Analysis of Stripe Domains in Nematic LCEs by Means of a Dynamic Numerical Framework

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DFG Deutsche Forschungsgemeinschaft

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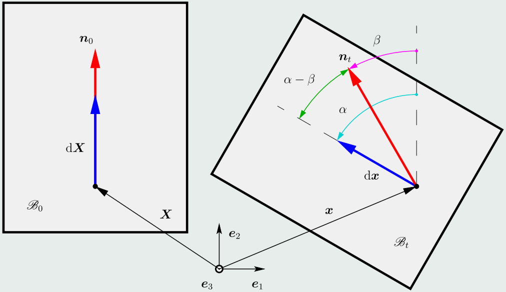
Introduction



[Concas and Groß (2024)]

- Orientation rate $\dot{\boldsymbol{\chi}} = \left(\dot{\boldsymbol{\alpha}} - \dot{\boldsymbol{\beta}}\right) \times \boldsymbol{\chi}$
- rate of rotational degree of freedom for the bulk elastomer $\dot{\boldsymbol{\alpha}} = -\frac{1}{2}\boldsymbol{\epsilon} : \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$
- dissipative rotation of the mesogens depending on the rate $\dot{\boldsymbol{\beta}}$

- semisoft model [Biggins et al. (2008)] with further free energy densities $\psi\left(\boldsymbol{F}, \boldsymbol{\chi}, \boldsymbol{\alpha}, \boldsymbol{\beta}\right)$ being invariants with respect to rotations occurring in the current $\mathscr{B}_t$ and reference configuration $\mathscr{B}_0$
- rotational viscosity [Concas and Groß (2024)] as tensor for penalizing any rotation of the mesogens about axes parallel to the plane geometry
- principle of virtual power for space-time discretization

Goals

- reproducing the stripe domains in liquid crystal elastomer (LCE) films under high strain rate stretch
- preserving mechanical balance laws, i.e. momentum and moment of momentum balances

Energy density formulation

$$\psi = \underbrace{c_1\left(\boldsymbol{I}:\boldsymbol{C}-3-2\log(J)\right)+\frac{\lambda}{2}\left(\left[\log(J)\right]^2+(J-1)^2\right)}_{\text{Neo-Hooke}} + c_3\boldsymbol{n}_0\cdot\boldsymbol{C}\boldsymbol{n}_0$$

$$+\, c_3\left(\boldsymbol{I}:\left(\boldsymbol{n}_t\otimes\boldsymbol{n}_t\right)-1\right)+\underbrace{(c_9+a)\left|\boldsymbol{F}^T\boldsymbol{n}_t\right|^2+(c_{10}-a)\left(\boldsymbol{n}_0\cdot\boldsymbol{F}^T\boldsymbol{n}_t\right)^2}_{\text{[Biggins et al. (2008)]}}$$

$$+\underbrace{c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]^2\left(\boldsymbol{n}_t\cdot\boldsymbol{B}\boldsymbol{n}_t\right)}_{\text{[Warner and Terentjev (2007)]}} + c_{14}\left[\left((\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right)\cdot\boldsymbol{F}\boldsymbol{n}_0\right]^2$$

Material parameters

- $c_1 = \frac{\mu}{2}$
- $c_3 = \frac{\mu(r-1)}{2}$
- $c_9 = \frac{\mu}{2}\left(\frac{1}{r}-1\right)$
- $c_{10} = \frac{\mu}{2}\left(2-\frac{1}{r}-r\right)$
- $a = 0.063\,\frac{\mu}{2}$ [de Luca et al. (2013)]
- $c_{13} = 0.025\,\frac{\mu}{2}$
- $c_{14} = -0.05\,\mu$
- $\lambda = \frac{2}{3}\mu\left(\frac{1+\nu}{1-2\nu}-1\right)$
- $r = \frac{\ell_\parallel}{\ell_\perp}$

Rotational interactive energy density in the reference configuration

$$\psi^{c13}\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]^2(\boldsymbol{n}_t\cdot\boldsymbol{B}\boldsymbol{n}_t)\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 0$$

$$\psi^{c14}\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\cdot\boldsymbol{F}\boldsymbol{n}_0\right]^2\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 0$$

