# POD Model Order Reduction for Coupled Systems of Integrated Circuits

## Application of Discrete Empirical Interpolation

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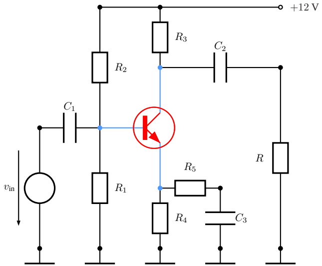
# Overview




## Goal:

Efficient simulation of circuits containing semiconductors.

Our approach:

- Describe the semiconductors by PDEs (drift-diffusion equations).
- Discretize PDEs in space by FEM, obtain a high dimensional DAE for the circuit. $\rightarrow$ “method of lines”.
- Integrate DAE for a set of inputs and take snapshots.
- Perform a POD on the snapshot trajectories, obtain a reduced state-space approximation.
- Again discretize in space; but: singular vectors as ansatz functions.

# Outline

Coupled model and FEM discretization

Construction of the reduced model

Application of Discrete Empirical Interpolation

Residual based parameter adaptivity

Outlook

# Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

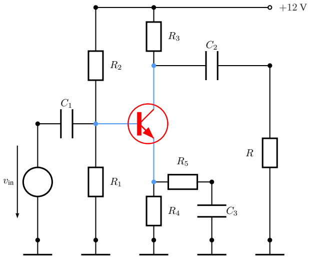
Kirchhoff's' laws (no semiconductors) read

$$Aj = 0, \quad v = A^\top e$$

$A$: (reduced) incidence matrix.

Voltage-current relations of components:

$$j_C = \frac{\mathrm{d}q_C}{\mathrm{d}t}(v_C, t), \; j_R = g(v_R, t), \; v_L = \frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L, t)$$



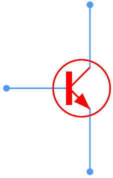

Modified Nodal Analysis: join all equations to DAE system

$$\begin{aligned} A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right) + A_L j_L(t) + A_V j_V(t) &= -A_I i_s(t), \\ \frac{\mathrm{d}\phi_L}{\mathrm{d}t}\left(j_L(t), t\right) - A_L^\top e(t) &= 0, \\ A_V^\top e(t) &= v_s(t). \end{aligned}$$

# Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

PDE-model (drift-diffusion equations) for semiconductors

$$
\begin{aligned}
\operatorname{div}(\varepsilon \nabla \psi) &= q(n-p-C), \\
-q\partial_t n + \operatorname{div} J_n &= \quad qR(n,p), \\
q\partial_t p + \operatorname{div} J_p &= -qR(n,p), \\
J_n &= \mu_n q(\quad U_T \nabla n - n\nabla\psi), \\
J_p &= \mu_p q(-U_T \nabla p - p\nabla\psi),
\end{aligned}
$$



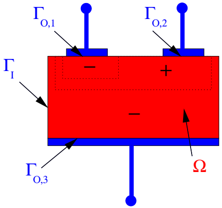
on $\Omega \times [0, T]$ with $\Omega \subset \mathbb{R}^d$ ($d = 1, 2, 3$).
Dirichlet boundary constraints at $\Gamma_{O,k}$:

$$\psi(t,x), \quad n(t,x) = \tilde{n}(x), \quad p(t,x) = \tilde{p}(x)$$

and Neumann boundary constraints at $\Gamma_I$:

$$\nabla\psi(t,x)\cdot\nu(x) = J_n \cdot \nu(x) = J_p(t,x)\cdot\nu(x) = 0$$

or mixed boundary conditions at MI contacts (MOSFETs).

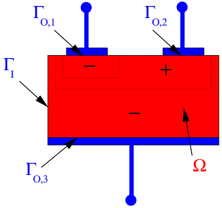


# Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

Coupling conditions:

$$j_{S,k}(t) = \int_{\Gamma_{O,k}} (J_n + J_p - \varepsilon \partial_t \nabla \psi) \cdot \boldsymbol{\nu} \, d\boldsymbol{\sigma},$$

$$\psi(t,x) = \psi_{bi}(x) + (A_S^\top e(t))_k \quad \text{for } (t,x) \in [0,T] \times \Gamma_{O,k},$$




and add current $j_S$ to Kirchhoff's current law:

$$A_C \frac{dq_C}{dt}\left(A_C^\top e, t\right) + A_R g\left(A_R^\top e, t\right) + A_L j_L + A_V j_V + A_S j_S = -A_I i_s,$$

$$\frac{d\phi_L}{dt}(j_L, t) - A_L^\top e = 0,$$

$$A_V^\top e = v_s.$$

Add DD-equations + coupling conditions for each semiconductor.

