Model Order Reduction of Electrical Circuits with Nonlinear Elements

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- Introduction, Motivation
- Model equations for electrical circuits with nonlinear elements
- Model order reduction
- Software package: PABTEC
- Numerical tests
- Summary

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Introduction

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- This leads to a system of model equations consisting up to millions or even more unknowns.
- Simulation of such large models is mostly impossible or, at least, unacceptably time and storage consuming.
- Model order reduction presents a way out of this dilemma.

- While the structural size of the electrical devices is decreasing, the complexity of the electrical circuits is increasing.
- This leads to a system of model equations consisting up to millions or even more unknowns.
- Simulation of such large models is mostly impossible or, at least, unacceptably time and storage consuming.
- Model order reduction presents a way out of this dilemma.
- A general idea of model order reduction is to replace a large-scale system by a much smaller model which approximates the input-output relation of the large-scale system within a required accuracy.

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Model equations

$$\begin{aligned} \mathcal{E}(x)\frac{\mathrm{d}}{\mathrm{d}t}x &= \mathcal{A}x + f(x) + \mathcal{B}u, \\ y &= \mathcal{B}^T x, \end{aligned}$$

Model equations

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states $x = \left[\ \eta^T \quad \imath_{\mathcal{L}}^T \quad \imath_{\mathcal{V}}^T\ \right]^T$

inputs $u = \left[\ \imath_{\mathcal{I}}^T \quad u_{\mathcal{V}}^T\ \right]^T$

outputs $y = -\left[\ u_{\mathcal{I}}^T \quad \imath_{\mathcal{V}}^T\ \right]^T$

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$$\mathcal{E}(x) = \begin{bmatrix} A_{\mathcal{C}}\mathcal{C}(A_{\mathcal{C}}^T\eta)A_{\mathcal{C}}^T & 0 & 0 \\ 0 & \mathcal{L}(\imath_{\mathcal{L}}) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} 0 & -A_{\mathcal{L}} & -A_{\mathcal{V}} \\ A_{\mathcal{L}}^T & 0 & 0 \\ A_{\mathcal{V}}^T & 0 & 0 \end{bmatrix},$$

$$f(x) = \begin{bmatrix} -A_{\mathcal{R}}g(A_{\mathcal{R}}^T\eta) \\ 0 \\ 0 \end{bmatrix},$$

$$\mathcal{B} = \begin{bmatrix} -A_{\mathcal{I}} & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix}.$$

Model equations

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$\mathcal{C}$, $\mathcal{R}$, $\mathcal{L}$, $\mathcal{V}$, $\mathcal{I}$ as index denote conductors, resistors, inductors, voltage sources, current sources

η vector of node potentials
$\imath_*$ vector of currents
u_* vector of volatages
A_* incidence matrices
$\mathcal{C}$ conductance matrix-valued function
$\mathcal{L}$ inductance matrix-valued function
g resistor relation

$$\begin{aligned} &\text{states} & x &= \begin{bmatrix} \eta^T & \imath_{\mathcal{L}}^T & \imath_{\mathcal{V}}^T \end{bmatrix}^T \\ &\text{inputs} & u &= \begin{bmatrix} \imath_{\mathcal{I}}^T & u_{\mathcal{V}}^T \end{bmatrix}^T \\ &\text{outputs} & y &= -\begin{bmatrix} u_{\mathcal{I}}^T & \imath_{\mathcal{V}}^T \end{bmatrix}^T \end{aligned}$$

$$\begin{aligned} \mathcal{E}(x) &= \begin{bmatrix} A_{\mathcal{C}}\mathcal{C}(A_{\mathcal{C}}^T\eta)A_{\mathcal{C}}^T & 0 & 0 \\ 0 & \mathcal{L}(\imath_{\mathcal{L}}) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \mathcal{A} &= \begin{bmatrix} 0 & -A_{\mathcal{L}} & -A_{\mathcal{V}} \\ A_{\mathcal{L}}^T & 0 & 0 \\ A_{\mathcal{V}}^T & 0 & 0 \end{bmatrix}, \\ f(x) &= \begin{bmatrix} -A_{\mathcal{R}}g(A_{\mathcal{R}}^T\eta) \\ 0 \\ 0 \end{bmatrix}, \\ \mathcal{B} &= \begin{bmatrix} -A_{\mathcal{I}} & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix}. \end{aligned}$$

Model equations - Assumption

We will assume that

(A1) the matrix $A_{\mathcal{V}}$ has full column rank,

(A2) the matrix $[A_{\mathcal{C}}, A_{\mathcal{L}}, A_{\mathcal{R}}, A_{\mathcal{V}}]$ has full row rank,

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(A3) the matrices $\mathcal{C}(A_{\mathcal{C}}^T \eta)$ and $\mathcal{L}(\imath_{\mathcal{L}})$ are positive definite for all admissible η and $\imath_{\mathcal{L}}$, and

(A4) the function $g(A_{\mathcal{R}}^T \eta)$ is monotonically increasing for all admissible η.

Model equations - Assumption

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- (A2) the matrix $[A_{\mathcal{C}}, A_{\mathcal{L}}, A_{\mathcal{R}}, A_{\mathcal{V}}]$ has full row rank,
- (A3) the matrices $\mathcal{C}(A_{\mathcal{C}}^T \eta)$ and $\mathcal{L}(\imath_{\mathcal{L}})$ are positive definite for all admissible η and $\imath_{\mathcal{L}}$, and
- (A4) the function $g(A_{\mathcal{R}}^T \eta)$ is monotonically increasing for all admissible η.

Assumptions (A1) and (A2) imply that the circuit does not contain loops of voltage sources and cutsets of current sources, respectively, while assumptions (A3) and (A4) on the capacitance and inductance matrices and the resistor relation mean that all circuit elements do not generate energy.

Model equations - Assumption

Furthermore, we assume without loss of generality that the circuit elements are ordered such that

$$A_{\mathcal{C}} = \left[\begin{array}{cc} A_{\bar{\mathcal{C}}} & A_{\widetilde{\mathcal{C}}} \end{array}\right], \qquad A_{\mathcal{L}} = \left[\begin{array}{cc} A_{\bar{\mathcal{L}}} & A_{\widetilde{\mathcal{L}}} \end{array}\right],$$
$$A_{\mathcal{R}} = \left[\begin{array}{cc} A_{\bar{\mathcal{R}}} & A_{\widetilde{\mathcal{R}}} \end{array}\right],$$

