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# Performance Evaluation of Global Investment Portfolios: The Role of Currencies


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**Abstract**

The aim of performance evaluation is to make qualified statements about the success of management in allocating and selecting investment portfolios. Over the last 20 to 30 years many approaches have been developed for directly comparing actively managed portfolios to passively managed benchmark portfolios. During the late 1980s the main interest was on finding and defining attribution variables (allocation and selection variables). Beginning in the 1990s theoretical focus shifted to global investment portfolios and the handling of currencies and currency management. Nevertheless, by focusing on local currency and exchange rate returns while ignoring interest rate differentials, conventional performance evaluation systems retained the independence of market and currency management, leading to improper results. This paper presents a method of performance measurement that employs the dependence of both management stages in terms of allocation processes, but allows their separation in terms of selection processes. As a consequence, interpretation of performance evaluation is much more reasonable, and leads to the same results, whether the investment process starts with market management followed by currency management, or vice versa. Numerical examples make this clear.

**Keywords:** Financial mathematics, performance evaluation, currency management.

**Mathematical subject classification:** 00A69 [General applied mathematics]

## 1 Introduction

*"The ability of the investment community to investigate the issues that are presented by global markets would be enhanced [...], if the investigation could be conducted within a consistent, general framework that accounts for the interaction of global asset returns and currency returns. In particular, such a framework would recognize that introducing currency considerations into portfolio analysis has implications for the manner in which the underlying assets are evaluated."* (Singer and Karnosky, 1994, p. 1)

Although in the 1980s and early 1990s many research papers dealing with performance evaluation were published (see, for example, Brinson and Fachler, 1985; Brinson, Hood, and Beebower, 1986; Hensel, Don Ezra, and Ilkiw, 1991; Brinson, Singer, and Beebower, 1991; Ankrim, 1992), none of these were concerned with "the interaction of global asset returns and currency returns". Later that view broadened, and currencies came into perspective. Consequently, in the mid-1990s previous analyses were redone with a focus on currency aspects (Ankrim and Hensel, 1994, Singer and Karnosky, 1994). One of these new works was that of Singer and Karnosky (henceforth S+K) published in 1994 (Singer and Karnosky, 1994). The authors write in the tradition of Brinson et al. (see above citations), i. e. they define allocation and selection variables to identify different kinds of management decisions. Furthermore and new, they divide the overall performance in a very unconventional way into market as well as currency returns, evaluate attribution variables accordingly, and are thereby able to distinguish between market and currency management decisions. Hence, "EVA", the portfolio's excess return over the benchmark's return or (economic) value added, is now split between market allocation return, currency allocation return, market security selection, and currency hedge selection.

Undoubtedly, the paper by S+K leads to new insights for coping with currency problems when analysing global investment portfolios. On the other hand, from a mathematical point of view, the evaluations in the paper are too imprecise. First, because in performance evaluation all returns are initially calculated in simple, instead of continously compounded terms, the correct way of defining overall performance is by multiplying market and currency returns rather than by adding these two (see, for example, Sharpe and Alexander, 1990, p. 780, (24.4)). Second, for reasons of arbitrage the forward currency return, expressed as the difference of Eurodeposit returns for each country, must be divided by a discount factor which depends on the foreign currency Eurodeposit return (see, for example, Fischer-Erlach, 1995). By overlooking these factors, S+K were mistakenly able to entirely separate market and currency returns within the overall performance. In terms of management decisions, this separation would mean that market and currency management were totally independent. However, consider the example of a market manager who already has decided where and when to invest. The task of the succeeding currency management would then be to handle the currency exposures generated by these choices. Therefore both management divisions are not independent.

While the idea of alloction separation envisioned by S+K is not possible, a different kind of separation, within asset selection, is, and is a valuable tool. The aim of this paper is to develop this approach together with a more precise performance measurement. Section 2 presents this method of performance measurement and compares it with a more conventional system. Decision trees are used to make portfolio decomposition – representing management decisions – more visible and therefore understandable. A numerical example follows in Section 3. Formulas for the attribution variables are presented in Section 4 with the newly defined performance measures. These are clarified with a numerical example in Section 5. Section 6 summarizes the results.

## 2 Portfolio Decomposition and Performance Measurement

One major difficulty of performance evaluation is that the process of managing a large investment portfolio depends on team work, but the task of performance evaluation is to show the success of each portfolio manager separately. A first step in solving this problem is to draw a diagram highlighting the entanglements. A detailed graph showing the structure of the portfolio can provide a clearer understanding of which weight and return variables must be evaluated. To express the effect of different performance measurement systems, we draw graphs of various portfolio decompositions.

One obvious dependency leading to complexity in portfolio evaluation is the one between market and currency management in global investment portfolios. To address this problem, consider the following restricted example: a portfolio with four markets (Japan, USA, United Kingdom, Euroland) and their four currencies (Yen, US-Dollar, Pound Sterling, Euro) with the base currency of the portfolio the US-Dollar. Moreover, in every country only one possible investment may be chosen. A decomposition of the portfolio should then show all markets, the portfolio invested in, and the resulting currency exposures (see Figs. 1 and 2).

Within the four-market, four-currency example, first consider the conventional decision tree. Here, conventional refers to a system measuring market return in local currency by comparing initial and final values of the respective securities (simple compounding of rates of return). Currency rates of return reflect those of forward currency contracts (if hedging or cross-hedging were involved) or exchange rate returns (for unhedged currency). Because we examine only a single period, weight variables retain their initial values.