# Mixed formulation

$E = -\nabla\psi$ plays dominant role in DD-equations.

## Mixed formulation

[Brezzi et al. '05]

Provide additional variable $g_\psi$ and equation

$$g_\psi = \nabla\psi.$$

Scaled DD equations then read:

$$\begin{aligned}
\lambda \operatorname{div} g_\psi &= n - p - C, \\
-\partial_t n + \nu_n \operatorname{div} J_n &= \quad R(n,p), \\
\partial_t p + \nu_p \operatorname{div} J_p &= -R(n,p), \\
g_\psi &= \quad \nabla\psi, \\
J_n &= \quad \nabla n - n g_\psi, \\
J_p &= -\nabla p - p g_\psi.
\end{aligned}$$

# Finite Element approximation

## Finite elements

- piecewise constant ansatz functions for $\psi$, $n$ and $p$.
  Basis functions: $\varphi_i, \quad i = 1, \dots, N, \quad N = |\mathcal{T}|$.
- Raviart-Thomas elements for $g_\psi$, $J_n$ and $J_p$.
  Basis functions: $\phi_j, \quad i = 1, \dots, M, \quad M = |\mathcal{E}| - |\mathcal{E}_N|$.

$$RT_0 := \{y : \Omega \to \mathbb{R}^d : \; y|_T(x) = a_T + b_T x, \; a_T \in \mathbb{R}^d, \; b_T \in \mathbb{R}, \\ [y]_E \cdot \boldsymbol{\nu}_E = 0, \text{ for all inner edges } E\}.$$

Galerkin formulation:

$$\psi^h(t,x) = \sum_{i=1}^{N} \psi_i(t)\varphi_i(x), \quad g_\psi^h(t,x) = \sum_{j=1}^{M} g_{\psi,j}(t)\phi_j(x),$$

and analogously for $n$, $p$, $J_n$, and $J_p$.

# Finite Element approximation

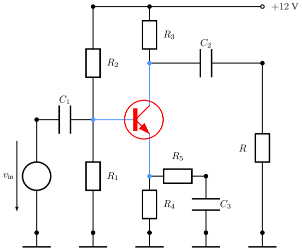
FEM yields a high-dimensional DAE for each semiconductor (method of lines)

$$
\begin{pmatrix} 0 \\ -M_L \dot{n}(t) \\ M_L \dot{p}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix}
+
\underbrace{\begin{pmatrix}
 & -M_L & M_L & \lambda D & & \\
 & & & & \nu_n D & \\
 & & & & & \nu_p D \\
D^\top & & & M_H & & \\
 & D^\top & & & M_H & \\
 & & -D^\top & & & M_H
\end{pmatrix}}_{A_{FEM}}
\begin{pmatrix} \psi(t) \\ n(t) \\ p(t) \\ g_\psi(t) \\ J_n(t) \\ J_p(t) \end{pmatrix}
$$

$$
+ \mathcal{F}(n^h, p^h, g_\psi^h) = b(A_S^\top e(t)),
$$

$$
\mathcal{F}(n^h, p^h, g_\psi^h) = \begin{pmatrix} 0 \\ -\int_\Omega R(n^h, p^h)\, \varphi \\ \int_\Omega R(n^h, p^h)\, \varphi \\ 0 \\ \int_\Omega n^h g_\psi^h \cdot \phi \\ \int_\Omega p^h g_\psi^h \cdot \phi \end{pmatrix}, \quad
b = \begin{pmatrix} -\int_\Omega C\, \varphi \\ 0 \\ 0 \\ \int_\Gamma \psi^h(A_S^\top e(t))\, \phi \cdot \nu \\ \int_\Gamma n^h\, \phi \cdot \nu \\ -\int_\Gamma p^h\, \phi \cdot \nu \end{pmatrix}.
$$

