathrm{d}V\, \mathrm{d}t$$

Thermal energy

(symmetry of virtual time shifts)

$$\Pi^{\mathrm{the}}(t_{n+1}) - \Pi^{\mathrm{the}}(t_n) = \int_{\mathcal{T}_n}\int_{\mathcal{B}_0} \left[-\frac{\partial \Psi}{\partial \Theta}\dot{\Theta} + D_\chi^{\mathrm{int}}\right] \mathrm{d}V\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\partial_{\dot{\Theta}} \mathcal{B}_0} \dot{\Theta}\, \tilde{h}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\partial_\Theta \mathcal{B}_0} \Theta\, \tilde{\lambda}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\partial_Q \mathcal{B}_0} \bar{Q}\, \mathrm{d}A\, \mathrm{d}t$$

Potential energy

(symmetry of virtual time shifts)

$$\Pi(t_{n+1}) - \Pi(t_n) = \int_{\mathcal{T}_n}\int_{\mathcal{B}_0} [\tilde{\boldsymbol{S}}_\chi : \frac{\partial}{\partial t}\left(\tilde{\boldsymbol{F}}^t \tilde{\boldsymbol{F}}_\chi\right) + \tilde{\boldsymbol{S}}_K : \frac{\partial}{\partial t}\left(\tilde{\boldsymbol{F}}^t \tilde{\boldsymbol{G}}_\chi\right) + \tilde{\boldsymbol{S}}_\chi : \tilde{\boldsymbol{F}}^t (\boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}}) \tilde{\boldsymbol{F}}_\chi]\, \mathrm{d}V\, \mathrm{d}t + \int_{\mathcal{T}_n}\int_{\mathcal{B}_0} \left[\tilde{\boldsymbol{S}} : \dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{F}} - \boldsymbol{B} \cdot \dot{\boldsymbol{\varphi}} - \boldsymbol{B}_\chi \cdot \dot{\boldsymbol{\chi}}\right] \mathrm{d}V\, \mathrm{d}t$$

Path-independent volume dead loads

$$\Pi^{\mathrm{ext}}(t) := -\int_{\mathcal{B}_0} \boldsymbol{B} \cdot \boldsymbol{\varphi}\, \mathrm{d}V\, \mathrm{d}t - \int_{\mathcal{B}_0} \boldsymbol{B}_\chi \cdot \boldsymbol{\chi}\, \mathrm{d}V$$

Balance laws of the weak formulation (IV)

Total energy (cf. Romero [2010])

$$\begin{aligned}\mathcal{H}(t_{n+1}) - \mathcal{H}(t_n) &= \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \tilde{\boldsymbol{R}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \tilde{\boldsymbol{Z}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t \\ &- \int_{\mathscr{T}_n}\int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t \\ &- \int_{\mathscr{T}_n}\int_{\partial_\Theta \mathscr{B}_0} \tilde{\lambda}\, \Theta\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_{\dot{\Theta}} \mathscr{B}_0} \tilde{h}\, \dot{\Theta}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_Q \mathscr{B}_0} \bar{Q}\, \mathrm{d}A\, \mathrm{d}t\end{aligned}$$

Lyapunov function (cf. Romero [2010])

$$\begin{aligned}\mathcal{F}(t_{n+1}) - \mathcal{F}(t_n) &= \int_{\mathscr{T}_n}\int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \tilde{\boldsymbol{R}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A\, \mathrm{d}t \\ &+ \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_\varphi \mathscr{B}_0} \tilde{\boldsymbol{Z}} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}A\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_{\dot{\Theta}} \mathscr{B}_0} \tilde{h}\, \dot{\Theta}\, \mathrm{d}A\, \mathrm{d}t \\ &- \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} \frac{\Theta_\infty}{\Theta}\, D^{\mathrm{tot}}\, \mathrm{d}V\, \mathrm{d}t + \int_{\mathscr{T}_n}\int_{\partial_Q \mathscr{B}_0} \frac{\Theta - \Theta_\infty}{\Theta}\, \bar{Q}\, \mathrm{d}A\, \mathrm{d}t\end{aligned}$$