Notation (See Appendix A for numerical examples.):

- $i$ market currency with $i = ¥, \$, £, €$,
- $j$ "hedge" currency with $j = ¥, \$, £, €$,
- $r_i$ rate of return in market $i$ measured in local currency,
- $f_{ij}$ rate of return from forward contracts hedging currency $i$ to currency $j$ ($f_{ii} = 0$),
- $e_{j\$}$ rate of change of the dollar relative to currency $j$ ($e_{\$\$} = 0$),
- $w_i$ weight of market $i$ as a fraction of the total ($-based) money value of the portfolio invested in market $i$ with $w_i \geq 0$, $\sum_i w_i = 1$,
- $w_{ij}$ weight of currency $i$ exchanged to currency $j$ as a fraction of the (local currency-based) money value of market $i$ with $w_{ij} \geq 0$, $\sum_j w_{ij} = 1$.

A decision tree forming the investment decisions within a conventional performance measurement system – with currency management after market management – is shown in Figure 1.

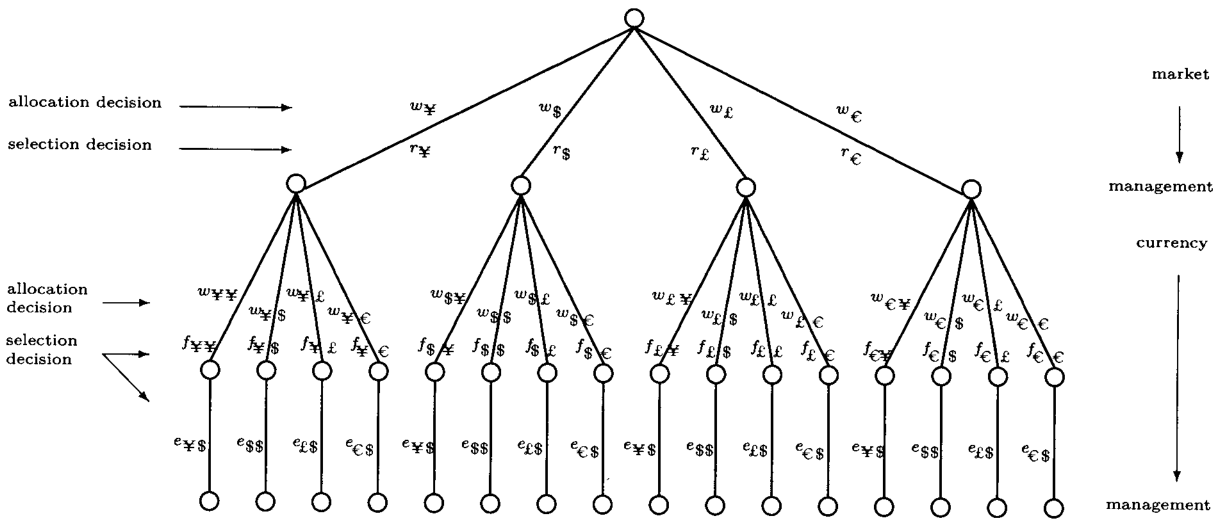


Figure 1: Investment alternatives within a conventional performance measurement system

The total rate of return of the portfolio, $R$, is the sum of all end knots in Figure 1:

$$R = \Big(\sum_i \sum_j w_i \cdot (1 + r_i) \cdot w_{ij} \cdot (1 + f_{ij}) \cdot (1 + e_{j\$})\Big) - 1. \tag{1}$$

There are two points to be made regarding Figure 1 and equation (1). First, although the individual branches of the tree must be added to get the overall performance of the portfolio, within one branch, market and currency return variables need to be multiplied (Sharpe and Alexander, 1990, p. 780, (24.4)). Market and currency returns here are comparable to returns from different time periods, both derived from the same inital outlay. Second, while continously compounded rates of return would be computationally easier (rates of return of different time periods only need to be added), the discrete nature of the process requires simple compounding for accuracy (otherwise it gets difficult to define appropriate weight variables).

The problem of performance evaluation, highlighted in Figure 1, is how to judge currency managers, once an independently acting market manager has made the allocation and selection decisions (first layer of nodes in tree). For example, suppose the market manager only invested in one market. Instead of 16 possible branches, the currency manager is left to optimize returns among four.

In their small book "Global Asset Management and Performance Attribution" (S+K, 1994) Singer and Karnosky introduce a performance measurement system that takes market and currency management divisions into considerations. Consider equation (1). Following S+K, let $c_i$ represent the rate of return from local Eurodeposit cash market $i$. Written in terms of $c_i$, $f_{ij}$, the rate of return from a currency forward contract, becomes:

$$f_{ij} = \frac{c_j - c_i}{1 + c_i}. \tag{2}$$

The use of forward currency contracts in this analysis is based on "covered interest parity". Equation (2) can therefore be derived through arbitrage considerations (see Appendix B). Although S+K claim to apply "covered interest parity" (S+K, 1994, p. 5, footnote 1) to define the forward currency return, their result for $f_{ij}$ omitts the denominator $(1 + c_i)$ in equation (2). This omission is what allows S+K to achieve the overly simplistic separation referred to above.