Stress tensor and vectors work conjugated to $\boldsymbol{F}$, $\boldsymbol{n}_t$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ for ψ^{c13}

$$\frac{\partial\psi^{c13}}{\partial\boldsymbol{F}}\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 2c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]^2(\boldsymbol{n}_t\otimes\boldsymbol{n}_t)\,\boldsymbol{F}\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = \boldsymbol{0}$$

$$\frac{\partial\psi^{c13}}{\partial\boldsymbol{n}_t}\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = -2c_{13}\,(\boldsymbol{n}_t\cdot\boldsymbol{B}\boldsymbol{n}_t)\left[\boldsymbol{\epsilon}\cdot(\boldsymbol{\alpha}-\boldsymbol{\beta})\right]\left[\boldsymbol{\epsilon}\cdot(\boldsymbol{\alpha}-\boldsymbol{\beta})\right]\boldsymbol{n}_t$$

$$+\,2c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]^2\boldsymbol{B}\boldsymbol{n}_t = \boldsymbol{0}$$

$$\frac{\partial\psi^{c13}}{\partial\boldsymbol{\alpha}}\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = -\frac{\partial\psi^{c13}}{\partial\boldsymbol{\beta}}\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = -2\,(\boldsymbol{n}_t\cdot\boldsymbol{B}\boldsymbol{n}_t)\left[\boldsymbol{\epsilon}\cdot\boldsymbol{n}_t\right]\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right] = \boldsymbol{0}$$

Rotational interactive energy density in the reference configuration

$$\psi^{c_{13}}\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]^2 (\boldsymbol{n}_t\cdot\boldsymbol{B}\boldsymbol{n}_t)\Big|_{\substack{\boldsymbol{B}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 0$$

$$\psi^{c_{14}}\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\cdot\boldsymbol{F}\boldsymbol{n}_0\right]^2\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 0$$

Stress tensor and vectors work conjugated to $\boldsymbol{F}$, $\boldsymbol{n}_t$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ for $\psi^{c_{14}}$

$$\frac{\partial\psi^{c_{14}}}{\partial\boldsymbol{F}}\Bigg|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 2c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\cdot\boldsymbol{F}\boldsymbol{n}_0\right]\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\otimes\boldsymbol{n}_0\right]\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = \boldsymbol{0}$$

$$\frac{\partial\psi^{c_{14}}}{\partial\boldsymbol{n}_t}\Bigg|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = 2c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\cdot\boldsymbol{F}\boldsymbol{n}_0\right]\left[\left[\boldsymbol{\epsilon}\cdot(\boldsymbol{\alpha}-\boldsymbol{\beta})\right]\boldsymbol{F}\boldsymbol{n}_0\right]\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = \boldsymbol{0}$$

$$\frac{\partial\psi^{c_{14}}}{\partial\boldsymbol{\alpha}}\Bigg|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = -\frac{\partial\psi^{c_{14}}}{\partial\boldsymbol{\beta}}\Bigg|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = -2c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})\times\boldsymbol{n}_t\right]\cdot\boldsymbol{F}\boldsymbol{n}_0\right]\left[\left[\boldsymbol{\epsilon}\cdot\boldsymbol{\chi}\right]\boldsymbol{F}\boldsymbol{n}_0\right]\Big|_{\substack{\boldsymbol{F}=\boldsymbol{I}\\ \boldsymbol{n}_t=\boldsymbol{n}_0\\ \boldsymbol{\alpha}=\boldsymbol{\beta}}} = \boldsymbol{0}$$

Invariance with respect to rotations in $\mathscr{B}_0$ and $\mathscr{B}_t$

Rotational interactive energy density

$$\psi^{c13} + \psi^{c14} = c_{13} \left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right]^2 (\boldsymbol{n}_t \cdot \boldsymbol{B}\boldsymbol{n}_t) + c_{14} \left[\left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \boldsymbol{F}\boldsymbol{n}_0\right]^2$$

Superimposed rotations i.e. change of observer in $\mathscr{B}_0$

- $\boldsymbol{F}_* = \boldsymbol{F}\boldsymbol{Q}$
- $\boldsymbol{B}_* = \boldsymbol{F}\boldsymbol{Q}\,(\boldsymbol{F}\boldsymbol{Q})^T = \boldsymbol{F}\boldsymbol{Q}\boldsymbol{Q}^T\boldsymbol{F}^T = \boldsymbol{F}\boldsymbol{F}^T$ inherently invariant in $\mathscr{B}_0$
- $\boldsymbol{n}_{0\,*} = \boldsymbol{Q}^T\boldsymbol{n}_0$
- $\boldsymbol{n}_{t\,*} = \boldsymbol{n}_t$ inherently invariant in $\mathscr{B}_0$
- $(\boldsymbol{\alpha} - \boldsymbol{\beta})_* = (\boldsymbol{\alpha} - \boldsymbol{\beta})$ inherently invariant in $\mathscr{B}_0$