# Full model




$$
\begin{aligned}
A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right)& \\
+ A_L j_L(t) + A_V j_V(t) + A_S j_S(t) &= -A_I i_s(t), \\
\frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L(t), t) - A_L^\top e(t) &= 0, \\
A_V^\top e(t) &= v_s(t), \\
j_S(t) - C_1 J_n(t) - C_2 J_p(t) - C_3 \dot{g}_\psi(t) &= 0,
\end{aligned}
$$

$$
\begin{pmatrix} 0 \\ -M_L \dot{n}(t) \\ M_L \dot{p}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} \psi(t) \\ n(t) \\ p(t) \\ g_\psi(t) \\ J_n(t) \\ J_p(t) \end{pmatrix} + \mathcal{F}(n^h, p^h, g_\psi^h) - b(A_S^\top e(t)) = 0.
$$

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# Snapshot-POD (Proper Orthogonal Decomposition) [L. Sirovich '87]

Full simulation yields snapshots (here: $y = \psi, n, p, \ldots$)

$$\{y(t_i,\cdot)\}_{i=1,\ldots,m} \subset \operatorname{span}\{\varphi_j\}_{j=1,\ldots,N}, \quad \text{with} \quad y(t_i,x) = \sum_{j=1}^{N} \vec{y}_j(t_i)\varphi_j(x).$$

Gather coefficients in matrix

$$Y := \big(\vec{y}(t_1),\ldots,\vec{y}(t_m)\big) \in \mathbb{R}^{N\times m}.$$

POD in Hilbert space $X$ as eigenvalue problem:

$$Kv^k = \sigma_k^2 v^k, \quad \text{with} \quad K_{ij} := \langle y(t_i,\cdot), y(t_j,\cdot)\rangle_X .$$

Note that $K = Y^\top M Y$ with $M_{ij} = \langle \varphi_i, \varphi_j\rangle_X$. Write POD in terms of SVD:

$$\tilde{U}\Sigma\tilde{V}^\top = L^\top Y, \quad \text{with} \quad LL^\top := M.$$

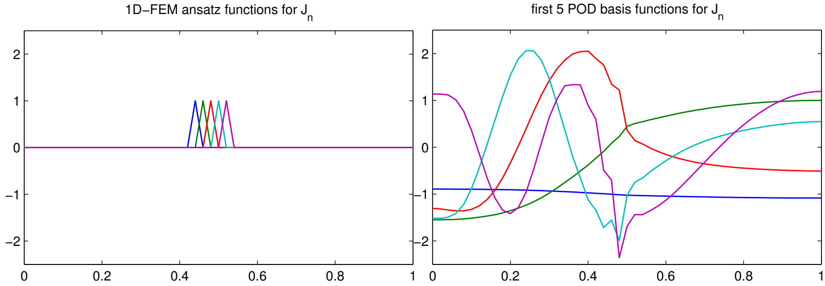
Then, the $s$-dimensional POD basis is

$$\left\{u^i := \sum_{j=1}^{N} \vec{u}^i_j \varphi_j(\cdot)\right\}_{i=1,\ldots,s}, \qquad U := (\vec{u}^1,\ldots,\vec{u}^s) := L^{-\top}\tilde{U}_{(:,1:s)}.$$

# Model Order Reduction

- Simulate the complete network at one or more reference parameters.
- Take snapshots of the state of each semiconductor at time points $t_i$.
- Perform POD component wise on $\psi$, $n$, $p$, $g_\psi$, $J_n$ and $J_p$.
- Use the POD basis functions as (non local) FEM ansatz/test functions:

$$\psi^{POD}(t,x) = \sum_{i=1}^{s} \gamma_{\psi,i}(t) u_\psi^i(x)$$



# Construction of reduced model

$$\underbrace{\begin{pmatrix} U_\psi^\top & & & & & \\ & U_n^\top & & & & \\ & & U_p^\top & & & \\ & & & U_{g_\psi}^\top & & \\ & & & & U_{J_n}^\top & \\ & & & & & U_{J_p}^\top \end{pmatrix}}_{U} \left[ \begin{pmatrix} 0 \\ -M_L U_n \dot{\gamma}_n(t) \\ M_L U_p \dot{\gamma}_p(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} U_\psi \gamma_\psi(t) \\ U_n \gamma_n(t) \\ U_p \gamma_p(t) \\ U_{g_\psi} \gamma_{g_\psi}(t) \\ U_{J_n} \gamma_{J_n}(t) \\ U_{J_p} \gamma_{J_p}(t) \end{pmatrix} \right.$$

$$\left. + \begin{pmatrix} 0 \\ -\int_\Omega R(n^{POD}, p^{POD})\, \varphi \\ \int_\Omega R(n^{POD}, p^{POD})\, \varphi \\ 0 \\ \int_\Omega n^{POD} g_\psi^{POD} \cdot \phi \\ \int_\Omega p^{POD} g_\psi^{POD} \cdot \phi \end{pmatrix} - b(A_S^\top e(t)) \right] = 0$$

By construction (POD in Hilbert space): $U_n^\top M_L U_n = I$ and $U_p^\top M_L U_p = I$.