Reorientation function

$$\mathcal{C}^{\mathrm{ori}}(t_{n+1}) - \mathcal{C}^{\mathrm{ori}}(t_n) \equiv \int_{\mathscr{T}_n}\int_{\mathscr{B}_0} 2\,\boldsymbol{\chi} \cdot \dot{\boldsymbol{\chi}}\, \mathrm{d}V\, \mathrm{d}t = \int_{\mathscr{T}_n}\int_{\partial_\chi \mathscr{B}_0} 2\,\boldsymbol{\chi} \cdot \tilde{\boldsymbol{\nu}}\, \mathrm{d}A\, \mathrm{d}t$$

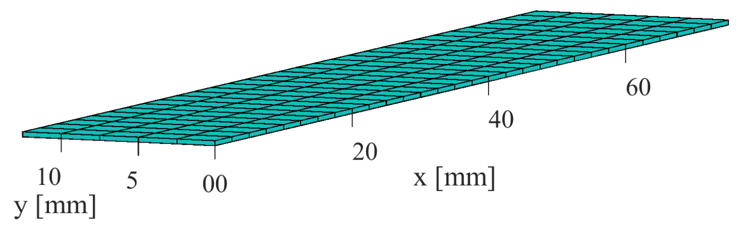
Thin LCE film subject to initial rotation

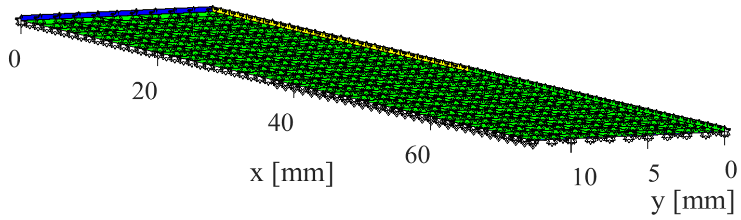
Boundary and initial conditions

121-em with H20-mixed-Bbar

Mesh and boundary conditions

$\boldsymbol{n}_0^A = \boldsymbol{e}_y$, $\boldsymbol{\omega}_0^A = 32\,\boldsymbol{e}_z [1/\mathrm{s}]$





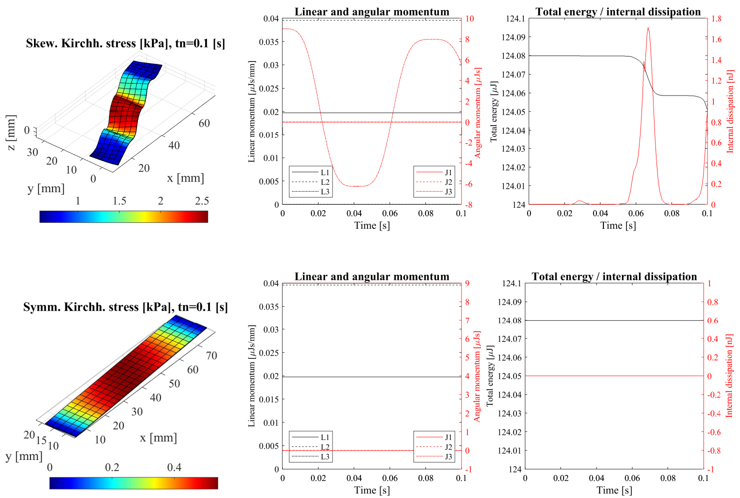
Activated Dirichlet and Neumann boundaries

Green bottom patches as boundary $\partial_\chi \mathcal{B}_0$: Fixed orientation $n_z^A = 0$