Rotational interactive energy density in $\mathscr{B}_0$

$$\psi_*^{c13} + \psi_*^{c14} = c_{13} \left[(\boldsymbol{\alpha} - \boldsymbol{\beta})_* \times \boldsymbol{n}_{t\,*}\right]^2 (\boldsymbol{n}_{t\,*} \cdot \boldsymbol{B}_*\boldsymbol{n}_{t\,*}) + c_{14} \left[\left[(\boldsymbol{\alpha} - \boldsymbol{\beta})_* \times \boldsymbol{n}_{t\,*}\right] \cdot \boldsymbol{F}_*\boldsymbol{n}_{0\,*}\right]^2$$

$$= c_{13} \left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right]^2 (\boldsymbol{n}_t \cdot \boldsymbol{B}\boldsymbol{n}_t) + c_{14} \left[\left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \boldsymbol{F}\underbrace{\boldsymbol{Q}\boldsymbol{Q}^T}_{\boldsymbol{I}}\boldsymbol{n}_0\right]^2$$

$$= c_{13} \left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right]^2 (\boldsymbol{n}_t \cdot \boldsymbol{B}\boldsymbol{n}_t) + c_{14} \left[\left[(\boldsymbol{\alpha} - \boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \boldsymbol{F}\boldsymbol{n}_0\right]^2$$

Superimposed rotations i.e. change of observer in $\mathscr{B}_t$

- $\boldsymbol{F}^* = \boldsymbol{Q}\boldsymbol{F}$
- $\boldsymbol{B}_* = \boldsymbol{Q}\boldsymbol{F}\left(\boldsymbol{Q}\boldsymbol{F}\right)^T = \boldsymbol{Q}\boldsymbol{F}\boldsymbol{F}^T\boldsymbol{Q}^T$
- $\boldsymbol{n}_0^* = \boldsymbol{n}_0$ inherently invariant in $\mathscr{B}_t$
- $\boldsymbol{n}_t^* = \boldsymbol{Q}\boldsymbol{n}_t$
- $(\boldsymbol{\alpha} - \boldsymbol{\beta})^* = \boldsymbol{Q}\left(\boldsymbol{\alpha} - \boldsymbol{\beta}\right)$

Rotational interactive energy density in $\mathscr{B}_t$

$$
\begin{aligned}
\psi^{c_{13}\,*} + \psi^{c_{14}\,*} &= c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})^* \times \boldsymbol{n}_t^*\right]^2 \left(\boldsymbol{n}_t^* \cdot \boldsymbol{B}^*\boldsymbol{n}_t^*\right) + c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta})^* \times \boldsymbol{n}_t^*\right] \cdot \boldsymbol{F}^*\boldsymbol{n}_0^*\right]^2 \\
&= c_{13}\boldsymbol{Q}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right]\boldsymbol{Q}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right]\left(\boldsymbol{Q}\boldsymbol{n}_t \cdot \boldsymbol{Q}\boldsymbol{B}\boldsymbol{Q}\boldsymbol{Q}\boldsymbol{n}_t\right) \\
&+ c_{14}\left[\boldsymbol{Q}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \boldsymbol{Q}\boldsymbol{F}\boldsymbol{n}_0\right]^2 \\
&= c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \underbrace{\boldsymbol{Q}^T\boldsymbol{Q}}_{\boldsymbol{I}}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right]\left(\boldsymbol{n}_t \cdot \underbrace{\boldsymbol{Q}^T\boldsymbol{Q}}_{\boldsymbol{I}}\boldsymbol{B}\underbrace{\boldsymbol{Q}^T\boldsymbol{Q}}_{\boldsymbol{I}}\boldsymbol{n}_t\right) \\
&+ c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \underbrace{\boldsymbol{Q}^T\boldsymbol{Q}}_{\boldsymbol{I}}\boldsymbol{F}\boldsymbol{n}_0\right]^2 \\
&= c_{13}\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right]^2\left(\boldsymbol{n}_t \cdot \boldsymbol{B}\boldsymbol{n}_t\right) + c_{14}\left[\left[(\boldsymbol{\alpha}-\boldsymbol{\beta}) \times \boldsymbol{n}_t\right] \cdot \boldsymbol{F}\boldsymbol{n}_0\right]^2
\end{aligned}
$$