# Construction of reduced model

Linear terms can be computed offline which yields

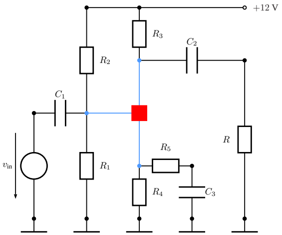
$$\begin{pmatrix} 0 \\ -\dot{\gamma}_n(t) \\ \dot{\gamma}_p(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{POD} \begin{pmatrix} \gamma_\psi(t) \\ \gamma_n(t) \\ \gamma_p(t) \\ \gamma_{g_\psi}(t) \\ \gamma_{J_n}(t) \\ \gamma_{J_p}(t) \end{pmatrix} + U^\top \mathcal{F}(n^{POD}, p^{POD}, g_\psi^{POD}) = U^\top b_{POD}(A_S^\top e(t)),$$

The Matrix $A_{POD} = U^\top A_{FEM} U$ contains dense blocks:

$$A_{POD} = \begin{pmatrix} & -U_\psi^\top M_L U_n & U_\psi^\top M_L U_p & \lambda U_\psi^\top D U_{g_\psi} & & \\ & & & & \nu_n U_n^\top D U_{J_n} & \\ & & & & & \nu_p U_p^\top D U_{J_p} \\ U_{g_\psi}^\top D^\top U_\psi & & & U_{g_\psi}^\top M_H U_{g_\psi} & & \\ & U_{J_n}^\top D^\top U_n & & & U_{J_n}^\top M_H U_{J_n} & \\ & & -U_{J_p}^\top D^\top U_p & & & U_{J_p}^\top M_H U_{J_p} \end{pmatrix}$$

By construction: $U_{g_\psi}^\top M_H U_{g_\psi} = I$, $U_{J_n}^\top M_H U_{J_n} = I$, and $U_{J_p}^\top M_H U_{J_p} = I$.

# Reduced model




$$
\begin{aligned}
A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right)& \\
+A_L j_L(t) + A_V j_V(t) + A_S j_S(t) &= -A_I i_s(t), \\
\frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L(t), t) - A_L^\top e(t) &= 0, \\
A_V^\top e(t) &= v_s(t),
\end{aligned}
$$

$$
j_S(t) - C_1 U_{J_n}\gamma_{J_n}(t) - C_2 U_{J_p}\gamma_{J_p}(t) - C_3 U_{g_\psi}\dot{\gamma}_{g_\psi}(t) = 0,
$$

$$
\begin{pmatrix} 0 \\ -\dot{\gamma}_n(t) \\ \dot{\gamma}_p(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{POD} \begin{pmatrix} \gamma_\psi(t) \\ \gamma_n(t) \\ \gamma_p(t) \\ \gamma_{g_\psi}(t) \\ \gamma_{J_n}(t) \\ \gamma_{J_p}(t) \end{pmatrix} + U^\top \mathcal{F}(n^{POD}, p^{POD}, g_\psi^{POD}) - U^\top b(A_S^\top e(t)) = 0.
$$

# Outline

# Computational complexity

For the sake of presentation we only consider the recombination term:

$$F_n(n(t), p(t)) = \begin{pmatrix} \int_\Omega R\left(\sum_{i=1}^N n_i(t)\varphi_i(x), \sum_{i=1}^N p_i(t)\varphi_i(x)\right)\ \varphi_1(x)\,dx \\ \vdots \\ \int_\Omega R\left(\sum_{i=1}^N n_i(t)\varphi_i(x), \sum_{i=1}^N p_i(t)\varphi_i(x)\right)\ \varphi_N(x)\,dx \end{pmatrix}.$$