We also assume that the linear and nonlinear elements are not mutually connected, i.e.,

$$\mathcal{C}(A_{\mathcal{C}}^T\eta) = \left[\begin{array}{cc} \bar{\mathcal{C}} & 0 \\ 0 & \widetilde{\mathcal{C}}(A_{\widetilde{\mathcal{C}}}^T\eta) \end{array}\right], \; \mathcal{L}(\imath_{\mathcal{L}}) = \left[\begin{array}{cc} \bar{\mathcal{L}} & 0 \\ 0 & \widetilde{\mathcal{L}}(\imath_{\widetilde{\mathcal{L}}}) \end{array}\right],$$
$$g(A_{\mathcal{R}}^T\eta) = \left[\begin{array}{c} \bar{\mathcal{G}} A_{\bar{\mathcal{R}}}^T\eta \\ \widetilde{g}(A_{\widetilde{\mathcal{R}}}^T\eta) \end{array}\right],$$

Model equations

Consequently, we have the model equations in the form

$$\begin{aligned}\mathcal{E}(x)\frac{\mathrm{d}}{\mathrm{d}t}x &= \mathcal{A}x + f(x) + \mathcal{B}u,\\ y &= \mathcal{B}^T x,\end{aligned}$$

with

$$\mathcal{E}(x) = \begin{bmatrix} A_{\bar{\mathcal{C}}}\bar{\mathcal{C}}A_{\bar{\mathcal{C}}}^T + A_{\widetilde{\mathcal{C}}}\widetilde{\mathcal{C}}(A_{\widetilde{\mathcal{C}}}^T\eta)A_{\widetilde{\mathcal{C}}}^T & 0 & 0 & 0\\ 0 & \bar{\mathcal{L}} & 0 & 0\\ 0 & 0 & \widetilde{\mathcal{L}}(\imath_{\widetilde{\mathcal{L}}}) & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} -A_{\widetilde{\mathcal{R}}}\widetilde{g}(A_{\widetilde{\mathcal{R}}}^T\eta)\\ 0\\ 0\\ 0 \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} A_{\bar{\mathcal{R}}}\bar{\mathcal{G}}A_{\bar{\mathcal{R}}}^T & -A_{\bar{\mathcal{L}}} & -A_{\widetilde{\mathcal{L}}} & -A_{\mathcal{V}}\\ A_{\bar{\mathcal{L}}}^T & 0 & 0 & 0\\ A_{\widetilde{\mathcal{L}}}^T & 0 & 0 & 0\\ A_{\mathcal{V}}^T & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} -A_{\mathcal{I}} & 0\\ 0 & 0\\ 0 & 0\\ 0 & -I \end{bmatrix}.$$

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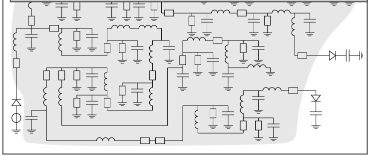
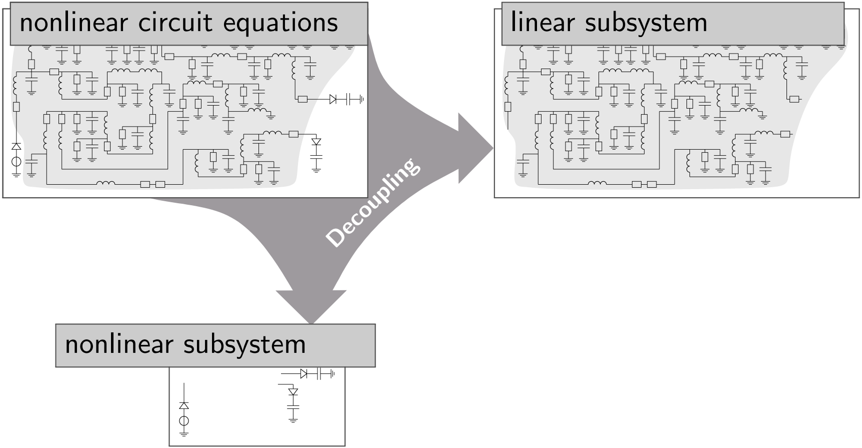
Model Order Reduction - Approach

nonlinear circuit equations



Model Order Reduction - Approach

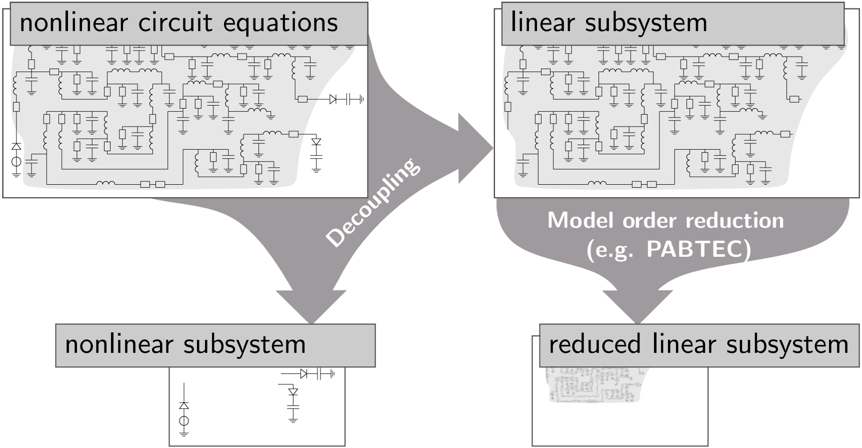


nonlinear circuit equations

linear subsystem

nonlinear subsystem

Model Order Reduction - Approach



nonlinear circuit equations

linear subsystem

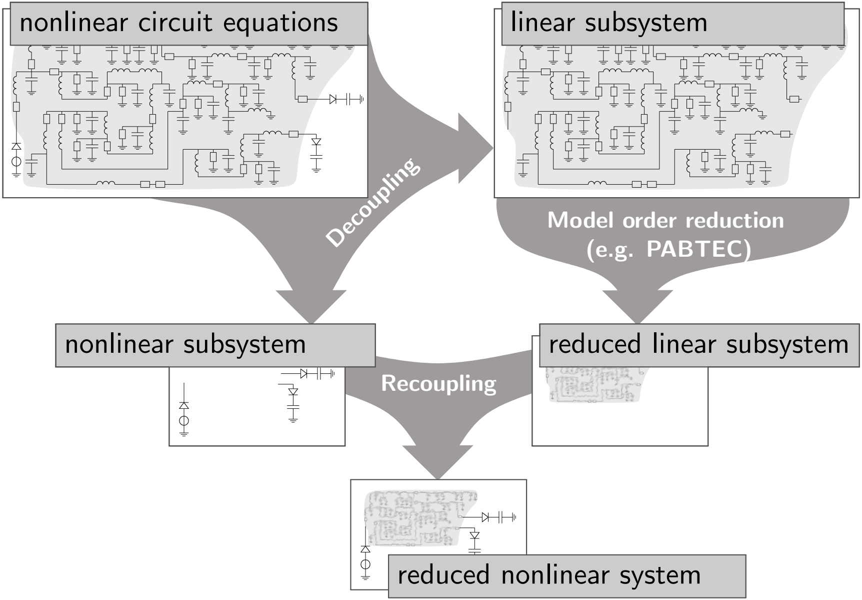
Decoupling

Model order reduction
(e.g. PABTEC)

nonlinear subsystem

reduced linear subsystem

Model Order Reduction - Approach



nonlinear circuit equations

linear subsystem

Decoupling

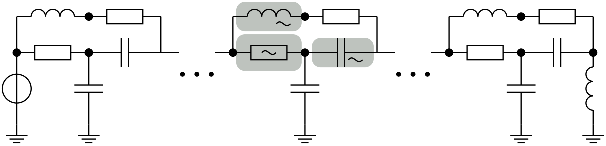
Model order reduction
(e.g. PABTEC)