Thin LCE film subject to initial rotation

Isothermal dynamical effects of the reorientation

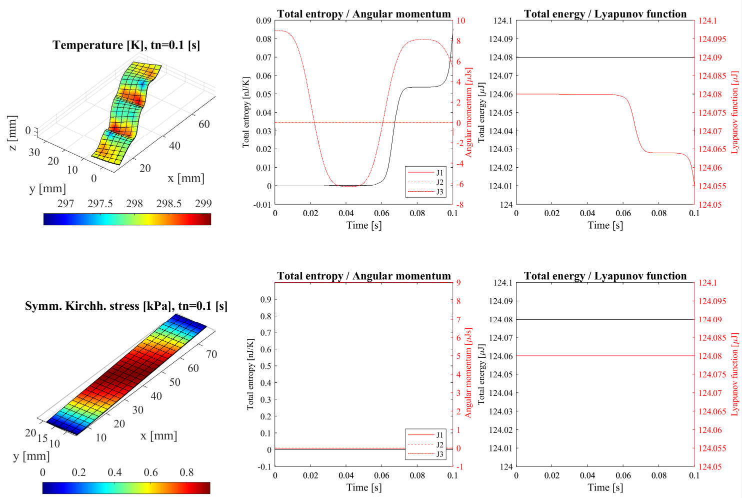
Balance laws with/without reorientation $(\mathrm{E} \approx 0.914\,[\mathrm{MPa}],\ \nu \approx 0.493)$



Thin LCE film subject to initial rotation

Non-isothermal dynamical effects of the reorientation

Balance laws with/without reorientation $(E \approx 0.914\,[\mathrm{MPa}],\ \nu \approx 0.493)$



Thin LCE film subject to initial rotation

Unsteady right-left-rotation due to reorientation

Movie I of a soft film $(\mathrm{E} \approx 0.914\,[\mathrm{MPa}],\ \nu \approx 0.493)$

Thin LCE film subject to initial rotation

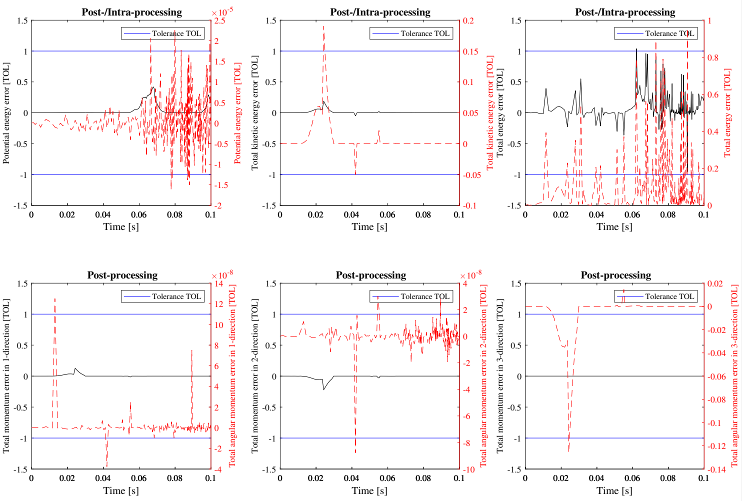
Unsteady right-left-rotation due to reorientation

Movie II of a soft film ($E \approx 0.914\,[\mathrm{MPa}]$, $\nu \approx 0.493$)

Thin LCE film subject to initial rotation

Unsteady right-left-rotation due to reorientation

Balance laws versus time $(E \approx 0.914\,[\mathrm{MPa}],\ \nu \approx 0.493)$



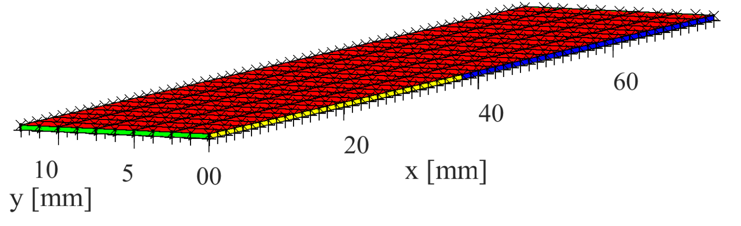
Thin LCE film subject to boundary load

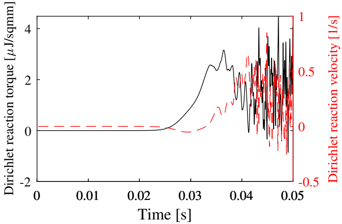
Boundary and initial conditions

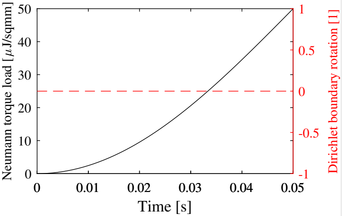
121-em with H20-mixed-Bbar

Boundary conditions and loads

$\boldsymbol{n}_0 = (\boldsymbol{e}_x + \boldsymbol{e}_y)/\sqrt{2}$







Activated Dirichlet and Neumann boundaries

Red top as $\partial_W \mathcal{B}_0$: $W_y^A = -\hat{W}^A(t)$ Green bottom as $\partial_\chi \mathcal{B}_0$: Fixed orientation $n_z^A = 0$