In the reduced model we compute

$$\underbrace{U_n^\top}_{\text{size } s_n \times N}\ \underbrace{F_n}_{N \text{ evaluations}}\ \big(\ \underbrace{U_n}_{\text{size } N \times s_n}\ \gamma_n(t),\ \underbrace{U_p}_{\text{size } N \times s_p}\ \gamma_p(t)\ \big),$$

and for example the derivative with respect to $\gamma_p$:

$$\underbrace{U_n^\top}_{\text{size } s_n \times N}\ \underbrace{\frac{\partial F_n}{\partial p}(U_n\gamma_n(t), U_p\gamma_p(t))}_{\text{size } N \times N, \text{ sparse}}\ \underbrace{U_p}_{\text{size } N \times s_p}.$$

Computational complexity of the POD-ROM is still $O(N)$.

# Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen ’09]

## DEIM

- Collect snapshots $\{F_n(n(t_k), p(t_k))\}$ of the nonlinearity.
- Do POD on snapshots $\{F_n(n(t_k), p(t_k))\}$, obtain basis $V_n \in \mathbb{R}^{N \times \tau_n}$.
- Ansatz:
$$F_n(U_n \gamma_n(t), U_p \gamma_p(t)) \approx V_n c(t)$$
is overdetermined.
- Select $\tau_n$ “useful” rows:
$$P_n^\top F_n(\ldots) \approx P_n^\top V_n c(t).$$
- If $P_n^\top V_n$ is regular:
$$F_n(\ldots) \approx V_n c(t) = V_n (P_n^\top V_n)^{-1} P_n^\top F_n(\ldots)$$

The regularity of $P_n^\top V_n$ can be guaranteed and the growth of a global error bound is limited, see [CS09].

# Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen '09]

We obtain an approximation

$$U_n^\top F_n(U_n\gamma_n(t), U_p\gamma_p(t)) \approx W_n P_n^\top F_n(U_n\gamma_n(t), U_p\gamma_p(t)),$$

with

$$W_n := (U_n^\top V_n(P_n^\top V_n)^{-1}) \in \mathbb{R}^{s_n \times \tau_n}.$$

# Discrete Empirical Interpolation Md. (DEIM) [S. Chaturantabut, D. Sorensen '09]

We obtain an approximation

$$U_n^\top F_n(U_n\gamma_n(t), U_p\gamma_p(t)) \approx W_n P_n^\top F_n(U_n\gamma_n(t), U_p\gamma_p(t)),$$

with

$$W_n := (U_n^\top V_n (P_n^\top V_n)^{-1}) \in \mathbb{R}^{s_n \times \tau_n}.$$

Require whole vector $U_n\gamma_n(t) \in \mathbb{R}^N$ for evaluation of $P_n^\top F_n(\ldots) \in \mathbb{R}^{\tau_n}$?
No, here each vector component depends only on some variables:

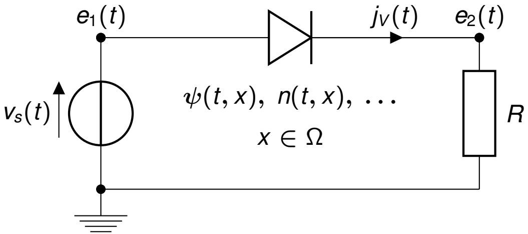
$$F_n(n(t), p(t)) = \begin{pmatrix} \int_\Omega R\left(\sum_{i=1}^N n_i(t)\varphi_i(x), \sum_{i=1}^N p_i(t)\varphi_i(x)\right)\ \varphi_1(x)\,dx \\ \vdots \\ \int_\Omega R\left(\sum_{i=1}^N n_i(t)\varphi_i(x), \sum_{i=1}^N p_i(t)\varphi_i(x)\right)\ \varphi_N(x)\,dx \end{pmatrix}.$$

# Numerical results

Our C++ implementations make use of various tools:

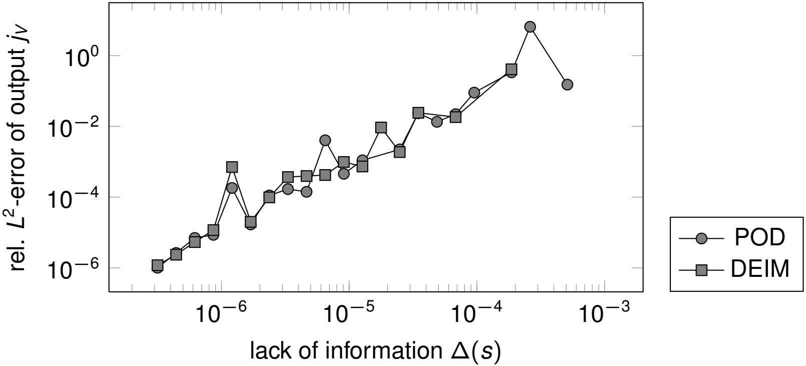
- deal.II (FEM)
- DASPK (DAE integrator, index-1 network assumed)
- ADOL-C (automatic derivatives of nonlinearities)
- SuperLU (linear solver)