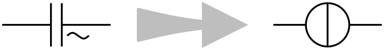
nonlinear subsystem

reduced linear subsystem

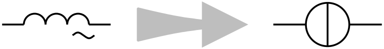
Recoupling

reduced nonlinear system





:-(possibly creation of LI-cutsets



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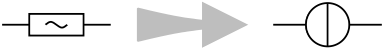

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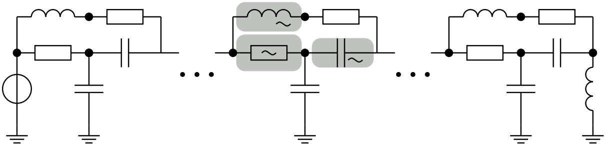
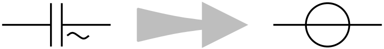
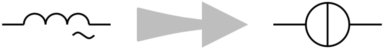
Model Order Reduction - Decoupling





:-/ introduction of additional variables



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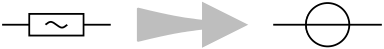
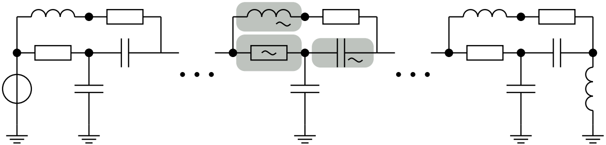
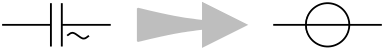


:-(possibly creation of CV-loops

Model Order Reduction - Decoupling





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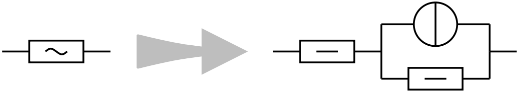
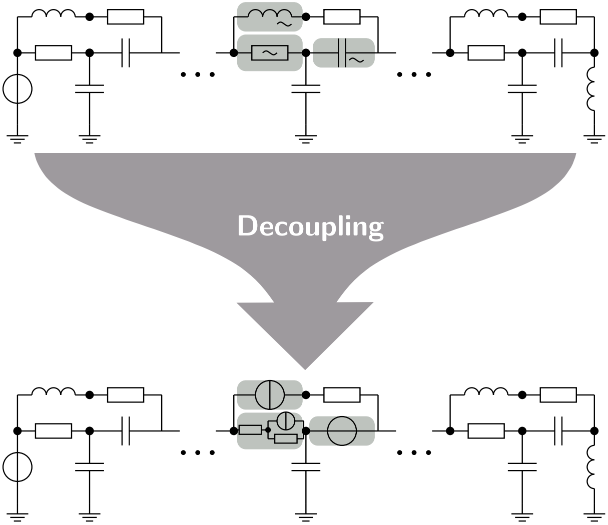


:-/ introduction of additional nodes

Model Order Reduction - Decoupling



Model Order Reduction - Decoupling

Let $A_{\widetilde{\mathcal{R}}} \in \{-1,0,1\}^{n_\eta, n_{\widetilde{\mathcal{R}}}}$ be an incidence matrix. Then the matrices $A^1_{\widetilde{\mathcal{R}}}$ and $A^2_{\widetilde{\mathcal{R}}}$ are uniquely defined with $A^1_{\widetilde{\mathcal{R}}} \in \{0,1\}^{n_\eta, n_{\widetilde{\mathcal{R}}}}$ and $A^2_{\widetilde{\mathcal{R}}} \in \{-1,0\}^{n_\eta, n_{\widetilde{\mathcal{R}}}}$ satisfying $A^1_{\widetilde{\mathcal{R}}} + A^2_{\widetilde{\mathcal{R}}} = A_{\widetilde{\mathcal{R}}}$.
Furthermore, let $\imath_{\widetilde{\mathcal{L}}} \in \mathbb{R}^{n_{\widetilde{L}}}$, $u_{\widetilde{\mathcal{C}}} \in \mathbb{R}^{n_{\widetilde{C}}}$, and $\imath_z \in \mathbb{R}^{n_{\widetilde{R}}}$ be defined by the relations

$$\widetilde{\mathcal{L}}(\imath_{\widetilde{\mathcal{L}}}) \frac{\mathrm{d}}{\mathrm{d}t} \imath_{\widetilde{\mathcal{L}}} = A^T_{\widetilde{\mathcal{L}}} \eta, \tag{1}$$

$$u_{\widetilde{\mathcal{C}}} = A^T_{\widetilde{\mathcal{C}}} \eta, \tag{2}$$

$$\imath_z = \Gamma_s G_1^{-1} (\widetilde{g}(A^T_{\widetilde{\mathcal{R}}} \eta) - \Gamma_{12} A^T_{\widetilde{\mathcal{R}}} \eta) \tag{3}$$

with the notation

$$\begin{aligned} \Gamma_s &= G_1 + G_2, \\ \Gamma_{12} &= G_1 (G_1 + G_2)^{-1} G_2. \end{aligned}$$

Model Order Reduction - Decoupling

Then the original system of model equations together with the relations

$$\eta_z = \Gamma_s^{-1}((G_1(A^1_{\widetilde{\mathcal{R}}})^T - G_2(A^2_{\widetilde{\mathcal{R}}})^T)\eta - \imath_z), \tag{4a}$$

$$\imath_{\widetilde{\mathcal{C}}} = \widetilde{\mathcal{C}}(u_{\widetilde{\mathcal{C}}})\frac{\mathrm{d}}{\mathrm{d}t}u_{\widetilde{\mathcal{C}}} \tag{4b}$$

is equivalent to the linear system

$$\begin{bmatrix} A_{\bar{\mathcal{C}}}\bar{\mathcal{C}}A_{\bar{\mathcal{C}}}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\mathcal{L}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}\eta \\ \frac{\mathrm{d}}{\mathrm{d}t}\eta_z \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\bar{\mathcal{L}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\mathcal{V}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\widetilde{\mathcal{C}}} \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & -A_{\bar{\mathcal{L}}} & -A_{\mathcal{V}} & -A_{\widetilde{\mathcal{C}}} \\ \mathcal{A}_{12}^T & -\Gamma_s & 0 & 0 & 0 \\ A_{\bar{\mathcal{L}}}^T & 0 & 0 & 0 & 0 \\ A_{\mathcal{V}}^T & 0 & 0 & 0 & 0 \\ A_{\widetilde{\mathcal{C}}}^T & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \eta_z \\ \imath_{\bar{\mathcal{L}}} \\ \imath_{\mathcal{V}} \\ \imath_{\widetilde{\mathcal{C}}} \end{bmatrix} \tag{5a}$$