Rearranging equation (1) using equation (2) gives:

$$\begin{aligned} R &= \Big(\sum_i \sum_j w_i \cdot (1 + r_i) \cdot w_{ij} \cdot \frac{1 + c_j}{1 + c_i} \cdot (1 + e_{j\$})\Big) - 1 \\ &= \Big(\sum_i w_i \cdot \frac{1 + r_i}{1 + c_i} \cdot \sum_j w_{ij} \cdot (1 + c_j) \cdot (1 + e_{j\$})\Big) - 1 \\ &= \Big(\sum_i w_i \cdot (1 + \mu_i) \cdot \sum_j w_{ij} \cdot (1 + \kappa_j)\Big) - 1, \end{aligned} \tag{3}$$

where:

$c_i$ reflects rolling short-term Eurodeposits of market $i$ over the investment horizon of the portfolio,

$\mu_i$ is the market premium for market $i$ with $\mu_i = \frac{r_i - c_i}{1 + c_i}$,

$\kappa_j$ is the currency premium for currency $j$ with $\kappa_j = (1 + c_j) \cdot (1 + e_{j\$}) - 1$.

Reconstructing the tree using the new variables, $\mu_i$ and $\kappa_j$, allows for a simpler and more powerful figure (see Fig. 2).

After moving down any of the available market branches, in Figure 2, only the same choices present themselves for the subsequent currency management selections. This means that the currency selection (choosing the best value out of $\{\kappa_{¥}, \ldots, \kappa_{€}\}$) is now independent of any previous market decision. However, the currency allocation (which means allocating money to the values of $\{w_{¥¥}, \ldots, w_{€€}\}$) is, as the first index indicates, still related to these market decisions. Expressed in a more theoretical way: because the values of market and currency management must be multiplied and not summed to get the performance of the whole branch, the tree cannot be separated into one for market management and one for currency management

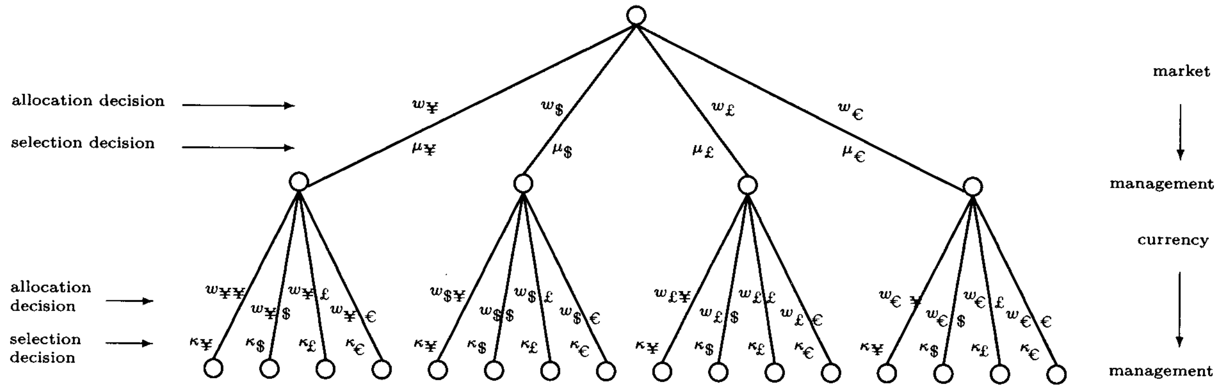


Figure 2: Investment alternatives within a new performance measurement system

(as S+K suggested, see S+K, 1994, p. 9). Both decisions are still tied together, but only through the allocation process.

## 3 Numerical Example

To highlight the differences between the conventional and new scheme, imagine building a "best" portfolio for each of the performance measurement systems, where "best" refers to the investment objective of maximizing overall portfolio return. For completness also include a third system which does not differentiate between market and currency returns and is therefore solely based on market returns in the base currency. Furthermore, introduce a benchmark portfolio consisting of dollar-equal investments in all markets. It is then possible to compare the evolved portfolios with that benchmark portfolio and draw conclusions about the several schemes.

In the four-country, four-currency example of Section 2 introduce some figures (see Table 1).

| Market | Market Returns in LC ($r_i$) | Exchange Rate Returns in BC ($e_{i\$}$) | Eurodeposit Returns in LC ($c_i$) |
|---|---|---|---|
| Japan | 9.50% | -1.00% | 9.00% |
| USA | 8.40 | — | 7.50 |
| UK | 10.50 | -3.00 | 11.25 |
| Euroland | 7.00 | 1.00 | 5.00 |

Table 1: Global Security Returns (S+K, 1994, p. 22)

This example goes back to a similar one by S+K (see S+K, 1994, p. 21ff.).

Notation:

*LC* local currency of a security (=home currency),

*BC* base currency of the portfolio (=US-Dollar),

$r_{ij}$ rate of return by currency transfers from one currency $i$ to another one $j$ ($i,j =$ ¥, \$, £, €), measured in US-Dollars; $r_{ij} = [(1+f_{ij}) \cdot (1+e_{j\$})] - 1$,

$R^B$ portfolio return of benchmark portfolio $B$.