Dissipative rotation of the mesogens modelled by β

cf. [Oates and Wang (2009), Groß et al. (2023)]

- Clausius-Planck inequality
  - $\left(\boldsymbol{N} - \frac{\partial \psi}{\partial \boldsymbol{F}} \right) : \dot{\boldsymbol{F}} + \boldsymbol{\eta} \cdot \dot{\boldsymbol{\chi}} - \frac{\partial \psi}{\partial \boldsymbol{\chi}} \cdot \dot{\boldsymbol{\chi}} - \frac{\partial \psi}{\partial \boldsymbol{\alpha}} \cdot \dot{\boldsymbol{\alpha}} - \frac{\partial \psi}{\partial \boldsymbol{\beta}} \cdot \dot{\boldsymbol{\beta}} \geq 0$
- Dissipation inequality
  - $\mathcal{D}^{\text{int}} = - \frac{\partial \psi}{\partial \boldsymbol{\beta}} \cdot \dot{\boldsymbol{\beta}} \geq 0$
  - $\mathcal{D}^{\text{int}} \stackrel{!}{=} \boldsymbol{V}_{\beta} \dot{\boldsymbol{\beta}} \cdot \dot{\boldsymbol{\beta}}$
- Rotational viscosity tensor for penalizing possible rotations of $\boldsymbol{\chi}$ about the $x-$ and $y-$directions

$$\boldsymbol{V}_{\beta} = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k_F \end{bmatrix} \text{ with } k >> k_F$$

Principle of virtual power

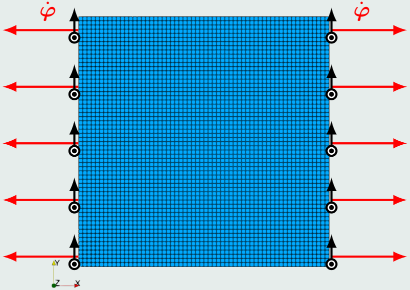
cf. [Concas and Groß (2024)]

$$\delta_* \dot{\mathcal{T}} \left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}} \right) + \delta_* \dot{\mathcal{T}}_n \left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_{\chi}, \dot{\mathbf{p}}_{\chi} \right) + \delta_* \dot{\Pi}^{\text{ext}} \left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R} \right) + \delta_* \dot{\Pi}_n^{\text{ext}} \left(\dot{\boldsymbol{\chi}} \right) + \delta_* \dot{\Pi}^{\text{int}} \left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\chi}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\beta}}, \boldsymbol{\tau}_{\chi}, \boldsymbol{w}_{\tau} \right) := 0$$

Numerical test: LCE film under tension

Boundary conditions

$0.0125 \times 0.0125 \times 0.0005$ m, 3.600-em H1-standard



$\dot{\varphi} = 0.008\, m/s$
$\boldsymbol{n}_0 = \boldsymbol{e}_y$
$\boldsymbol{y} = 0$
$\boldsymbol{z} = 0$

- high strain rate $(1.28\, 1/s)$ approaching Mbanga et al. (2010) $(1.33\, 1/s)$
- specimen with $L/H = 1$ cf. Zhang et al. (2020) and references therein

Parameters

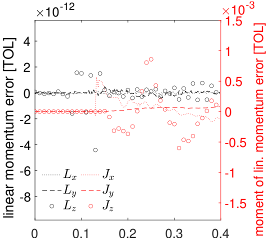
cf. [de Luca et al. (2013), Groß et al. (2022)]

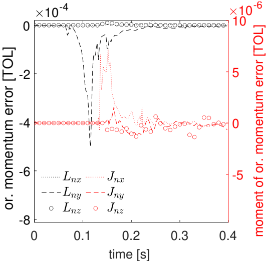
- $cG(k=2)$
- $h_n = 0.002\, s$
- $t_{end} = 0.04\, s$ (stretch up to 50% of the initial length)
- $TOL = 10^{-5}$
- convergence criterion: $||\boldsymbol{R}|| < TOL$
- $E = 0.914\, MPa$
- $\nu = 0.493$
- $\rho = 1760\, kg/m^3$
- $r(T = 60°C) = 1.88$
- $a(T = 60°C) = 0.063$
- $\boldsymbol{V}_\beta$ with $k_F = 200$ and $k = 10^8$