$$+ \begin{bmatrix} -A_{\mathcal{I}} & -A^2_{\widetilde{\mathcal{R}}} & -A_{\widetilde{\mathcal{L}}} & 0 & 0 \\ 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} -A_{\mathcal{I}}^T & 0 & 0 & 0 & 0 \\ -(A^2_{\widetilde{\mathcal{R}}})^T & -I & 0 & 0 & 0 \\ -A_{\widetilde{\mathcal{L}}}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} \eta \\ \eta_z \\ \imath_{\bar{\mathcal{L}}} \\ \imath_{\mathcal{V}} \\ \imath_{\widetilde{\mathcal{C}}} \end{bmatrix} \tag{5b}$$

Model Order Reduction - Decoupling

Then the original system of model equations together with the relations

$$\begin{aligned} \eta_z &= \Gamma_s^{-1}\big((G_1(A^1_{\widetilde{\mathcal{R}}})^T - G_2(A^2_{\widetilde{\mathcal{R}}})^T)\eta - \imath_z\big), && (4a) \\ \imath_{\widetilde{\mathcal{C}}} &= \widetilde{\mathcal{C}}(u_{\widetilde{\mathcal{C}}})\frac{\mathrm{d}}{\mathrm{d}t}u_{\widetilde{\mathcal{C}}} && (4b) \end{aligned}$$

is equivalent to the linear system

$$\begin{bmatrix} A_{\bar{\mathcal{C}}}\bar{\mathcal{C}}A_{\bar{\mathcal{C}}}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\mathcal{L}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}\eta \\ \frac{\mathrm{d}}{\mathrm{d}t}\eta_z \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\bar{\mathcal{L}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\mathcal{V}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\imath_{\widetilde{\mathcal{C}}} \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & -A_{\bar{\mathcal{L}}} & -A_{\mathcal{V}} & -A_{\widetilde{\mathcal{C}}} \\ \mathcal{A}_{12}^T & -\Gamma_s & 0 & 0 & 0 \\ A_{\bar{\mathcal{L}}}^T & 0 & 0 & 0 & 0 \\ A_{\mathcal{V}}^T & 0 & 0 & 0 & 0 \\ A_{\widetilde{\mathcal{C}}}^T & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \eta_z \\ \imath_{\bar{\mathcal{L}}} \\ \imath_{\mathcal{V}} \\ \imath_{\widetilde{\mathcal{C}}} \end{bmatrix} \qquad (5a)$$

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with

$$\begin{aligned} \mathcal{A}_{11} &= -A_{\bar{\mathcal{R}}}\mathcal{G}A_{\bar{\mathcal{R}}}^T - A^1_{\widetilde{\mathcal{R}}}G_1(A^1_{\widetilde{\mathcal{R}}})^T \\ &\quad -A^2_{\widetilde{\mathcal{R}}}G_2(A^2_{\widetilde{\mathcal{R}}})^T, \\ \mathcal{A}_{12} &= A^1_{\widetilde{\mathcal{R}}}G_1^T - A^2_{\widetilde{\mathcal{R}}}(G_2)^T \end{aligned}$$

Model Order Reduction - Decoupling

Then the original system of model equations together with the relations

$$\begin{aligned} \eta_z &= \Gamma_s^{-1}((G_1(A^1_{\widetilde{\mathcal{R}}})^T - G_2(A^2_{\widetilde{\mathcal{R}}})^T)\eta - \imath_z), && (4a) \\ \imath_{\widetilde{\mathcal{C}}} &= \widetilde{\mathcal{C}}(u_{\widetilde{\mathcal{C}}})\frac{\mathrm{d}}{\mathrm{d}t}u_{\widetilde{\mathcal{C}}} && (4b) \end{aligned}$$

is equivalent to the linear system

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$$+ \begin{bmatrix} -A_{\mathcal{I}} & -A^2_{\widetilde{\mathcal{R}}} & -A_{\widetilde{\mathcal{L}}} & 0 & 0 \\ 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

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Keep in mind

$$\begin{aligned} y_2 &= -(A^2_{\widetilde{\mathcal{R}}})^T\eta - \eta_z \\ y_3 &= -A^T_{\widetilde{\mathcal{L}}}\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{x} = \hat{\mathcal{A}}\hat{x} + \left[\begin{array}{ccccc} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{array}\right] \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix}, \tag{6a}$$

$$\hat{y} = \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}. \tag{6b}$$

Keep in mind

$$\begin{aligned} y_2 &= -(A^2_{\widetilde{\mathcal{R}}})^T\eta - \eta_z \\ y_3 &= -A^T_{\widetilde{\mathcal{L}}}\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{x} = \hat{\mathcal{A}}\hat{x} + \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \\ \hat{y}_5 \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}.$$

Keep in mind

$$\begin{aligned} y_2 &= -(A^2_{\widetilde{\mathcal{R}}})^T\eta - \eta_z \\ y_3 &= -A^T_{\widetilde{\mathcal{L}}}\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{x} = \hat{\mathcal{A}}\hat{x} + \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} \approx \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \\ \hat{y}_5 \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}.$$

Keep in mind

$$\begin{aligned} y_2 &= -(A_{\widetilde{\mathcal{R}}}^2)^T\eta - \eta_z \\ y_3 &= -A_{\widetilde{\mathcal{L}}}^T\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{x} = \hat{\mathcal{A}}\hat{x} + \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} \approx \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}.$$

Keep in mind

$$\begin{aligned} y_2 &= -(A_{\widetilde{\mathcal{R}}}^2)^T\eta - \eta_z \\ y_3 &= -A_{\widetilde{\mathcal{L}}}^T\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}} \frac{\mathrm{d}}{\mathrm{d}t} \hat{x} = \hat{\mathcal{A}} \hat{x} + \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} \\ -(A^2_{\widetilde{\mathcal{R}}})^T \eta - \eta_z \\ -A^T_{\widetilde{\mathcal{L}}} \eta \\ \\ -\imath_{\widetilde{\mathcal{C}}} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} \approx \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}.$$

Keep in mind

$$\begin{aligned} y_2 &= -(A^2_{\widetilde{\mathcal{R}}})^T \eta - \eta_z \\ y_3 &= -A^T_{\widetilde{\mathcal{L}}} \eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Model reduction of the linear subsystem