From the return data of Table 1, compare the three different performance measurement systems introduced in the previous section, the conventional (system I) and the new one (system II, see Figure 1 and 2 respectively), as well as a third one (system III) without explicit currency management (see Table 2).

| | Market Management | Currency Management |
|---|---|---|
| System I | Local Currency Market Returns | Exchange Rates of Return and Forward Currency Rates of Return |
| System II | Local Currency Market Premiums | Currency Premiums |
| System III | Base Currency Market Returns | — |

Table 2: Different Performance Measurement Systems

**Conventional Performance Measurement System, System I**

Assume the portfolio management to be divided into market and currency management. Whereas decisions of the market management are based on expectations about local currency market returns, currency management decisions are due to exchange rate returns and forward currency rates of return. With the relevant data of Table 1, all possible outcomes of the system are as follows (see Table 3 – highest values framed).

| System I | Market Returns in LC ($r_i$) | Currency Returns in BC ($r_{ij}$) | | | |
|---|---|---|---|---|---|
| | $j$: | ¥ | \$ | £ | € |
| Japan (¥) | 9.5000% | -1.0000% | -1.3761% | -0.9977% | -2.7064% |
| USA (\$) | 8.4000 | 0.3814 | 0.0000 | 0.3837 | -1.3488 |
| UK (£) | [10.5000] | -3.0022 | -3.3708 | [-3.0000] | -4.6742 |
| Euroland (€) | 7.0000% | 2.7714% | 2.3810% | 2.7738% | 1.0000% |

Table 3: Performance Measurement with System I

Ignoring risk considerations and taking the above values as given (the expected ones were realized), to form a return maximizing portfolio $A_I$ for system I means to optimize the return of equation (1). Because portfolio management is divided into market and currency management, this is done in two succeding steps (considering currency management before market management in the given constellation, leads to portfolio $A_{II}$ (see Table 6)).

$$\left.\begin{array}{lll} \text{Step 1:} & \max\limits_{x_{¥},\dots,x_{€}} & \sum\limits_i x_i \cdot (1+r_i) \\ \text{(market management)} & \text{s.t.} & x_i \in [0,1],\ \sum\limits_i x_i = 1 \end{array}\right\} \Longrightarrow x_{i^\star} = x_{£} = w_{£} = 1,$$

$$\left.\begin{array}{lll} \text{Step 2:} & \max\limits_{y_{£¥},\dots,y_{£€}} & \sum\limits_j y_{£j} \cdot (1+r_{£j}) \\ \text{(currency management)} & \text{s.t.} & y_{£j} \in [0,1],\ \sum\limits_j y_{£j} = 1 \end{array}\right\} \Longrightarrow y_{£j^\star} = y_{££} = w_{££} = 1.$$

$$\Longrightarrow \quad R_{A_I} = w_{£} \cdot (1+r_{£}) \cdot w_{££} \cdot (1+r_{££}) - 1.$$

Notation:

| | |
|---|---|
| $A_i$ | optimal portfolio for system $i$, $i = I, II, III$, |
| $R^{A_i}$ | return of portfolio $A_i$, |
| $x_{i^\star}, y_{i^\star j^\star}, y_{j^\star}$ | optimal values of the optimization problems. |

Because the second step is optimized under the outcome of the first, the total value $R_{A_I}$ can only by chance reach the maximal value of equation (1):

$$\underbrace{\left(\max_{x_i} \sum_i x_i \cdot (1+r_i)\right)}_{\text{(Step 1)}} \cdot \underbrace{\left(\max_{y_{i^\star j}} \sum_j y_{i^\star j} \cdot (1+r_{i^\star j})\right)}_{\text{(Step 2)}} \leq \max_{x_i, y_{ij}} \left(\sum_i \sum_j x_i \cdot (1+r_i) \cdot y_{ij} \cdot (1+r_{ij})\right).$$

The solution of the maximization in two separate steps is a Portfolio, $A_I$, fully invested in the United Kingdom with unhedged local currency, British Pounds Sterling (see Table 4).

| **Portfolio $A_I$** | Market Weight ($w_i$) | Currency Weight ($w_{ij}$) | Market Return in LC ($r_i$) | Curr. Return in BC ($r_{ij}$) | Evaluation |
|---|---|---|---|---|---|
| Market: UK | 100.00% | — | 10.50% | — | 1.105 $= w_{£}(1+r_{£})$ |
| curr. exp.: £ | — | 100.00 | — | -3.00 | 0.970 $= w_{££}(1+r_{££})$ |
| Sum | 100.00% | 100.00% | 10.50% | -3.00% | 7.185%$= R^{A_I}$ |

Table 4: Portfolio $A_I$

### A New Performance Measurement System, System II

Again assume the portfolio management to be divided into market and currency management. In contrast to system I, decisions of the market management are now based on expectations about local currency market premiums, and currency management decisions on expectations about currency premiums. According to Section 2, market premium here represents the discounted difference between local currency market return and local currency Eurodeposit return, whereas currency premium stands for any Eurodeposit return in base currency. As before, calculate the possible outcomes of the system under the data of Table 1 (see Table 5 – again the highlighted values are the best ones).

| **System II** $i$ | Market Premium in LC ($\mu_i$) | Currency Premium in BC ($\kappa_j$) | $j$ |
|---|---|---|---|
| Japan (¥) | 0.4587% | 7.9100% | ¥ |
| USA ($) | 0.8372 | 7.5000 | $ |
| UK (£) | -0.6742 | 7.9125 | £ |
| Euroland (€) | 1.9048% | 6.0500% | € |

Table 5: Performance Measurement with System II

Forming a return maximizing portfolio $A_{II}$ for system II means, in this case, optimizing the return of equation (4). Again, this can be done in two separate optimization steps. Yet, the great advantage of the following formulation is that step 2 is independent of step 1.

