Principle of Virtual Power and Drilling Degrees of Freedom for Modelling the Behaviour of Liquid Crystal Elastomer Films

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ACEX2022 - Florence, Italy 3-7 July, 2022

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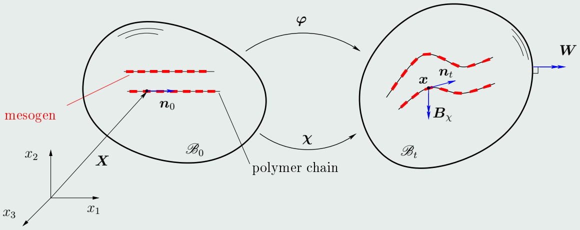
Deutsche Forschungsgemeinschaft

Acknowledgments: This research is provided by DFG grant GR 3297/7-1.

Motivation

- modeling of reorientation response of mesogens in liquid crystal elastomer (LCE) films under dynamic loads

Goals



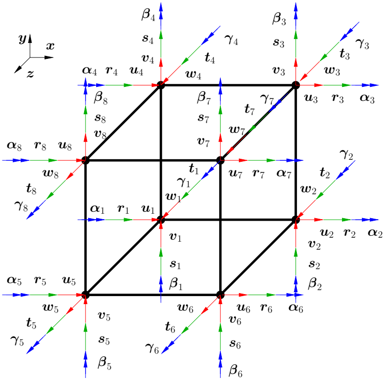
- formulation of the free energy density including drilling degrees of freedom
- principle of virtual power for space-time discretisation
- formulation of distinct momentum balances and angular momentum balances for the LCE material and the nematic director, according to Anderson et al. [1999] and their fulfillment by means of the continuous Galerkin method

Energy density formulation cf. Anderson et al. [1999]

$$\psi = \underbrace{c_1\left(\boldsymbol{I}:\boldsymbol{C}-3-2\log(J)\right)+\frac{\lambda}{2}\left(\left[\log(J)\right]^2+(J-1)^2\right)+c_3\boldsymbol{n}_0\cdot\boldsymbol{C}\boldsymbol{n}_0}_{\text{elastic energy density}}$$
$$\underbrace{+\,c_9\left|\boldsymbol{F}^T\boldsymbol{n}_t\right|^2+c_{10}\left(\boldsymbol{n}_0\cdot\boldsymbol{F}^T\boldsymbol{n}_t\right)^2}_{\text{interactive energy density}}$$

Material parameters cf. Anderson et al. [1999], de Luca et al. [2013]

- $c_1 = \frac{\mu}{2}$
- $c_3 = \frac{\mu(r-1)}{2}$
- $c_9 = \frac{\mu}{2}\left(\frac{1}{r}-1\right)$
- $c_{10} = \frac{\mu}{2}\left(2-\frac{1}{r}-r\right)$
- $\lambda = \frac{2}{3}\mu\left(\frac{1+\nu}{1-2\nu}-1\right)$
- $\ell_{0\parallel} = \ell_\parallel$
- $\ell_{0\perp} = \ell_\perp$
- $r = r_0 = \frac{\ell_\parallel}{\ell_\perp}$



Drilling degrees of freedom

- kinematic reorientation process for $\dot{\boldsymbol{\alpha}} = -\frac{1}{2}\boldsymbol{\epsilon} : \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$
- velocity of the nematic director $\dot{\boldsymbol{\chi}} = \dot{\boldsymbol{\alpha}} \times \boldsymbol{\chi}$
- axial vector of skew-symmetric Kirchhoff stress tensor $\boldsymbol{w}_{\tau} = 2\boldsymbol{\omega} = \boldsymbol{\tau}^{t}_{\text{skw}} : \boldsymbol{\epsilon}$ as first Lagrange multiplier
- reorientation stress vector $\boldsymbol{\tau}_{\chi}$ associated to the nematic director as second Lagrange multiplier

Principle of virtual power

- $\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}} \left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{p}}, \dot{\mathbf{v}}, \dot{\boldsymbol{\chi}}, \dot{\mathbf{p}}_{\chi}, \dot{\mathbf{v}}_{\chi}, \dot{\boldsymbol{\alpha}}, \boldsymbol{w}_{\tau}, \boldsymbol{\tau}_{\chi}, \boldsymbol{R} \right) = 0$
- $\mathcal{H} := \mathcal{T} + \mathcal{T}_n + \Pi^{\text{ext}} + \Pi^{\text{ext}}_n + \Pi^{\text{int}}$