Application of a model order reduction methode like PABTECL yields the reduced-order model

$$\hat{\mathcal{E}}\frac{\mathrm{d}}{\mathrm{d}t}\hat{x} = \hat{\mathcal{A}}\hat{x} + \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_4 & \hat{\mathcal{B}}_5 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ \imath_z \\ \imath_{\widetilde{\mathcal{L}}} \\ u_{\mathcal{V}} \\ u_{\widetilde{\mathcal{C}}} \end{bmatrix},$$

$$\begin{bmatrix} \\ -(A^2_{\widetilde{\mathcal{R}}})^T\eta - \eta_z \\ -A^T_{\widetilde{\mathcal{L}}}\eta \\ \\ -\imath_{\widetilde{\mathcal{C}}} \end{bmatrix} \approx \begin{bmatrix} \hat{\mathcal{C}}_1 \\ \hat{\mathcal{C}}_2 \\ \hat{\mathcal{C}}_3 \\ \hat{\mathcal{C}}_4 \\ \hat{\mathcal{C}}_5 \end{bmatrix} \hat{x}.$$

Keep in mind

$$\begin{aligned} y_2 &= -(A^2_{\widetilde{\mathcal{R}}})^T\eta - \eta_z \\ y_3 &= -A^T_{\widetilde{\mathcal{L}}}\eta \\ y_5 &= -\imath_{\widetilde{\mathcal{C}}} \end{aligned}$$

Model Order Reduction - Recoupling

We have

$$-(A_{\widetilde{\mathcal{R}}}^2)^T\eta - \eta_z \approx \hat{\mathcal{C}}_2\hat{x}, \tag{12a}$$

$$-A_{\widetilde{\mathcal{L}}}^T\eta \approx \hat{\mathcal{C}}_3\hat{x}, \tag{12b}$$

$$-\imath_{\widetilde{\mathcal{C}}} \approx \hat{\mathcal{C}}_5\hat{x}. \tag{12c}$$

Then we get from (1), (3), (4a), and (4b) the relations

$$\widetilde{\mathcal{L}}(\hat{\imath}_{\widetilde{\mathcal{L}}})\frac{\mathrm{d}}{\mathrm{d}t}\hat{\imath}_{\widetilde{\mathcal{L}}} = -\hat{\mathcal{C}}_3\hat{x} \tag{13a}$$

$$\widetilde{\mathcal{C}}(\hat{u}_{\widetilde{\mathcal{C}}})\frac{\mathrm{d}}{\mathrm{d}t}\hat{u}_{\widetilde{\mathcal{C}}} = -\hat{\mathcal{C}}_5\hat{x}, \tag{13b}$$

$$0 = -G_1\hat{\mathcal{C}}_2\hat{x} - G_1\hat{u}_{\widetilde{\mathcal{R}}} + \widetilde{g}(\hat{u}_{\widetilde{\mathcal{R}}}) \tag{13c}$$

and

$$\imath_z = \Gamma_s G_1^{-1}\widetilde{g}(u_{\widetilde{\mathcal{R}}}) - G_2 u_{\widetilde{\mathcal{R}}}, \tag{14}$$

where $\hat{\imath}_{\widetilde{\mathcal{L}}}$, $\hat{u}_{\widetilde{\mathcal{C}}}$, and $\hat{u}_{\widetilde{\mathcal{R}}}$ are approximations for $\imath_{\widetilde{\mathcal{L}}}$, $u_{\widetilde{\mathcal{C}}}$, and $u_{\widetilde{\mathcal{R}}}$, respectively.

Model Order Reduction - Recoupling

Now, adding (13a), (13b), and (13c) to (6) and using in addition to $\hat{x}$ also the approximations $\hat{\imath}_{\widetilde{\mathcal{L}}}$, $\hat{u}_{\widetilde{\mathcal{C}}}$, and $\hat{u}_{\widetilde{\mathcal{R}}}$ as state variables, then we get with (14) the descriptor system

$$\begin{bmatrix} \hat{\mathcal{E}} & 0 & 0 & 0 \\ 0 & \widetilde{\mathcal{L}}(\hat{\imath}_{\widetilde{\mathcal{L}}}) & 0 & 0 \\ 0 & 0 & \widetilde{\mathcal{C}}(\hat{u}_{\widetilde{\mathcal{C}}}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}\hat{x} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{\imath}_{\widetilde{\mathcal{L}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{u}_{\widetilde{\mathcal{C}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{A}} & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_5 & -\hat{\mathcal{B}}_2 G_2 \\ -\hat{\mathcal{C}}_3 & 0 & 0 & 0 \\ -\hat{\mathcal{C}}_5 & 0 & 0 & 0 \\ -G_1\hat{\mathcal{C}}_2 & 0 & 0 & -G_1 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\imath}_{\widetilde{\mathcal{L}}} \\ \hat{u}_{\widetilde{\mathcal{C}}} \\ \hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ u_{\mathcal{V}} \end{bmatrix} + \begin{bmatrix} \hat{\mathcal{B}}_2 \Gamma_s G_1^{-1} \widetilde{g}(\hat{u}_{\widetilde{\mathcal{R}}}) \\ 0 \\ 0 \\ \widetilde{g}(\hat{u}_{\widetilde{\mathcal{R}}}) \end{bmatrix},$$

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_4 \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{C}}_1 & 0 & 0 & 0 \\ \hat{\mathcal{C}}_4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\imath}_{\widetilde{\mathcal{L}}} \\ \hat{u}_{\widetilde{\mathcal{C}}} \\ \hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix}$$

Model Order Reduction - Reduced decoupled system

With row manipulations of the state equations we obtain, finally, the nonlinear descriptor system

$$\begin{bmatrix} \hat{\mathcal{E}} & 0 & 0 & 0 \\ 0 & \widetilde{\mathcal{L}}(\hat{\imath}_{\widetilde{\mathcal{L}}}) & 0 & 0 \\ 0 & 0 & \widetilde{\mathcal{C}}(\hat{u}_{\widetilde{\mathcal{C}}}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}\hat{x} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{\imath}_{\widetilde{\mathcal{L}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{u}_{\widetilde{\mathcal{C}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{A}} + \hat{\mathcal{B}}_2\Gamma_s\hat{\mathcal{C}}_2 & \hat{\mathcal{B}}_3 & \hat{\mathcal{B}}_5 & \hat{\mathcal{B}}_2 G_1 \\ -\hat{\mathcal{C}}_3 & 0 & 0 & 0 \\ -\hat{\mathcal{C}}_5 & 0 & 0 & 0 \\ -G_1\hat{\mathcal{C}}_2 & 0 & 0 & -G_1 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\imath}_{\widetilde{\mathcal{L}}} \\ \hat{u}_{\widetilde{\mathcal{C}}} \\ \hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix}$$

$$+ \begin{bmatrix} \hat{\mathcal{B}}_1 & \hat{\mathcal{B}}_4 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \imath_{\mathcal{I}} \\ u_{\mathcal{V}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \widetilde{g}(\hat{u}_{\widetilde{\mathcal{R}}}) \end{bmatrix},$$