LCE film under tension - momentum and angular momentum balances' errors

$c_{13} = 0,\ c_{14} = 0$ [Biggins et al. (2008)]

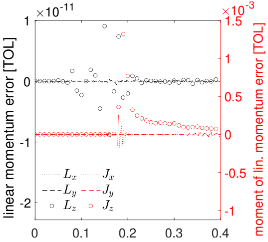
- $\psi = \psi^{NH} + \psi^{c3} + \psi^{c9} + \psi^{c10}$

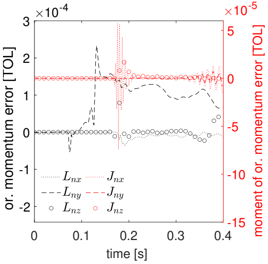




$c_{13} \neq 0,\ c_{14} = 0$ [Warner and Terentjev(2008)]

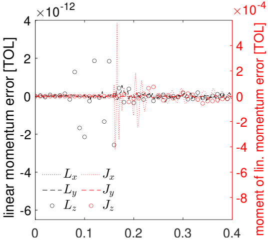
- $\psi = \psi^{NH} + \psi^{c3} + \psi^{c9} + \psi^{c10} + \psi^{c13}$

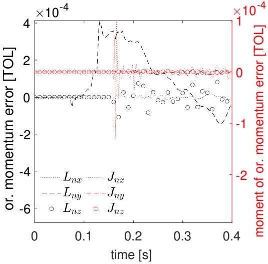




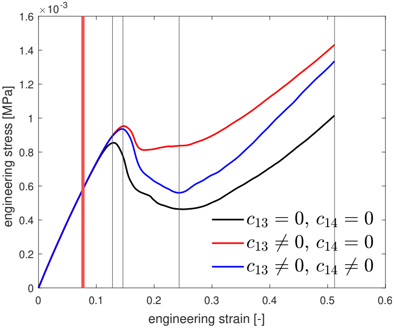
$c_{13} \neq 0,\ c_{14} \neq 0$ new ψ^{c14}

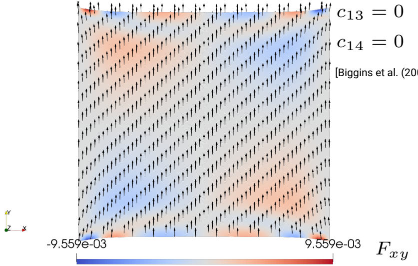
- $\psi = \psi^{NH} + \psi^{c3} + \psi^{c9} + \psi^{c10} + \psi^{c13} + \psi^{c14}$





LCE film under tension - stripe domains and semisoft elasticity

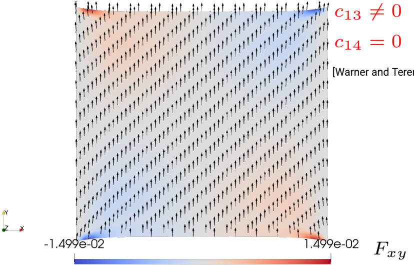




$c_{13} = 0$

$c_{14} = 0$

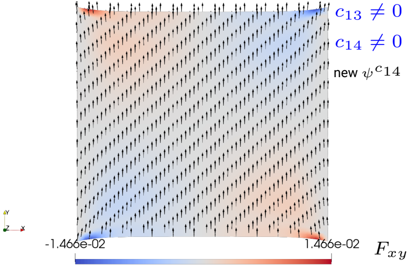
[Biggins et al. (2008)]



$c_{13} \neq 0$

$c_{14} = 0$

[Warner and Terentjev (2007)]