Principle of virtual power

$$\delta_* \dot{\mathcal{T}}\left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}\right) + \delta_* \dot{\mathcal{T}}_n\left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_\chi, \dot{\mathbf{p}}_\chi\right) + \delta_* \dot{\Pi}^{\text{ext}}\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R}\right) + \delta_* \dot{\Pi}^{\text{ext}}_n\left(\dot{\boldsymbol{\chi}}\right) + \delta_* \dot{\Pi}^{\text{int}}\left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\chi}}, \dot{\boldsymbol{\alpha}}, \boldsymbol{\tau}_\chi, \boldsymbol{w}_\tau\right) := 0$$

Deformational and orientational kinetic power cf. Anderson et al. [1999]

- Functional of the deformational kinetic power

$$\dot{\mathcal{T}}\left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}\right) := \int_{\mathscr{B}_0} \dot{\mathbf{v}} \cdot \rho_0 \mathbf{v} \, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\mathbf{p}} \cdot \left[\mathbf{v} - \dot{\boldsymbol{\varphi}}\right] \mathrm{d}V - \int_{\mathscr{B}_0} \mathbf{p} \cdot \left[\dot{\mathbf{v}} - \ddot{\boldsymbol{\varphi}}\right] \mathrm{d}V$$

- Virtual deformational kinetic power

$$\delta_* \dot{\mathcal{T}}\left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}\right) := \int_{\mathscr{B}_0} \dot{\mathbf{p}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \left(\rho_0 \mathbf{v} - \mathbf{p}\right) \cdot \delta_* \dot{\mathbf{v}} \, \mathrm{d}V - \int_{\mathscr{B}_0} \left[\mathbf{v} - \dot{\boldsymbol{\varphi}}\right] \cdot \delta_* \dot{\mathbf{p}}$$

- Functional of the orientational kinetic power

$$\dot{\mathcal{T}}_n\left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_\chi, \dot{\mathbf{p}}_\chi\right) := \int_{\mathscr{B}_0} \dot{\mathbf{v}}_\chi \cdot \left[\rho_0 l_\chi^2\right] \mathbf{v}_\chi \, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\mathbf{p}}_\chi \cdot \left[\mathbf{v}_\chi - \dot{\boldsymbol{\chi}}\right] \mathrm{d}V - \int_{\mathscr{B}_0} \mathbf{p}_\chi \cdot \left[\dot{\mathbf{v}}_\chi - \ddot{\boldsymbol{n}}_t\right] \mathrm{d}V$$

- Virtual orientational kinetic power

$$\delta_* \dot{\mathcal{T}}_n\left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_\chi, \dot{\mathbf{p}}_\chi\right) := \int_{\mathscr{B}_0} \dot{\mathbf{p}}_\chi \cdot \delta_* \dot{\boldsymbol{\chi}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \left(\rho_0 l_\chi^2 \mathbf{v}_\chi - \mathbf{p}_\chi\right) \cdot \delta_* \dot{\mathbf{v}}_\chi \, \mathrm{d}V - \int_{\mathscr{B}_0} \left[\mathbf{v}_\chi - \dot{\boldsymbol{\chi}}\right] \cdot \delta_* \dot{\mathbf{p}}_\chi$$

Variational-based weak formulation (II)

Principle of virtual power

$$\delta_* \dot{\mathcal{T}}\left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}\right) + \delta_* \dot{\mathcal{T}}_n\left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_\chi, \dot{\mathbf{p}}_\chi\right) + \delta_* \dot{\Pi}^{\text{ext}}\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R}\right) + \delta_* \dot{\Pi}_n^{\text{ext}}\left(\dot{\boldsymbol{\chi}}\right) + \delta_* \dot{\Pi}^{\text{int}}\left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\chi}}, \dot{\boldsymbol{\alpha}}, \boldsymbol{\tau}_\chi, \boldsymbol{w}_\tau\right) := 0$$

External deformational and orientational power cf. Anderson et al. [1999]

- Functional of the external deformational power

$$\dot{\Pi}^{\text{ext}}\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R}\right) := -\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \cdot \dot{\boldsymbol{\varphi}} \,\mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}} \,\mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \cdot \left(\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}\right) \,\mathrm{d}A$$

