Finding the ground state of N Coulomb charges on a sphere - a PDE based approach.

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Finding the ground state of N Coulomb charges on a sphere is known as Thomsons's problem. The ground state differs from the hexagonal structure in the plane by the presence of disclinations. The Euler theorem states, that the disclination charge of any triangulation of a sphere must be 12. As this can be achieved in many different ways, makes finding the minimum energy configuration an extremely difficult problem for large N. The Thomson problem is also related to the arrangement of colloidal particles in colloidosomes, proteins in viral capsides or self-assembled spherules on core/shell microstructures.

We introduce a new approach to solve such generalized Thomson problems, which also considers the dynamic evolution and rearrangement of particles on a curved surface. Our approach is based on a free energy functional. The H^{-1} gradient flow of the energy leads to a 6th order PDE, which is solved on a surface using parametric finite elements. We are able to treat problems with several thousand particles.

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