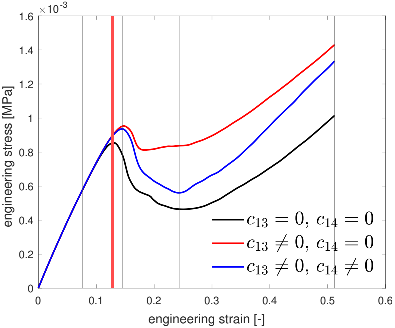


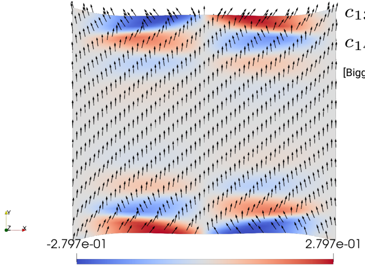
$c_{13} \neq 0$

$c_{14} \neq 0$

new ψ^{c14}

LCE film under tension - stripe domains and semisoft elasticity



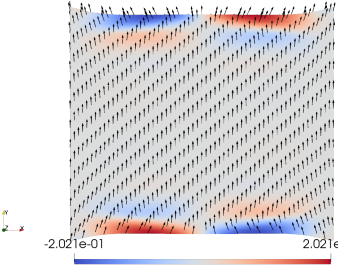


F_{xy}

$c_{13} = 0$

$c_{14} = 0$

[Biggins et al. (2008)]

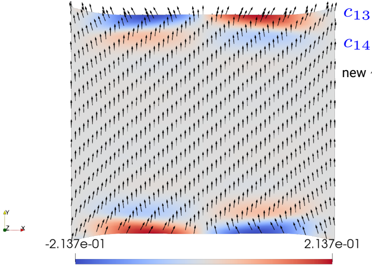


F_{xy}

$c_{13} \neq 0$

$c_{14} = 0$

[Warner and Terentjev (2007)]