- Virtual external deformational power

$$\delta_* \dot{\Pi}^{\text{ext}}\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R}\right) := -\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \cdot \delta_* \dot{\boldsymbol{\varphi}} \,\mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \,\mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \cdot \delta_* \dot{\boldsymbol{\varphi}} \,\mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \delta_* \boldsymbol{R} \cdot \left[\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}\right] \,\mathrm{d}A$$

- Functional of the external orientational power

$$\dot{\Pi}_n^{\text{ext}}\left(\dot{\boldsymbol{\chi}}\right) := -\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B}_\chi \cdot \dot{\boldsymbol{\chi}} \,\mathrm{d}V - \int_{\partial_W \mathscr{B}_0} \boldsymbol{W} \cdot \dot{\boldsymbol{\chi}} \,\mathrm{d}A$$

- Virtual external orientational power

$$\delta_* \dot{\Pi}_n^{\text{ext}}\left(\dot{\boldsymbol{\chi}}\right) := -\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B}_\chi \cdot \delta_* \dot{\boldsymbol{\chi}} \,\mathrm{d}V - \int_{\partial_W \mathscr{B}_0} \boldsymbol{W} \cdot \delta_* \dot{\boldsymbol{\chi}} \,\mathrm{d}A$$

Variational-based weak formulation (III)

Principle of virtual power

$$\delta_* \dot{\mathcal{T}}\left(\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}\right) + \delta_* \dot{\mathcal{T}}_n\left(\dot{\boldsymbol{\chi}}, \dot{\mathbf{v}}_\chi, \dot{\mathbf{p}}_\chi\right) + \delta_* \dot{\Pi}^{\text{ext}}\left(\dot{\boldsymbol{\varphi}}, \boldsymbol{R}\right) + \delta_* \dot{\Pi}_n^{\text{ext}}\left(\dot{\boldsymbol{\chi}}\right) + \delta_* \dot{\Pi}^{\text{int}}\left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\chi}}, \dot{\boldsymbol{\alpha}}, \boldsymbol{\tau}_\chi, \boldsymbol{w}_\tau\right) := 0$$

Internal power

- Functional of the internal power

$$\dot{\Pi}^{\text{int}}\left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{\chi}}, \dot{\boldsymbol{\alpha}}, \boldsymbol{\tau}_\chi, \boldsymbol{w}_\tau\right) := \mathcal{P}^{\text{int}}$$

$$\begin{aligned}\mathcal{P}^{\text{int}} := &\int_{\mathscr{B}_0} \left[\frac{\partial \psi}{\partial \boldsymbol{F}} : \dot{\boldsymbol{F}} + \frac{\partial \psi}{\partial \boldsymbol{\chi}} \cdot \dot{\boldsymbol{\chi}}\right] \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\tau}_\chi \cdot \left[\dot{\boldsymbol{\chi}} + \boldsymbol{\epsilon} : \boldsymbol{\chi} \otimes \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V \\ &+ \int_{\mathscr{B}_0} \boldsymbol{w}_\tau \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\boldsymbol{F}} \boldsymbol{F}^{-1} + \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V\end{aligned}$$

- Virtual internal power

$$\begin{aligned}\delta_* \mathcal{P}^{\text{int}} := &\int_{\mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{F}} : \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\varphi}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2} \boldsymbol{w}_\tau \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{F}^{-t} : \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\varphi}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \, \mathrm{d}V \\ &+ \int_{\mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\chi}} \cdot \delta_* \dot{\boldsymbol{\chi}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\tau}_\chi \cdot \delta_* \dot{\boldsymbol{\chi}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{\epsilon} : \boldsymbol{\tau}_\chi \otimes \boldsymbol{\chi} \, \mathrm{d}V \\ &+ \int_{\mathscr{B}_0} \boldsymbol{w}_\tau \cdot \delta_* \dot{\boldsymbol{\alpha}} \, \mathrm{d}V + \int_{\mathscr{B}_0} \delta_* \boldsymbol{\tau}_\chi \cdot \left[\dot{\boldsymbol{\chi}} + \boldsymbol{\epsilon} : \boldsymbol{\chi} \otimes \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V \\ &+ \int_{\mathscr{B}_0} \delta_* \boldsymbol{w}_\tau \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\boldsymbol{F}} \boldsymbol{F}^{-1} + \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V\end{aligned}$$