Basic test circuit with one diode:



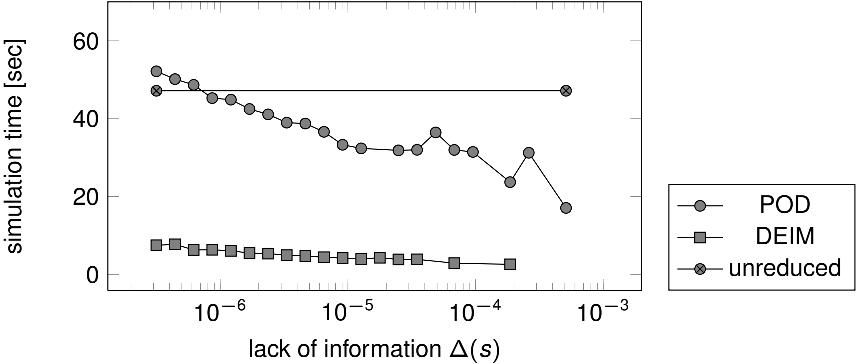
# Numerical results

Relative error between reduced and unreduced problem at the fixed frequency $5 \cdot 10^9$ $[Hz]$.



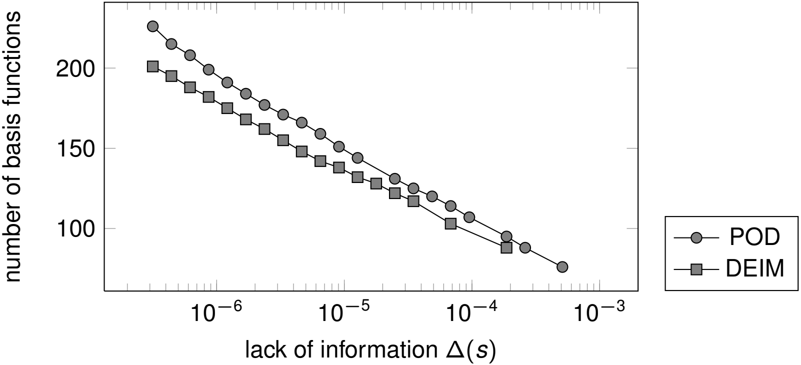
# Numerical results

Time consumption for simulation runs.



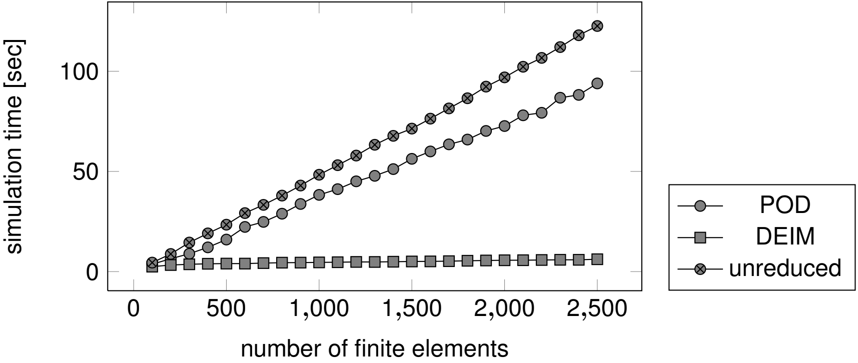
# Numerical results

The number of required POD basis function and DEIM interpolation indices grows only logarithmically with the requested information content.