Step 1: (market management)

$$\left.\begin{array}{ll} \max\limits_{x_{¥},\dots,x_{€}} & \sum\limits_i x_i \cdot (1+\mu_i) \\ \text{s.t.} & x_i \in [0,1],\ \sum\limits_i x_i = 1 \end{array}\right\} \Longrightarrow \quad x_{i^\star} = x_{€} = w_{€} = 1,$$

Step 2: (currency management)

$$\left.\begin{array}{ll} \max\limits_{y_{¥},\dots,y_{€}} & \sum\limits_j y_j \cdot (1+\kappa_j) \\ \text{s.t.} & y_j \in [0,1],\ \sum\limits_j y_j = 1 \end{array}\right\} \Longrightarrow \quad y_{j^\star} = y_{£} = w_{€£} = 1.$$

$$\Longrightarrow \quad R_{A_{II}} = w_{€} \cdot (1+\mu_{€}) \cdot w_{€£} \cdot (1+\kappa_{£}) - 1.$$

The total value of $R_{A_{II}}$ is therefore equal to the maximized value of equation (4) which was only a reformulation of equation (1) with new performance measures:

$$\underbrace{\max_{x_i}\Big(\sum_i x_i \cdot (1+\mu_i)\Big)}_{\text{(Step 1)}} \cdot \underbrace{\max_{y_j}\Big(\sum_j y_j \cdot (1+\kappa_j)\Big)}_{\text{(Step 2)}} = \max_{x_i, y_{ij}}\Big(\sum_i \sum_j x_i \cdot (1+r_i) \cdot y_{ij} \cdot (1+r_{ij})\Big).$$

A portfolio, fully invested in the markets of Euroland, and utilizing the British cash market for investing, would in this case hit the target of maximizing overall portfolio performance. The corresponding portfolio, $A_{II}$, would be the following (see Table 6 – for the purpose of a better comparison, the values of $A_{II}$ are also in "ordinary" return variables instead of premiums).

The advantage of the new performance measurement system is now obvious. Because currency management was done without restrictions due to market management, the overall performance was improved by nearly 40%, achieving the maximal possible value 9.968%.

| **Portfolio $A_{II}$** | Market Weight ($w_i$) | Currency Weight ($w_{ij}$) | Market Ret. in LC ($r_i$) | Curr. Ret. in BC ($r_{ij}$) | Evaluation | |
|---|---|---|---|---|---|---|
| Market: Eurol. | 100.00% | — | 7.000% | — | 1.07000 | $= w_{€}(1+r_{€})$ |
| curr. exp.: £ | — | 100.00 | — | 2.774 | 1.02774 | $= w_{€£}(1+r_{€£})$ |
| Sum | 100.00% | 100.00% | 7.000% | 2.774% | 9.96800%= $R^{A_{II}}$ | |

Table 6: Portfolio $A_{II}$

**Performance Measurement without Currency Management, System III**

For this performance measurement system assume there is no differentiation between market and currency management. Investment decision are based on expectations about market returns in base currency; currencies remain unhedged. The corresponding investment alternatives from Table 1 are shown in Table 7.

| **System III** | Japan ¥ | USA $ | UK £ | Euroland € |
|---|---|---|---|---|
| Market Return in BC | 8.405% | 8.400% | 7.185% | 8.070% |

Table 7: Performance Measurement with System III

Because the highest rate of return is found in Japan, the construction of a return-optimized portfolio, $A_{III}$, within system III leads to a 100% engagement in Japan and an unhedged yen currency. As with portfolio $A_{II}$, for reasons of comparison, market and currency returns are shown separately in Table 8.

| **Portfolio $A_{III}$** | Market Weight ($w_i$) | Currency Weight ($w_{ij}$) | Market Return in LC ($r_i$) | Curr. Ret. in BC ($r_{ij}$) | Evaluation | |
|---|---|---|---|---|---|---|
| Market: Japan | 100.00% | — | 9.50% | — | 1.095 | $= w_{¥}(1+r_{¥})$ |
| curr. exp.: ¥ | — | 100.00 | — | -1.00 | 0.990 | $= w_{¥¥}(1+r_{¥¥})$ |
| Sum | 100.00% | 100.00% | 9.50% | -1.00% | 8.405%= $R^{A_{III}}$ | |

Table 8: Portfolio $A_{III}$

The optimization of equation (1), within system III, is, as with system I, only suboptimal:

$$\max_{x_i}\left(\sum_i x_i \cdot (1+r_i)\cdot(1+e_{i\$})\right) \leq \max_{x_i,y_{ij}}\left(\sum_i\sum_j x_i \cdot (1+r_i)\cdot y_{ij}\cdot(1+r_{ij})\right).$$

**Benchmark Portfolio**

In order to analyse performance, construct a benchmark portfolio. The benchmark portfolio consists of a passive market portfolio, equally dollar-invested in all possible markets without currency hedging (see Table 9).