Space-time weak forms

Weak balance of linear momentum

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot [\dot{\mathbf{p}} - \rho_0 \boldsymbol{B}] \, \mathrm{d}V \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \frac{1}{2} \boldsymbol{w}_\tau \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{F}^{-t} : \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\varphi}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \, \mathrm{d}V \, \mathrm{d}t$$
$$+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{F}} : \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\varphi}} \cdot \delta_* \dot{\boldsymbol{\varphi}} \, \mathrm{d}V \, \mathrm{d}t - \int_{t_n}^{t_{n+1}} \int_{\partial_T \mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \bar{\boldsymbol{T}} \, \mathrm{d}A \, \mathrm{d}t$$
$$- \int_{t_n}^{t_{n+1}} \int_{\partial_\varphi \mathscr{B}_0} \delta_* \dot{\boldsymbol{\varphi}} \cdot \boldsymbol{R} \, \mathrm{d}A \, \mathrm{d}t = 0$$

Weak balance of orientational momentum

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot [\dot{\mathbf{p}}_\chi - \rho_0 \boldsymbol{B}_\chi] \, \mathrm{d}V \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \frac{\partial \psi}{\partial \boldsymbol{\chi}} \cdot \delta_* \dot{\boldsymbol{\chi}} \, \mathrm{d}V \, \mathrm{d}t$$
$$+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \boldsymbol{\tau}_\chi \, \mathrm{d}V \, \mathrm{d}t - \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathscr{B}_0} \delta_* \dot{\boldsymbol{\chi}} \cdot \boldsymbol{W} \, \mathrm{d}A \, \mathrm{d}t = 0$$

Weak balance of reorientation stress

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot [\boldsymbol{\tau}_\chi \times \boldsymbol{\chi} + \boldsymbol{w}_\tau] \, \mathrm{d}V \, \mathrm{d}t = 0$$

Weak balance of orientation rate

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \boldsymbol{\tau}_\chi \cdot [\dot{\boldsymbol{\chi}} - \dot{\boldsymbol{\alpha}} \times \boldsymbol{\chi}] \, \mathrm{d}V \, \mathrm{d}t = 0$$

Weak continuum rotation equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \boldsymbol{w}_\tau \cdot \left[\dot{\boldsymbol{\alpha}} + \frac{1}{2} \boldsymbol{\epsilon} : \dot{\boldsymbol{F}} \boldsymbol{F}^{-1} \right] \mathrm{d}V \, \mathrm{d}t = 0$$

Balance laws of the weak formulation (I)

Reorientation function

$$\mathcal{C}^{\text{ori}}\left(t_{n+1}\right)-\mathcal{C}^{\text{ori}}\left(t_{n}\right)=\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\left[\|\boldsymbol{\chi}\|^2-1\right]\,\mathrm{d}V\,\mathrm{d}t$$

Linear momentum $\delta_*\dot{\boldsymbol{\varphi}}=\boldsymbol{c}=\text{const.}$

$$\begin{aligned}\boldsymbol{L}\left(t_{n+1}\right)-\boldsymbol{L}\left(t_{n}\right) &= \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\rho_0\boldsymbol{B}\,\mathrm{d}V\,\mathrm{d}t\\ &+\int_{t_n}^{t_{n+1}}\int_{\partial_T\mathscr{B}_0}\bar{\boldsymbol{T}}\,\mathrm{d}A\,\mathrm{d}t+\int_{t_n}^{t_{n+1}}\int_{\partial_\varphi\mathscr{B}_0}\boldsymbol{R}\,\mathrm{d}A\,\mathrm{d}t\end{aligned}$$

Orientational momentum $\delta_*\dot{\boldsymbol{\chi}}=\boldsymbol{c}=\text{const.}$

$$\begin{aligned}\boldsymbol{L}_n\left(t_{n+1}\right)-\boldsymbol{L}_n\left(t_{n}\right) &= \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\rho_0\boldsymbol{B}_\chi\,\mathrm{d}V\,\mathrm{d}t+\int_{t_n}^{t_{n+1}}\int_{\partial_W\mathscr{B}_0}\boldsymbol{W}\,\mathrm{d}A\,\mathrm{d}t\\ &-\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\left[\boldsymbol{\tau}_\chi+\frac{\partial\psi}{\partial\boldsymbol{\chi}}\right]\mathrm{d}V\,\mathrm{d}t\end{aligned}$$