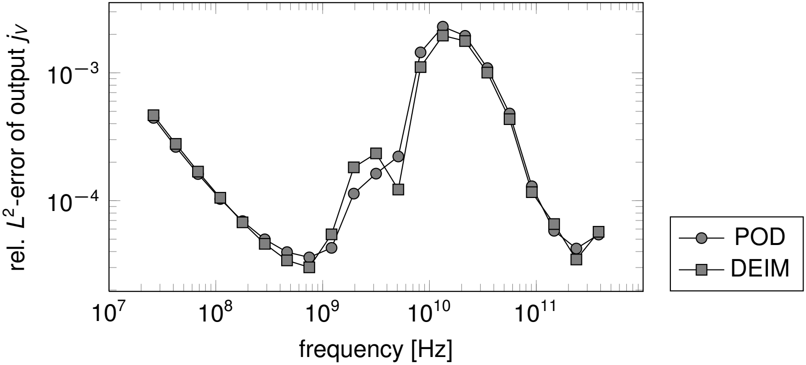
# Numerical results

Computation times of the unreduced and the reduced order models plotted versus the number of finite elements. Number of variables in reduced system is $\approx 180$.



# Numerical results

The reduced models are compared with the unreduced model at various input frequencies.

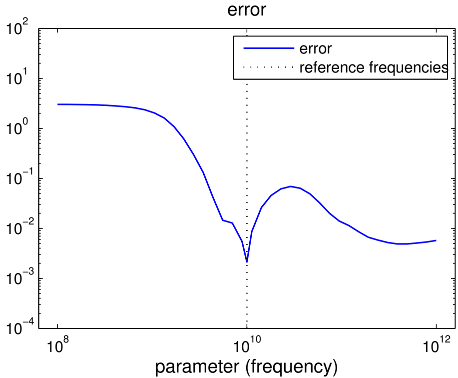


# Outline

# Reduced model over parameter space

Construction of reduced model requires snapshots from full simulations at reference parameters.
Is the model valid over a large parameter space?

reference parameter: $P_1 := \{f_1\} := \{10^{10}[Hz]\}$
parameter space $\mathcal{P} = [10^8, 10^{12}]$



# Reduced model over parameter space - sampling

## Goal

Find new sampling parameter $f_{k+1}$ (reference frequency) without simulating the full, unreduced system. Set $P_{k+1} := P_k \cup \{f_{k+1}\}$.

- We do not consider the PDE discretization error.
- Rigorous upper bound for the error not available

$$\|\mathcal{E}(f; P_k)\| = \|y^h(f) - y^{POD}(f; P_k)\| \leq ?(s)$$

where $y^h := (\psi^h, n^h, p^h, g^h_\psi, J^h_n, J^h_p)^\top$, $y^{POD} := (\psi^{POD}, n^{POD}, \ldots)^\top$.

- Rigorous RB methods, Greedy algorithm [see e.g. A. Patera, G. Rozza '07]: a-posteriori error estimates required.
- Linear ODEs [see e.g. B. Haasdonk, M. Ohlberger '09]: build difference between residual and unreduced equation to derive an ODE for the error.

# Residual based sampling

Define residual $\mathcal{R}(z^{POD}(f;P_k))$: insert $z^{POD}(f;P_k)$ into unreduced equation,

$$\mathcal{R} := \begin{pmatrix} 0 \\ -M_L \dot{n}^{POD}(t) \\ M_L \dot{p}^{POD}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} \psi^{POD}(t) \\ n^{POD}(t) \\ p^{POD}(t) \\ g_\psi^{POD}(t) \\ J_n^{POD}(t) \\ J_p^{POD}(t) \end{pmatrix} + \mathcal{F}(n^{POD}, p^{POD}, g_\psi^{POD}) - b(e^{POD}(t)).$$

Residual is scale-variant!
Scale with block diagonal matrix-valued function

$$D(f) := \mathrm{diag}(\ d_\psi(f)I,\ d_n(f)I,\ d_p(f)I,\ d_{g_\psi}(f)I,\ d_{J_n}(f)I,\ d_{J_p}(f)I\ )$$

and choose $d_\psi(f)$ such that it holds

$$d_\psi(f_j) \cdot \|\mathcal{R}_\psi(y^{POD}(f_j;P_k))\| = \frac{\|\psi^h(f_j) - \psi^{POD}(f_j;P_k)\|}{\|\psi^h(f_j)\|}, \quad \forall f_j \in P_k.$$