| **Benchmark** $B$ | Market Weight ($w_i$) | Currency Weight ($w_{ij}$) | Mark. Ret. in LC ($r_i$) | Curr. Ret. in BC ($r_{ij}$) | Evaluation |
|---|---|---|---|---|---|
| Markt: Japan | 25.00% | — | 9.50% | — | $0.27375 = w_{¥}(1 + r_{¥})$ |
| $i = ¥, j = ¥$ | — | 100.00 | — | -1.00 | $0.99000 = w_{¥¥}(1 + r_{¥¥})$ |
| | | | | | $0.2710125 = 0.27375 \cdot 0.99$ |
| Markt: USA | 25.00 | — | 8.40 | — | $0.27100 = w_{\$}(1 + r_{\$})$ |
| $i = \$, j = \$$ | — | 100.00 | — | 0.00 | $1.00000 = w_{\$\$}(1 + r_{\$\$})$ |
| | | | | | $0.2710000 = 0.271 \cdot 1.00$ |
| Markt: UK | 25.00 | — | 10.50 | — | $0.27625 = w_{£}(1 + r_{£})$ |
| $i = £, j = £$ | — | 100.00 | — | -3.00 | $0.97000 = w_{££}(1 + r_{££})$ |
| | | | | | $0.2679625 = 0.27625 \cdot 0.97$ |
| Markt: Eurol. | 25.00 | — | 7.00 | — | $0.26750 = w_{€}(1 + r_{€})$ |
| $i = €, j = €$ | — | 100.00 | — | 1.00 | $1.01000 = w_{€€}(1 + r_{€€})$ |
| | | | | | $0.2701750 = 0.26750 \cdot 1.01$ |
| Sum | 100.00% | 100.00% | — | — | $= 8.015\% = R^B$ |

Table 9: Benchmark Portfolio $B$ (= Market Portfolio)

The interesting differences in return between the active portfolios and the benchmark portfolio can be evaluated as shown in Table 10.

| | Portfolio $A_I$ | Portfolio $A_{II}$ | Portfolio $A_{III}$ | Benchmark $B$ |
|---|---|---|---|---|
| Portfolio Return | 7.185% | 9.968% | 8.405% | 8.015% |
| (Economic) Value Added | -0.830% | +1.953% | +0.390% | — |

Table 10: Outline of portfolio returns and respective EVA's

The superior returns of $A_{II}$ are a direct consequence of the advantageous, and appropriate, separation of tasks, as described. Furthermore, the traditional approach results in a portfolio ($A_I$) which underperforms even the benchmark!

# 4 Attribution Analysis

In order to directly compare returns from an actively managed portfolio ($A$) to a passive benchmark portfolio ($B$) this section introduces "allocation" ($\alpha$) and "selection" variables ($\sigma$) for both market and currency management. Separating out the individual contributions of management (market management, $M$, before currency management, $C$), these variables, $\alpha_i^{\mathrm{M}}$, $\sigma_i^{\mathrm{M}}$, $\alpha_i^{\mathrm{C}}$, $\sigma_i^{\mathrm{C}}$, are defined as follows:

- Market Allocation, $\alpha_i^{\mathrm{M}}$,

$$\alpha_i^{\mathrm{M}} = \left(w_i^{\mathrm{A}} - w_i^{\mathrm{B}}\right) \cdot \left[\left((1+\mu_i^{\mathrm{B}}) \cdot \sum_{j=1}^{m} w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})\right) - (1+R^{\mathrm{B}})\right]. \tag{4}$$

- Market Selection, $\sigma_i^{\mathrm{M}}$,

$$\sigma_i^{\mathrm{M}} = w_i^{\mathrm{A}} \cdot \left((1+\mu_i^{\mathrm{A}}) - (1+\mu_i^{\mathrm{B}})\right) \cdot \left(\sum_{j=1}^{m} w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})\right). \tag{5}$$

- Currency Allocation, $\alpha_i^{\mathrm{C}}$,

$$\alpha_i^{\mathrm{C}} = w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \left[\sum_{j=1}^{m} \left(w_{ij}^{\mathrm{A}} - w_{ij}^{\mathrm{B}}\right) \cdot (1+\kappa_j^{\mathrm{B}})\right]. \tag{6}$$

- Currency Selection, $\sigma_i^{\mathrm{C}}$,

$$\sigma_i^{\mathrm{C}} = w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \left[\sum_{j=1}^{m} w_{ij}^{\mathrm{A}} \cdot \left((1+\kappa_j^{\mathrm{A}}) - (1+\kappa_j^{\mathrm{B}})\right)\right]. \tag{7}$$

Notation:

| | |
|---|---|
| $i$ | market currencies in active and passive portfolio, $i = 1, \ldots, n$, |
| $j$ | hedge currencies, $j = 1, \ldots, m$, |
| A | superscript for returns and weights of an actively managed portfolio, |
| B | superscript for return and weight variables of the benchmark portfolio. |

The derivation of these formulas is straightforward. Each measure is built by the return-difference of two newly created active and passive portfolios.

- Market Allocation

  The first decision of active portfolio management is to allocate money to different markets. To measure the effect of deviating from the benchmark's investment policy in market allocation and to suppress the influence of other active portfolio management decisions made thereafter, an active portfolio is created which differ from the benchmark only in market allocating. It consists of active market allocation strategies, passive market selection as well as passive currency strategies. The return of this newly created "active" portfolio is compared to the return of the benchmark portfolio. The difference marks the total amount active market allocation contributed to the EVA.

$$\underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{B}}) \cdot \sum_j w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of active portfolio)}} - \underbrace{\sum_i w_i^{\mathrm{B}} \cdot (1+\mu_i^{\mathrm{B}}) \cdot \sum_j w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of passive portfolio)}}.$$

  For an interpretation of the adjustment term $(1 + R^B)$ in (4) see Section 5.