Balance laws of the weak formulation (II)

Moment of linear momentum

$\delta_* \dot{\boldsymbol{\varphi}} = \boldsymbol{c} \times \boldsymbol{\varphi},\ \delta_* \dot{\boldsymbol{\alpha}} = \boldsymbol{c}$

$$\begin{aligned}\boldsymbol{J}(t_{n+1}) - \boldsymbol{J}(t_n) = &- \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \left(\boldsymbol{F} \times \frac{\partial \psi}{\partial \boldsymbol{F}} \right) \mathrm{d}V \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} (\boldsymbol{\chi} \times \boldsymbol{\tau}_\chi) \, \mathrm{d}V \, \mathrm{d}t \\ &+ \int_{t_n}^{t_{n+1}} \int_{\partial_T \mathscr{B}_0} [\boldsymbol{\varphi} \times \bar{\boldsymbol{T}}] \, \mathrm{d}A \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\partial_\varphi \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{R}] \, \mathrm{d}A \, \mathrm{d}t \\ &+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}] \, \mathrm{d}V \, \mathrm{d}t\end{aligned}$$

Moment of orientational momentum

$\delta_* \dot{\boldsymbol{\chi}} = \boldsymbol{c} \times \boldsymbol{\chi}$

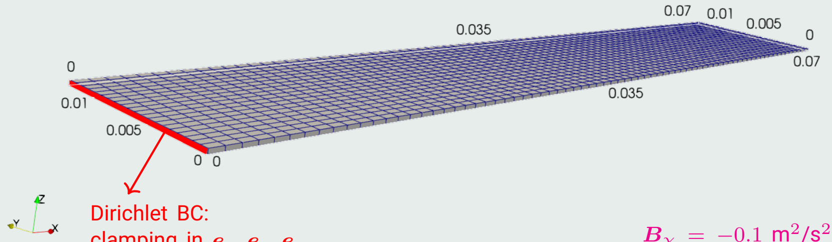
$$\begin{aligned}\boldsymbol{J}_n(t_{n+1}) - \boldsymbol{J}_n(t_n) = &- \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \left(\boldsymbol{\chi} \times \frac{\partial \psi}{\partial \boldsymbol{\chi}} \right) \mathrm{d}V \, \mathrm{d}t - \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} (\boldsymbol{\chi} \times \boldsymbol{\tau}_\chi) \, \mathrm{d}V \, \mathrm{d}t \\ &+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\chi} \times \rho_0 \boldsymbol{B}_\chi] \, \mathrm{d}V \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathscr{B}_0} [\boldsymbol{\chi} \times \boldsymbol{W}] \, \mathrm{d}A \, \mathrm{d}t\end{aligned}$$

Total moment of momentum

$$\begin{aligned}\boldsymbol{J}(t_{n+1}) - \boldsymbol{J}(t_n) + \boldsymbol{J}_n(t_{n+1}) - \boldsymbol{J}_n(t_n) = &+ \int_{t_n}^{t_{n+1}} \int_{\partial_\varphi \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{R}] \, \mathrm{d}A \, \mathrm{d}t \\ + \int_{t_n}^{t_{n+1}} \int_{\partial_T \mathscr{B}_0} [\boldsymbol{\varphi} \times \bar{\boldsymbol{T}}] \, \mathrm{d}A \, \mathrm{d}t &+ \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathscr{B}_0} [\boldsymbol{\chi} \times \boldsymbol{W}] \, \mathrm{d}A \, \mathrm{d}t \\ + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\chi} \times \rho_0 \boldsymbol{B}_\chi] \, \mathrm{d}V \, \mathrm{d}t &+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}] \, \mathrm{d}V \, \mathrm{d}t\end{aligned}$$