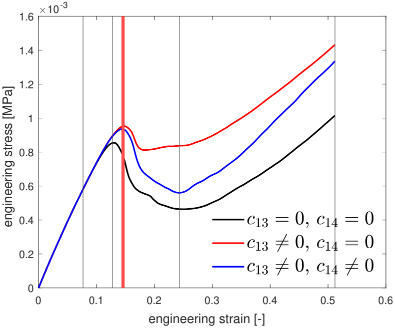
F_{xy}

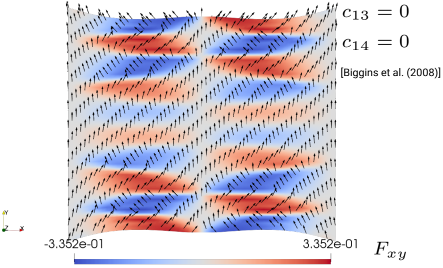
$c_{13} \neq 0$

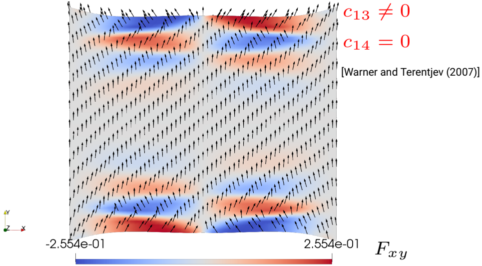
$c_{14} \neq 0$

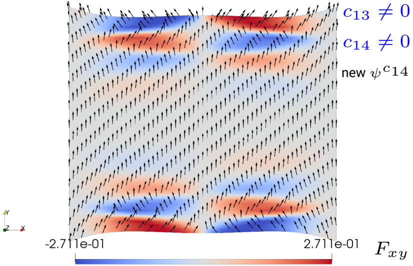
new $\psi^{c_{14}}$

LCE film under tension - stripe domains and semisoft elasticity

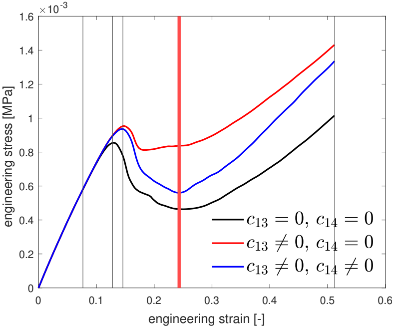


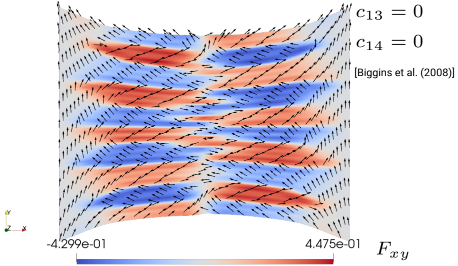


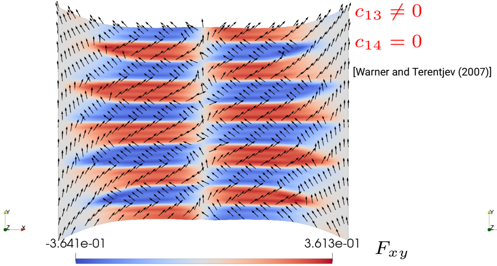


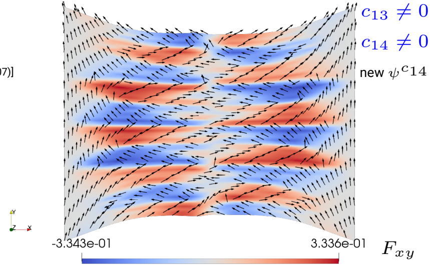


LCE film under tension - stripe domains and semisoft elasticity









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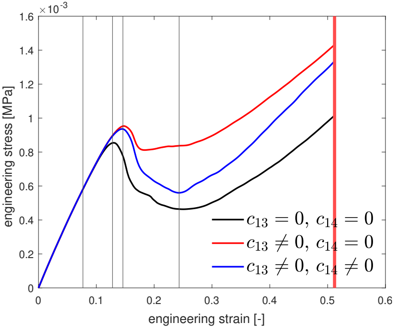

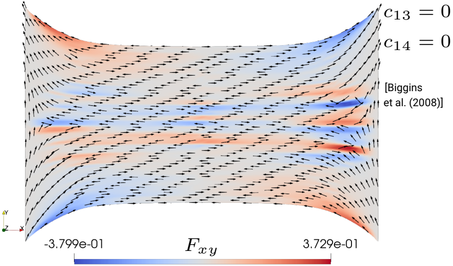

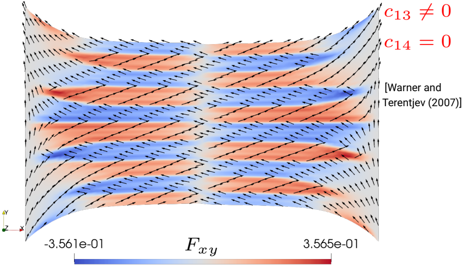

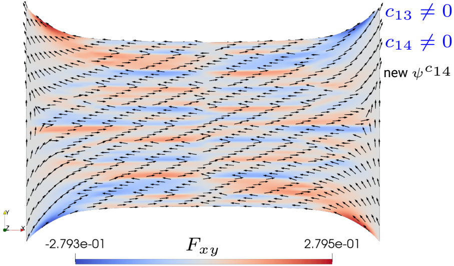

Conclusions

- dynamic and variational-consistent numerical framework of Concas and Groß (2024) for reproducing soft elasticity under high strain rates
- implementation of a new invariant strain energy ψ^{c14} in additions to the semi-soft free energy density of Biggins et al. (2008) and the rotational free energy density of Warner and Terentjev (2007) ψ^{c13}
- mechanical and orientational momentum balances are preserved
- the new free energy density ψ^{c14} allow to reach full rotation of the mesogens
- striped pattern depends on local values of the entry F_{xy} of the deformation gradient $\boldsymbol{F}$
- symmetric distribution of F_{xy} over the geometry leads to opposite rotations of the mesogens with respect to the central vertical axis of the geometry

Future work

- experimental validation of numerical results
- modelling thermo-mechanical effects in the context of dynamic

Linear momentum $\delta_* \dot{\boldsymbol{\varphi}} = \boldsymbol{c} = $ const.

$$\boldsymbol{L}(t_{n+1}) - \boldsymbol{L}(t_n) = \iint_{\mathscr{T}_n \times \mathscr{B}_0} \rho_0 \boldsymbol{B}_\varphi \, \mathrm{d}V \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_T \mathscr{B}_0} \boldsymbol{T} \, \mathrm{d}A \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_\varphi \mathscr{B}_0} \boldsymbol{R} \, \mathrm{d}A \, \mathrm{d}t$$

Orientational momentum $\delta_* \dot{\boldsymbol{\chi}} = \boldsymbol{c} = $ const.