- Market Selection
  Decisions of active market allocation are given. A new "passive" portfolio is created which undertakes these decisions, but equals the benchmark portfolio in every other strategy. A new "active" portfolio is built, following the given market allocation, and copying active market selection strategies, but neglecting active currency strategies by choosing the passive ones.

$$\underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_j w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of active portfolio)}} \quad - \quad \underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{B}}) \cdot \sum_j w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of passive portfolio)}}.$$

- Currency Allocation
  Now active and passive portfolio take the values of the original active portfolio in market management as given. In terms of currency management, the new passive portfolio equals the benchmark portfolio, whereas the new active portfolio copies active currency allocation, and suppresses the influence of active currency selection by choosing the benchmark values.

$$\underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_j w_{ij}^{\mathrm{A}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of active portfolio)}} \quad - \quad \underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_j w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of passive portfolio)}}.$$

- Currency Selection
  Currency selection is the last step in the management process, therefore all active decisions made before are taken as given. Active and passive portfolio differ therefore only in currency selection where the passive one chooses the benchmark values, the active one the one of the original active portfolio.

$$\underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_j w_{ij}^{\mathrm{A}} \cdot (1+\kappa_j^{\mathrm{A}})}_{\text{(return of active portfolio)}} \quad - \quad \underbrace{\sum_i w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_j w_{ij}^{\mathrm{A}} \cdot (1+\kappa_j^{\mathrm{B}})}_{\text{(return of passive portfolio)}}.$$

Without any residual term left, and therefore in contrast to many other definitions of these variables, the values of (4)-(7) sum up to the exact value of EVA as the difference between overall performance of the original active and passive portfolio (see also Paape, 2000, p. 109):

$$\begin{aligned}
&\sum_{i=1}^{n} \left(\alpha_i^{\mathrm{M}} + \sigma_i^{\mathrm{M}} + \alpha_i^{\mathrm{C}} + \sigma_i^{\mathrm{C}}\right) = \\
&= \sum_{i=1}^{n} \Bigg[\left(w_i^{\mathrm{A}} - w_i^{\mathrm{B}}\right) \cdot \left((1+\mu_i^{\mathrm{B}}) \cdot \sum_{j=1}^{m} w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}}) - (1+R^{\mathrm{B}})\right)\Bigg] + \\
&\quad + \Bigg[w_i^{\mathrm{A}} \cdot \left((1+\mu_i^{\mathrm{A}}) - (1+\mu_i^{\mathrm{B}})\right) \cdot \sum_{j=1}^{m} w_{ij}^{\mathrm{B}} \cdot (1+\kappa_j^{\mathrm{B}})\Bigg] + \\
&\quad + \Bigg[w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_{j=1}^{m} \left(w_{ij}^{\mathrm{A}} - w_{ij}^{\mathrm{B}}\right) \cdot (1+\kappa_j^{\mathrm{B}})\Bigg] + \\
&\quad + \Bigg[w_i^{\mathrm{A}} \cdot (1+\mu_i^{\mathrm{A}}) \cdot \sum_{j=1}^{m} w_{ij}^{\mathrm{A}} \cdot \left((1+\kappa_j^{\mathrm{A}}) - (1+\kappa_j^{\mathrm{B}})\right)\Bigg] =
\end{aligned}$$

$$
\begin{aligned}
= & \sum_{i=1}^{n}\Big[\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{B}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)-\Big(w_i^{\mathrm{B}}\cdot(1+\mu_i^{\mathrm{B}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)\Big]+ \\
& +\Big[\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)-\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{B}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)\Big]+ \\
& +\Big[\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{A}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)-\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)\Big]+ \\
& +\Big[\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{A}}\cdot(1+\kappa_j^{\mathrm{A}})\Big)-\Big(w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{A}}\cdot(1+\kappa_j^{\mathrm{B}})\Big)\Big] = \\
= & \sum_{i=1}^{n}\Big[w_i^{\mathrm{A}}\cdot(1+\mu_i^{\mathrm{A}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{A}}\cdot(1+\kappa_j^{\mathrm{A}})\Big]-\sum_{i=1}^{n}\Big[w_i^{\mathrm{B}}\cdot(1+\mu_i^{\mathrm{B}})\cdot\sum_{j=1}^{m}w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})\Big] = \\
= & R^{\mathrm{A}}-R^{\mathrm{B}}=EVA.
\end{aligned}
$$

## 5 Numerical Example

Using the attribution variables of the previous section, one can analyse the performance of the three active portfolios of Section 3, by comparing these with the benchmark portfolio of the same section. Market selection should be zero for every portfolio since each market had only one security to invest in (see Table 1). The currency selection variables are also zero since the given database did not permit selecting between various Eurodeposit returns, either (Eurodeposit returns vary with the maturity of the chosen contract). In both cases active portfolio management cannot deviate from the given benchmark policy which causes the corresponding attribution variables to be zero. Hence, in the following evaluations only market and currency allocation variables matter.