Mesh and boundary condition

$0.0125 \times 0.075 \times 0.0003$ m, 1500-em H1-standard

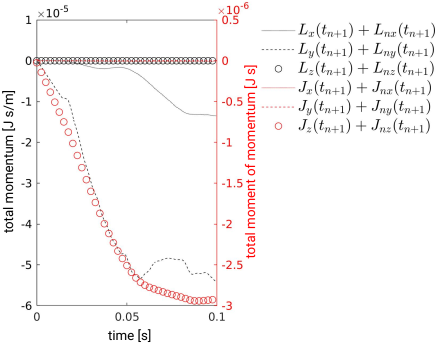


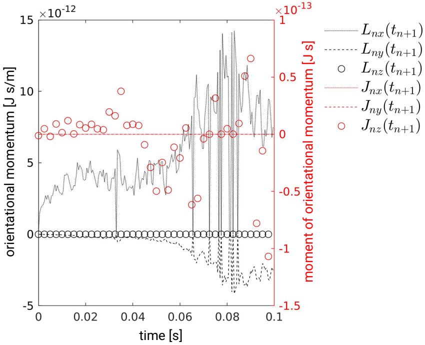
Dirichlet BC:
clamping in $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$

$$\boldsymbol{B}_\chi = -0.1 \text{ m}^2/\text{s}^2 \; \boldsymbol{e}_y$$
$$\boldsymbol{n}_0 = \boldsymbol{e}_x$$

Material parameters cf. de Luca et al. [2013], Groß et al. [2022] (submitted)

- $cG(k = 2)$
- $h_n = 0.0005\, s$ and $t_{end} = 0.1\, s$
- $TOL = 10^{-8}$
- convergence criterion: $\|\boldsymbol{R}\| < TOL$
- $\mu(T = 60°C) = 3.43 \cdot 10^4 \; N/m^2$
- $\nu = 0.49$
- $\rho = 1760\, kg/m^3$
- $r(T = 60°C) = 1.88$





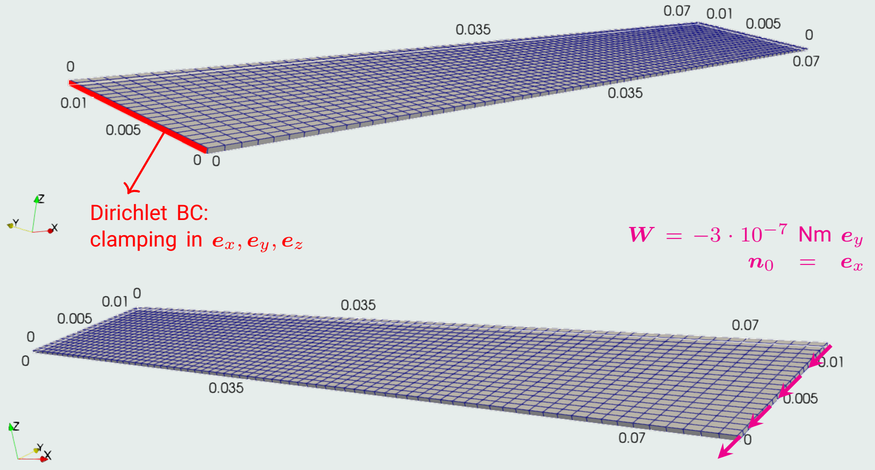
Total linear and orientational momenta

$$
\boldsymbol{L}\left(t_{n+1}\right) + \boldsymbol{L}_n\left(t_{n+1}\right) = \boldsymbol{L}\left(t_n\right) + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \,\mathrm{d}V\,\mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \,\mathrm{d}A\,\mathrm{d}t
+ \int_{t_n}^{t_{n+1}} \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \,\mathrm{d}A\,\mathrm{d}t + \boldsymbol{L}_n\left(t_n\right) + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \rho_0 \boldsymbol{B}_\chi \,\mathrm{d}V\,\mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathscr{B}_0} \boldsymbol{W} \,\mathrm{d}A\,\mathrm{d}t
- \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \left[\boldsymbol{\tau}_\chi + \frac{\partial \psi}{\partial \boldsymbol{\chi}} \right] \mathrm{d}V\,\mathrm{d}t
$$

Numerical test I: LCE film under body load

Numerical test II: LCE film under surface load

Mesh and boundary condition — $0.0125 \times 0.075 \times 0.0003$ m, 1500-em H1-standard