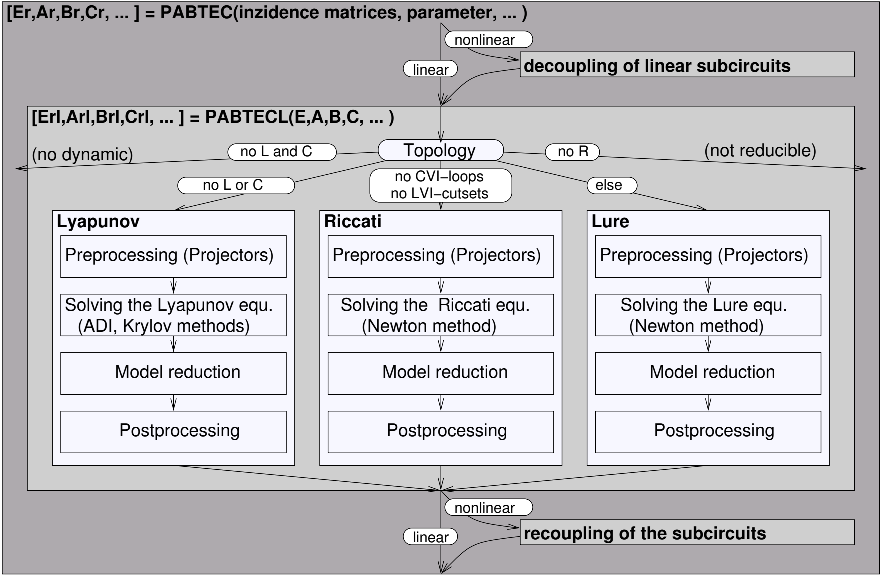
$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_4 \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{C}}_1 & 0 & 0 & 0 \\ \hat{\mathcal{C}}_4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{\imath}_{\widetilde{\mathcal{L}}} \\ \hat{u}_{\widetilde{\mathcal{C}}} \\ \hat{u}_{\widetilde{\mathcal{R}}} \end{bmatrix}$$

that approximate the original nonlinear system of model equations.

Contents:

- Introduction, Motivation
- Model equations for electrical circuits with nonlinear elements
- Model order reduction
- Software package: PABTEC
- Numerical tests
- Summary

MATLAB-Toolbox: PABTEC



[Er,Ar,Br,Cr, ...] = PABTEC(inzidence matrices, parameter, ...)

nonlinear

decoupling of linear subcircuits

linear

[Erl,Arl,Brl,Crl, ...] = PABTECL(E,A,B,C, ...)

(no dynamic)

no L and C

Topology

no R

(not reducible)

no L or C

no CVI–loops
no LVI–cutsets

else

Lyapunov

Preprocessing (Projectors)

Solving the Lyapunov equ.
(ADI, Krylov methods)

Model reduction

Postprocessing

Riccati

Preprocessing (Projectors)

Solving the Riccati equ.
(Newton method)

Model reduction

Postprocessing

Lure

Preprocessing (Projectors)

Solving the Lure equ.
(Newton method)

Model reduction

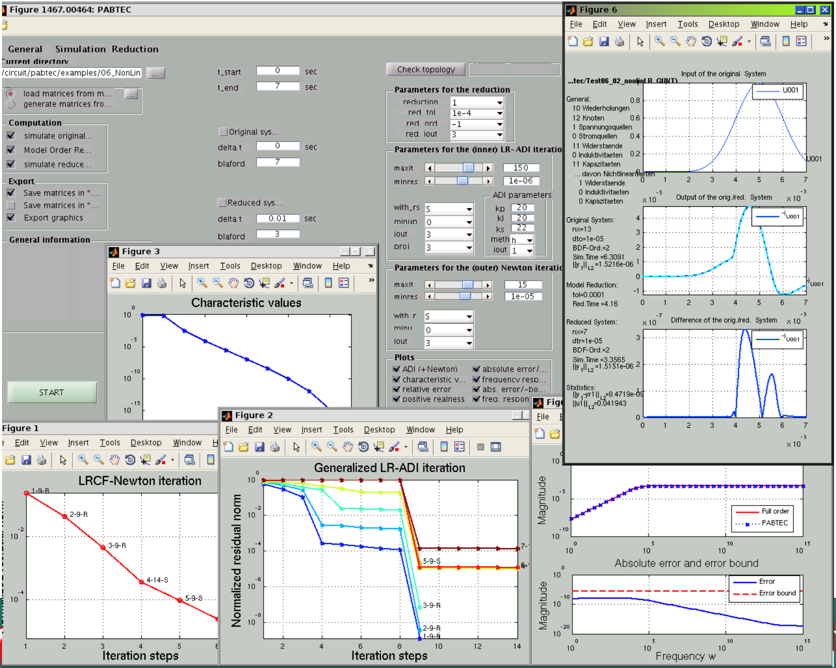
Postprocessing

nonlinear

recoupling of the subcircuits

linear

MATLAB-Toolbox: PABTEC



Current directory: /circuit/pabtec/examples/06_NonLin

● load matrices from m...
○ generate matrices fro...

Computation
- ☑ simulate original...
- ☑ Model Order Re...
- ☑ simulate reduce...

Export
- ☑ Save matrices in *....
- ☐ Save matrices in *...
- ☑ Export graphics

General information

t_start 0 sec
t_end 7 sec

☐ Original sys...
delta t 0 sec
blaford 7

☐ Reduced sys...
delta t 0.01 sec
blaford 3

Parameters for the reduction
reduction 1
red. tol 1e-4
red. ord -1
red. iout 3

Parameters for the (inner) LR-ADI iteratio
maxit 150
minres 1e-06
with_rs S
minun 0
iout 3
proj 3

ADI parameters
kp 20
kl 20
ks 22
meth h
iout 1

Parameters for the (outer) Newton iteratio
maxit 15
minres 1e-05
with_r S
minu 0
iout 3

Plots
- ☑ ADI (+Newton)
- ☑ characteristic v...
- ☑ relative error
- ☑ positive realness
- ☑ absolute error/...
- ☑ frequency resp...
- ☑ abs. error/-bo...
- ☑ freq. respon

...tec/Test06_02_nonlinLB_GUI(T)

General:
10 Wiederholungen
12 Knoten
1 Spannungsquellen
0 Stromquellen
11 Widerstaende
0 Induktivitaeten
11 Kapazitaeten
... davon Nichtlinearitaeten
1 Widerstaende
0 Induktivitaeten
0 Kapazitaeten

Original System:
nx=13
dto=1e-05
BDF-Ord.=2
Sim.Time =6.3091
$\|y_1\|_{L2}$=1.5216e-06

Model Reduction:
tol=0.0001
Red.Time =4.16

Reduced System:
nx=7
dtr=1e-05
BDF-Ord.=2
Sim.Time =3.3565
$\|y_1\|_{L2}$=1.5151e-06