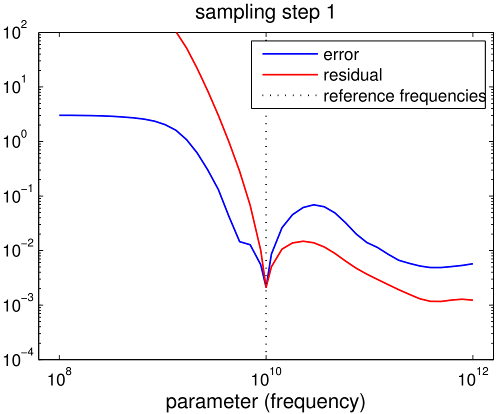
# Residual based sampling

## Algorithm: sampling

1. Select $f_1 \in \mathcal{P}$, $P_{test} \subset \mathcal{P}$, $tol > 0$, and set $k := 1$, $P_1 := \{f_1\}$.
2. Simulate the unreduced model at $f_1$ and calculate the reduced model with POD basis functions $U_1$.
3. Calculate weight functions $d_{(\cdot)}(f) > 0$ for all $f \in P_k$.
4. Calculate the scaled residual $\|D(f)\mathcal{R}(z^{POD}(f, P_k))\|$ for all $f \in P_{test}$.
5. Check termination conditions, e.g.
    - $\max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_k))\| < tol$,
    - no progress in weighted residual.
6. Calculate $f_{k+1} := \arg\max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_k))\|$.
7. Simulate the unreduced model at $f_{k+1}$ and create a new reduced model with POD basis $U_{k+1}$ using also the already available information at $f_1$, $\ldots$, $f_k$.
8. Set $P_{k+1} := P_k \cup \{f_{k+1}\}$, $k := k + 1$ and goto 3.

# Numerical example - sampling step 1

Let $f_1 := 10^{10}[Hz]$, $P_1 := \{10^{10}[Hz]\}$, $\mathcal{P} = [10^8, 10^{12}]$.

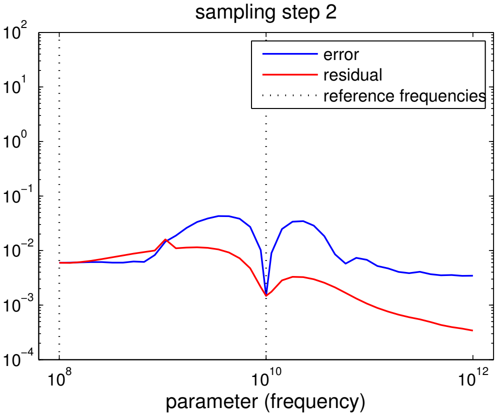



$f_2 = \arg\max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_1))\| = 10^8[Hz]$

$P_2 = \{10^8[Hz],\ 10^{10}[Hz]\}$

# Numerical example - sampling step 2

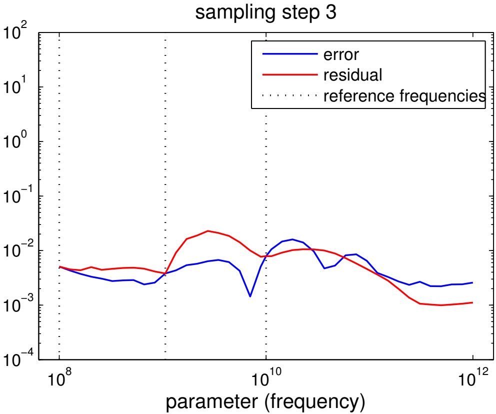
$P_2 = \{10^8[Hz],\ 10^{10}[Hz]\}$




$f_3 = \arg\max_{f \in P_{test}} \|D(f)\mathcal{R}(z^{POD}(f, P_2))\| = 1.0608 \cdot 10^9[Hz]$
$P_3 = \{10^8[Hz],\ 1.0608 \cdot 10^9[Hz],\ 10^{10}[Hz]\}$

# Numerical example - sampling step 3

$$P_3 = \{10^8[Hz],\ 1.0608 \cdot 10^9[Hz],\ 10^{10}[Hz]\}$$




Terminate with “no progress in residual”.

# Outlook

- Reduction of the DD-equations and the circuit:
  first results available; joint work with subproject 3 of the SyreNe research network [A. Steinbrecher, T. Stykel]
- More realistic device models will be tackled in the BMBF research network MoreSim4Nano (Model Reduction for Fast Simulation of new Semiconductor Structures in Nano- and Microsystems-Technology)
- Final presentation of the SyreNe research network at ModRed 2010, TU Berlin, 2-4 December 2010

Thank you for your attention.

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