$$\boldsymbol{L}_n(t_{n+1}) - \boldsymbol{L}_n(t_n) = \iint_{\mathscr{T}_n \times \mathscr{B}_0} \rho_0 \boldsymbol{B}_\chi \, \mathrm{d}V \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_W \mathscr{B}_0} \boldsymbol{W} \, \mathrm{d}A \, \mathrm{d}t - \iint_{\mathscr{T}_n \times \mathscr{B}_0} \left[\boldsymbol{\tau}_\chi + \frac{\partial \psi}{\partial \boldsymbol{\chi}} \right] \mathrm{d}V \, \mathrm{d}t$$

Moment of linear momentum $\delta_* \dot{\boldsymbol{\varphi}} = \boldsymbol{c} \times \boldsymbol{\varphi}$, $\delta_* \dot{\boldsymbol{\alpha}} = \boldsymbol{c}$, $\delta_* \dot{\boldsymbol{\beta}} = \boldsymbol{c}$

$$\begin{aligned} \boldsymbol{J}(t_{n+1}) - \boldsymbol{J}(t_n) = & - \iint_{\mathscr{T}_n \times \mathscr{B}_0} \left(\boldsymbol{F} \times \frac{\partial \psi}{\partial \boldsymbol{F}} \right) \mathrm{d}V \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}_\varphi] \, \mathrm{d}V \, \mathrm{d}t \\ & + \iint_{\mathscr{T}_n \times \partial_T \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{T}] \, \mathrm{d}A \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_\varphi \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{R}] \, \mathrm{d}A \, \mathrm{d}t \\ & + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\beta}} \, \mathrm{d}V \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \boldsymbol{\Sigma}_\beta \, \mathrm{d}V \, \mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\alpha}} \, \mathrm{d}V \, \mathrm{d}t \end{aligned}$$

Moment of orientational momentum

$\delta_* \dot{\boldsymbol{\chi}} = \boldsymbol{c} \times \boldsymbol{\chi}$

$$\boldsymbol{J}_\chi(t_{n+1}) - \boldsymbol{J}_\chi(t_n) = - \iint_{\mathscr{T}_n \times \mathscr{B}_0} \left(\boldsymbol{\chi} \times \frac{\partial \psi}{\partial \boldsymbol{\chi}} \right) \mathrm{d}V\,\mathrm{d}t - \iint_{\mathscr{T}_n \times \mathscr{B}_0} (\boldsymbol{\chi} \times \boldsymbol{\tau}_\chi)\,\mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} [\boldsymbol{\chi} \times \rho_0 \boldsymbol{B}_\chi] + \mathrm{d}V\,\mathrm{d}t \iint_{\mathscr{T}_n \times \partial_W \mathscr{B}_0} [\boldsymbol{\chi} \times \boldsymbol{W}]\,\mathrm{d}A\,\mathrm{d}t$$

Total moment of momentum

$$\boldsymbol{J}(t_{n+1}) - \boldsymbol{J}(t_n) + \boldsymbol{J}_n(t_{n+1}) - \boldsymbol{J}_n(t_n) = - \iint_{\mathscr{T}_n \times \mathscr{B}_0} \left(\boldsymbol{F} \times \frac{\partial \psi}{\partial \boldsymbol{F}} \right) \mathrm{d}V\,\mathrm{d}t - \iint_{\mathscr{T}_n \times \mathscr{B}_0} \left(\boldsymbol{\chi} \times \frac{\partial \psi}{\partial \boldsymbol{\chi}} \right) \mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\alpha}}\,\mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\beta}}\,\mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} \boldsymbol{\Sigma}_\beta\,\mathrm{d}V\,\mathrm{d}t - \iint_{\mathscr{T}_n \times \mathscr{B}_0} (\boldsymbol{\chi} \times \boldsymbol{\tau}_\chi)\,\mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} [\boldsymbol{\chi} \times \rho_0 \boldsymbol{B}_\chi] + \mathrm{d}V\,\mathrm{d}t \iint_{\mathscr{T}_n \times \partial_W \mathscr{B}_0} [\boldsymbol{\chi} \times \boldsymbol{W}]\,\mathrm{d}A\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \mathscr{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}_\varphi]\,\mathrm{d}V\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_T \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{T}]\,\mathrm{d}A\,\mathrm{d}t + \iint_{\mathscr{T}_n \times \partial_\varphi \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{R}]\,\mathrm{d}A\,\mathrm{d}t$$