Statistics:
$\|y_1-yr1\|_{L2}$=8.4719e-09
$\|u1\|_{L2}$=0.041943

Contents:

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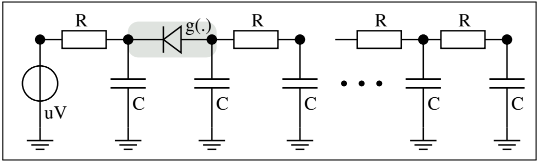
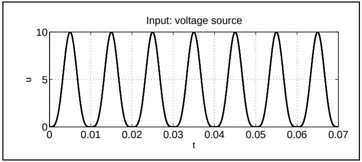
Example 01 - Problem $n = 503$



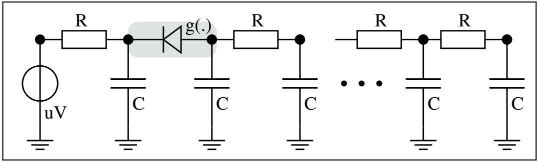


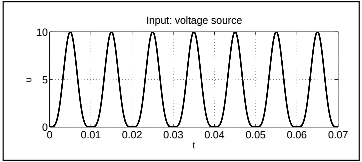
502	nodes	1	voltage source
501	linear capacities	1	output
500	linear resistors	1	diode

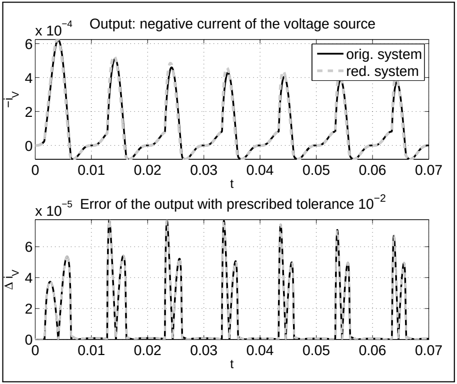
State dimension of the model equations $n = 503$

Simulation is done for $t \in [0s, 0.07s]$ using BDF method of order 2 with fixed stepsize of length $1 \cdot 10^{-5}$. The computations are done with MATLAB.

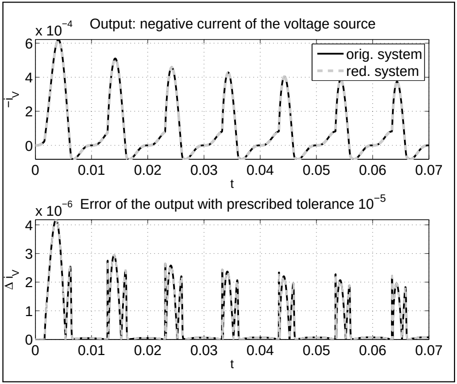
Example 01 - Simulation results $n = 503$




Input: voltage source

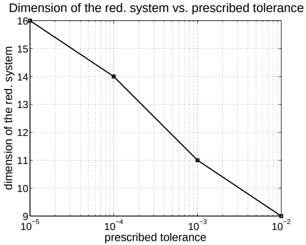

Output: negative current of the voltage source

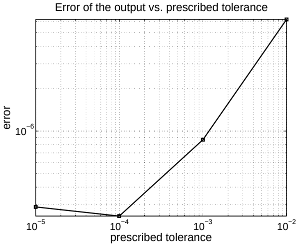
Error of the output with prescribed tolerance 10^{-2}

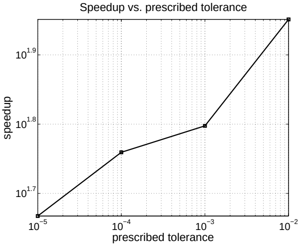

Output: negative current of the voltage source

Error of the output with prescribed tolerance 10^{-5}

Example 01 - Efficiency $n = 503$

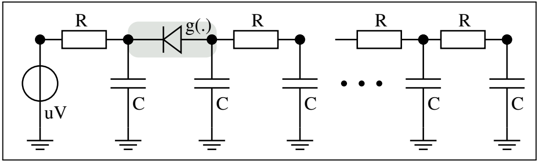


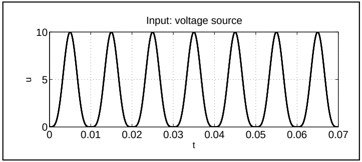




dimension of the original system	503	503	503	503
simulation time for the original system	6772s	6772s	6772s	6772s
prescribed tolerance for the model reduction	1e-02	1e-03	1e-04	1e-05
time for the model reduction	7s	8s	27s	46s
dimension of the reduced system	9	11	14	16
simulation time for the reduced system	76s	108s	118s	146s
obtained error of the output of the red. system	6.2e-06	8.7e-07	2.5e-07	2.9e-07
speedup	89.4	62.7	57.5	46.5

Example 01 - Problem $n = 1503$





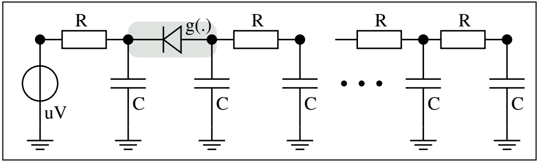
1502	nodes	1	voltage source
1501	linear capacities	1	output
1500	linear resistors	1	diode

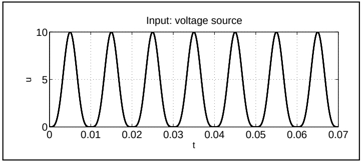
State dimension of the model equations $n = 1503$

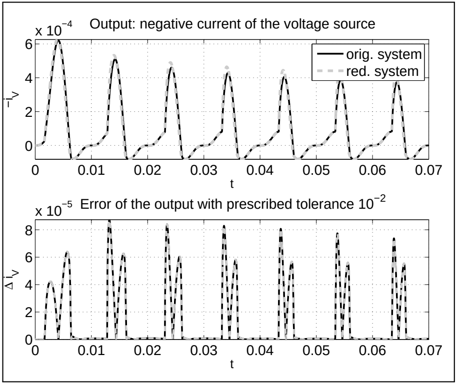
Simulation is done for $t \in [0s, 0.07s]$ using BDF method of order 2 with fixed stepsize of length $1 \cdot 10^{-5}$. The computations are done with MATLAB.