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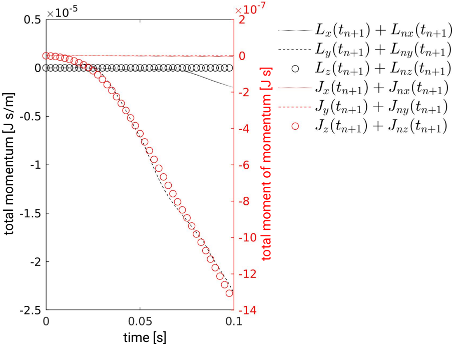
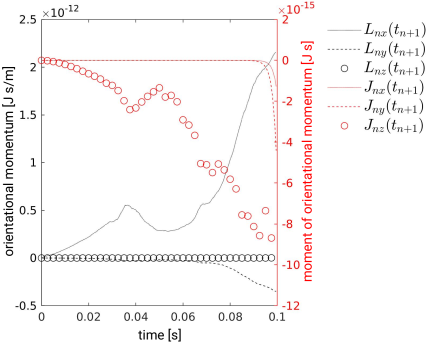
Numerical test II: LCE film under surface load





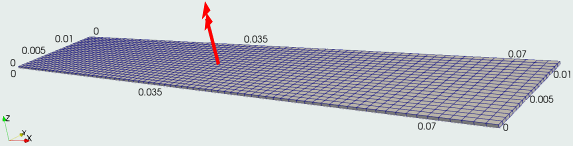
Total moment of momentum

$$
\begin{aligned}
&\boldsymbol{J}(t_{n+1}) + \boldsymbol{J}_n(t_{n+1}) = \boldsymbol{J}(t_n) + \boldsymbol{J}_n(t_n) + \int_{t_n}^{t_{n+1}} \int_{\partial_\varphi \mathscr{B}_0} [\boldsymbol{\varphi} \times \boldsymbol{R}] \, \mathrm{d}A \, \mathrm{d}t \\
&+ \int_{t_n}^{t_{n+1}} \int_{\partial_T \mathscr{B}_0} [\boldsymbol{\varphi} \times \bar{\boldsymbol{T}}] \, \mathrm{d}A \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathscr{B}_0} [\boldsymbol{\chi} \times \boldsymbol{W}] \, \mathrm{d}A \, \mathrm{d}t \\
&+ \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\chi} \times \rho_0 \boldsymbol{B}_\chi] \, \mathrm{d}V \, \mathrm{d}t + \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}] \, \mathrm{d}V \, \mathrm{d}t
\end{aligned}
$$

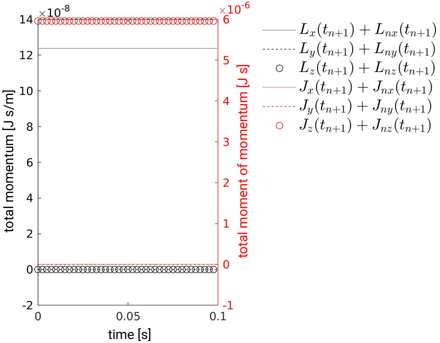
Numerical test II: LCE film under surface load

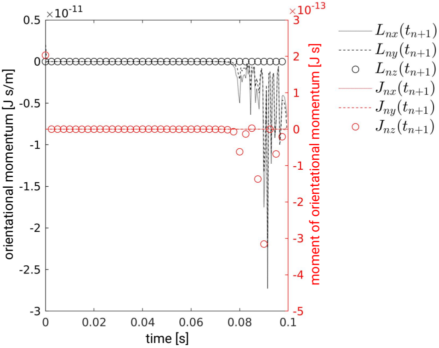
Mesh and boundary condition

$0.0125 \times 0.075 \times 0.0003$ m, 1500-em H1-standard



$$\boldsymbol{\omega}_0 = 25 \text{ rad/s } \boldsymbol{e}_z$$
$$\boldsymbol{n}_0 = \boldsymbol{e}_y$$
no BC





Numerical test III: LCE film under initial rotation

Conclusions

- formulation of the weak forms by applying the principle of virtual power
- modelling of reorientation process of nematic director through drilling degrees of freedom
- formulation of separate momentum and angular momentum balance laws associated to the deformation mapping and orientation mapping, respectively
- conserving of all momentum balances is achieved

Future work

- development of a mixed finite element formulation, with introduction of further variables, e.g. gradient of the orientation mapping
- comparison between the present standard finite element formulation with the mixed finite element formulation
- improvement of the present formulation for modelling the motion actuation of nematic LCE