Thin LCE film subject to boundary load

Contraction with folding motion

Movie I of a stiffer film $(\mathrm{E} \approx 9.140\,[\mathrm{MPa}],\ \nu \approx 0.493)$

Thin LCE film subject to boundary load

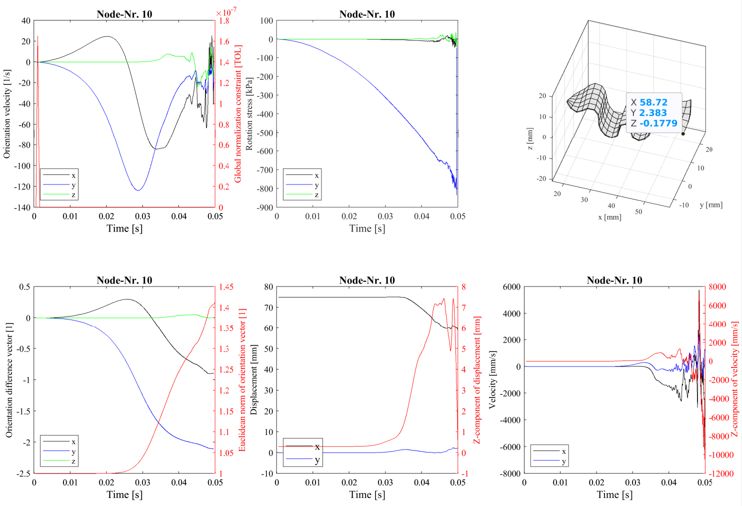
Contraction with folding motion

Movie II of a stiffer film ($E \approx 9.140\,[\mathrm{MPa}]$, $\nu \approx 0.493$)

Thin LCE film subject to boundary load

Contraction with folding motion

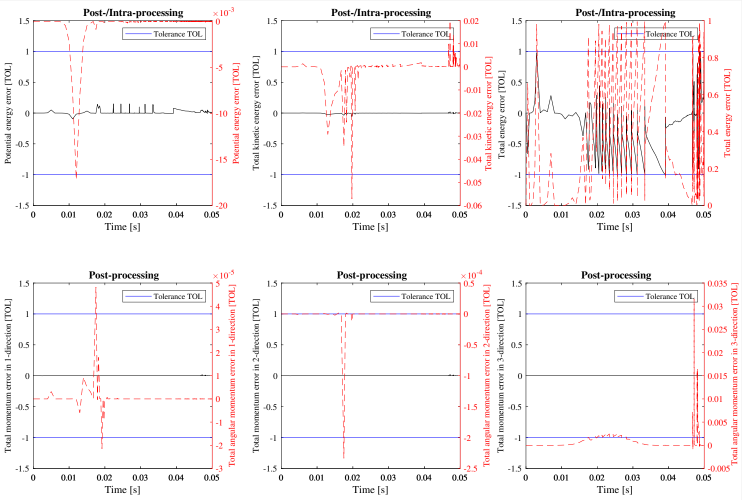
Nodal time evolutions ($E \approx 9.140$ [MPa], $\nu \approx 0.493$)



Thin LCE film subject to boundary load

Contraction with folding motion

Time adaptivity based on balance laws $(E \approx 9.140\,[\mathrm{MPa}],\ \nu \approx 0.493)$