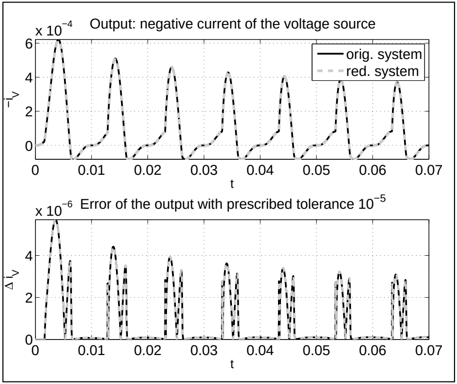
Example 01 - Simulation results

$n = 1503$



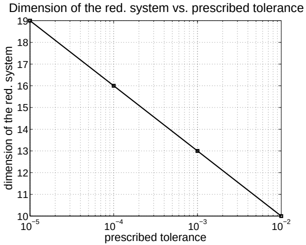


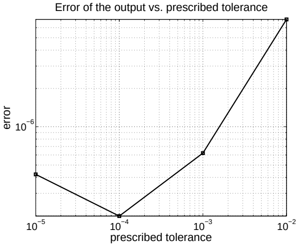


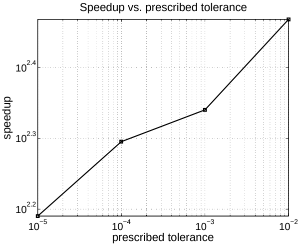


Example 01 - Efficiency

$n = 1503$

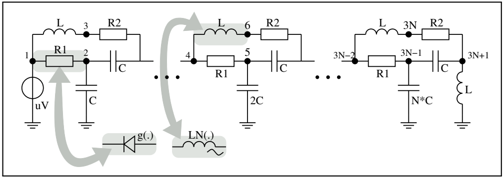

Dimension of the red. system vs. prescribed tolerance

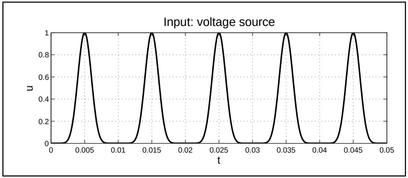

Error of the output vs. prescribed tolerance


Speedup vs. prescribed tolerance

dimension of the original system	1503	1503	1503	1503
simulation time for the original system	24012s	24012s	24012s	24012s
prescribed tolerance for the model reduction	1e-02	1e-03	1e-04	1e-05
time for the model reduction	15s	24s	42s	61s
dimension of the reduced system	10	13	16	19
simulation time for the reduced system	82s	110s	122s	155s
obtained error of the output of the red. system	7.0e-06	6.2e-07	2.0e-07	4.2e-07
speedup	294.0	219.0	197.4	155.0

Example 02 - Problem



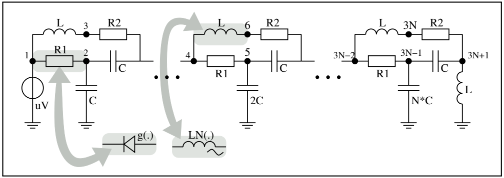


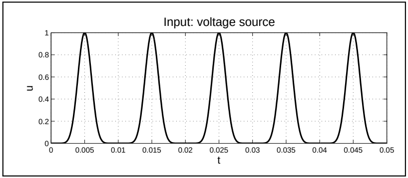
3001	nodes	1	voltage source
2000	linear capacities	1	output
1990	linear resistors	10	diode
991	linear inductors	10	nonlinear inductors

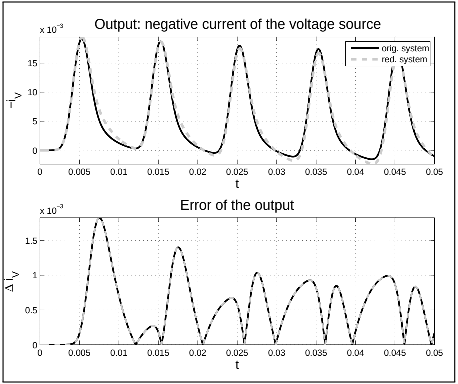
State dimension of the model equations $n = 4003$

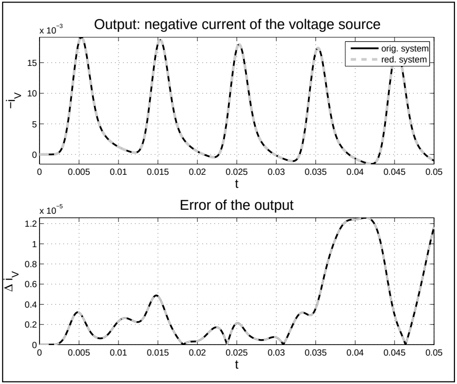
Simulation is done for $t \in [0s, 0.05s]$ using BDF method of order 2 with fixed stepsize of length $5 \cdot 10^{-5}$. The computations are done with MATLAB.

Example 02 - Simulation results

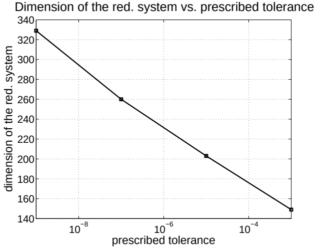


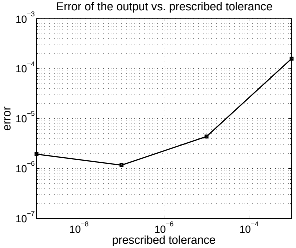


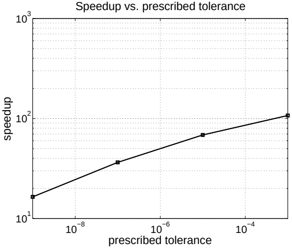




Example 02 - Efficiency


Dimension of the red. system vs. prescribed tolerance


Error of the output vs. prescribed tolerance


Speedup vs. prescribed tolerance

dimension of the original system	4003	4003	4003	4003
simulation time for the original system	4557s	4557s	4557s	4557s
prescribed tolerance for the model reduction	1e-03	1e-05	1e-07	1e-09
time for the model reduction	902s	822s	834s	900s
dimension of the reduced system	149	203	260	329
simulation time for the reduced system	43s	67s	125s	277s
obtained error of the output of the red. system	1.6e-04	4.4e-06	1.16e-06	1.9e-06
speedup	107.2	68.5	36.5	16.5

Contents:

- Introduction, Motivation
- Model equations for electrical circuits with nonlinear elements
- Model order reduction
- Software package: PABTEC
- Numerical tests
- Summary

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    - model reduction of the remained linear part
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Summary

- We developed a model order reduction approach for the model equations of nonlinear circuits.
- The developed model reduction technique bases on ...
  - decoupling of linear and nonlinear subcircuits
  - model reduction of the remained linear part
  - recoupling of the reduced linear subcircuit with the unchanged nonlinear subcircuit
- The efficency and applicability of the proposed model reduction approach was demonstrated on several numerical examples.

Workshop: MODRED 2010

MODEL REDUCTION FOR COMPLEX DYNAMICAL SYSTEMS

December 2-4, 2010

TU Berlin, Germany

Invited speakers:	Michel S. Nakhla	(Carleton University, Ottawa)
	Joel R. Phillips	(Cadence Berkeley Laboratories, San Jose)
	Timo Reis	(TU Hamburg-Harburg)

Important date: Registration: November 1, 2010

http://www3.math.tu-berlin.de/modred2010

Thank you for your attantion.