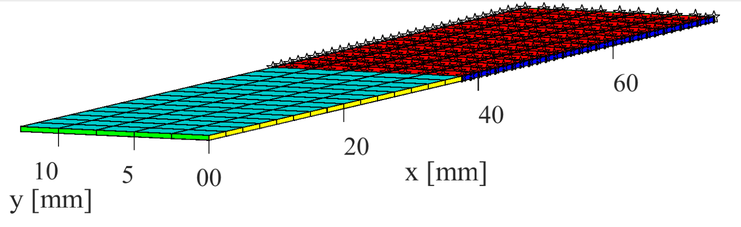
Thin LCE film subject to volume load

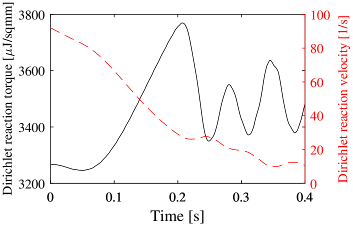
Boundary and initial conditions

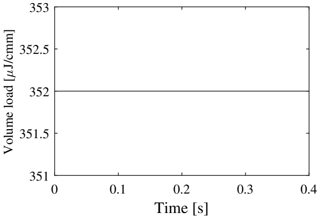
121-em with H20-mixed-Bbar

Boundary conditions and loads

$\boldsymbol{n}_0 = \boldsymbol{e}_x$







Activated Dirichlet and Neumann boundaries

Red top as $\partial_\chi \mathcal{B}_0$: Orientation $\boldsymbol{n}^A = \boldsymbol{n}_0^A$ Blue sides as $\partial_\Theta \mathscr{B}_0$: Temperature $\Theta^A = \Theta_\infty$

Thin LCE film subject to volume load

Steady clockwise rotation due to reorientation

Movie I of the stiff film $(\mathrm{E} \approx 9.140\,[\mathrm{MPa}],\ \nu \approx 0.493)$

Thin LCE film subject to volume load

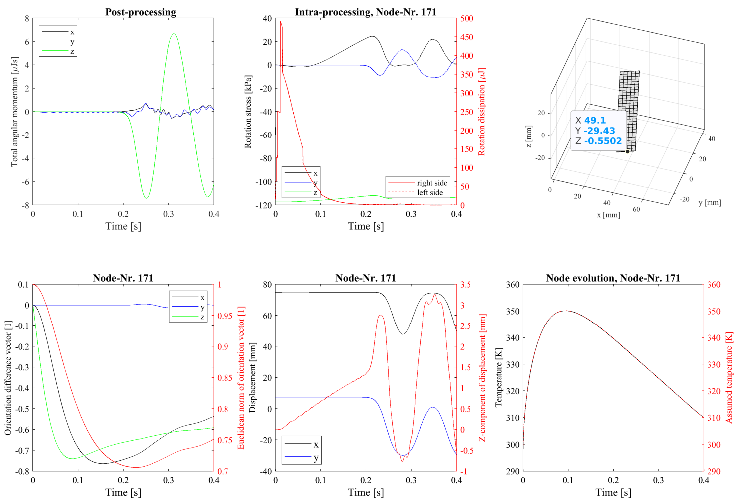
Steady clockwise rotation due to reorientation

Movie II of the stiff film ($\mathrm{E} \approx 9.140\,[\mathrm{MPa}]$, $\nu \approx 0.493$)

Thin LCE film subject to volume load

Steady clockwise rotation due to reorientation

Nodal time evolutions and balance laws $(E \approx 9.140\,[\mathrm{MPa}],\ \nu \approx 0.493)$



Summary

1. Motivation: Non-isothermal simulations of motion actuations
   - with liquid crystal elastomers as actuator material by using of
   - boundary and volume loads (thermal and photochemical fields).
2. Numerical goals: Dynamic finite element simulations
   - with the approach of a mixed finite element method and
   - reorientations by using of drilling degrees of freedom.
3. Numerical strategy: Boundary and volume actuation by
   - introducing an independent global orientation field,
   - formulating local rotations by drilling degrees of freedom,
   - using of local evolution equations for stress-induced motions.
4. Numerical results: Motion actuation with
   - Neumann boundary loads and volume-specific loads
   - activates deformation modes and rigid-body modes.
5. Next step:
   - Adapting orientation loads for simulating thermal actuations