### Analysis of Portfolio $A_I$

Portfolio $A_I$ attempted to produce a return-maximized portfolio in the context of a "conventional" performance measurement system (see Table 3 and 4). It invested in the market with the highest local currency market return in Table 1 (UK) and tried to do the best with the resulting currency exposure (unhedged British Pounds). Nevertheless it failed to beat the benchmark portfolio and produced an EVA of −0.83% (see page 10):

$$EVA^{A_I} = R^{A_I} - R^{B} = 7.185\% - 8.015\% = -0.830\%. \tag{8}$$

The "attribution" of $EVA^{A_I}$ through market and currency allocation variables is as follows.

**Market Allocation $A_I$:**

| | $\left(w_i^{A_I} - w_i^{\mathrm{B}}\right)$ | $\cdot$ | $\left((1+\mu_i^{\mathrm{B}})\right.$ | $\cdot$ | $\sum_{j=1}^{m} w_{ij}^{\mathrm{B}}\cdot(1+\kappa_j^{\mathrm{B}})$ | $-$ | $\left.(1+R^{\mathrm{B}})\right)$ | = | $\alpha_i^{\mathrm{M}}$ |
|---|---|---|---|---|---|---|---|---|---|
| $i =$ ¥ : | −25% | $\cdot$ | 1.004587 | $\cdot$ | $(1\cdot 1.079100+0+0+0)$ | − | 1.08015 | = | −0.0975% |
| $i =$ \$ : | −25% | $\cdot$ | 1.008372 | $\cdot$ | $(0+1\cdot 1.075000+0+0)$ | − | 1.08015 | = | −0.0962% |
| $i =$ £ : | +75% | $\cdot$ | 0.993258 | $\cdot$ | $(0+0+1\cdot 1.079125+0)$ | − | 1.08015 | = | −0.6225% |
| $i =$ € : | −25% | $\cdot$ | 1.019048 | $\cdot$ | $(0+0+0+1\cdot 1.060500)$ | − | 1.08015 | = | −0.0138% |
| Sum | | | | | | | | | −0.8300% |

**Currency Allocation $A_I$:**

$$
\begin{array}{lccccccc}
 & w_i^{A_I} & \cdot & (1+\mu_i^{A_I}) & \cdot & \sum_{j=1}^{m}\left(w_{ij}^{A_I}-w_{ij}^{\mathrm{B}}\right)\cdot(1+\kappa_j^{\mathrm{B}}) & = & \alpha_i^{\mathrm{C}} \\
i=¥: & 0\% & \cdot & 1.004587 & \cdot & \left((0-1)\cdot 1.079100+0+0+0\right) & = & 0\% \\
i=\$: & 0\% & \cdot & 1.008372 & \cdot & \left(0+(0-1)\cdot 1.075000+0+0\right) & = & 0\% \\
i=£: & 100\% & \cdot & 0.993258 & \cdot & \left(0+0+(1-1)\cdot 1.079125+0\right) & = & 0\% \\
i=€: & 0\% & \cdot & 1.019048 & \cdot & \left(0+0+0+(0-1)\cdot 1.060500\right) & = & 0\% \\
\hline
\text{Sum} & & & & & & & 0\%
\end{array}
$$

**Summary $A_I$:**

| $A_I$ | market allocation | market selection | currency allocation | currency selection | Sum |
|---|---|---|---|---|---|
| Japan | -0.0975% | 0% | 0% | 0% | -0.0975% |
| USA | -0.0962 | 0 | 0 | 0 | -0.0962 |
| UK | -0.6225 | 0 | 0 | 0 | -0.6225 |
| Euroland | -0.0138 | 0 | 0 | 0 | -0.0138 |
| Sum | -0.8300% | 0% | 0% | 0% | -0.8300% |

Table 11: Attribution Analysis for Portfolio $A_I$

**Interpretation $A_I$:**
Because the active currency management did not change the given benchmark policy of not hedging at all, the currency allocation variables are zero. Thus, only market allocation variables are non-zero. Of special significance is the sign of these variables. If this is negative, portfolio management failed to improve on the benchmark portfolio. This was the case for portfolio $A_I$. Table 11 shows that the principle reason for $A_I$'s poor result stemmed from the overweighting in British securities which performed worse than the overall benchmark portfolio (see the adjustment term $(1+R^B)$ in (4)). A minor effect was contributed through the underweightings in other markets whose returns performed better than the overall benchmark portfolio.

**Analysis of Portfolio $A_{II}$**
Portfolio $A_{II}$ resulted from creating a return-maximized portfolio in the context of the new system of Section 2 (see Table 6). It invested in the market with the highest local currency market premium (Euroland) and the highest currency premium (British Pounds) in Table 5. The highest currency premium reflects the highest cash return possible in the markets. The portfolio was able to outperform the benchmark portfolio by an added value of +1.953% (see page 10):

$$EVA^{A_{II}} = R^{A_{II}} - R^B = 9.968\% - 8.015\% = +1.953\%. \tag{9}$$

**Summary $A_{II}$:**

| $A_{II}$ | market allocation | market selection | currency allocation | currency selection | Sum |
|---|---|---|---|---|---|
| Japan | -0.0975% | 0% | 0% | 0% | -0.0975% |
| USA | -0.0962 | 0 | 0 | 0 | -0.0962 |
| UK | 0.2075 | 0 | 0 | 0 | 0.2075 |
| Euroland | 0.0412 | 0 | 1.8980 | 0 | 1.9392 |
| Sum | 0.0550% | 0% | 1.8980% | 0% | 1.9530% |

Table 12: Attribution Analysis for Portfolio $A_{II}$

**Interpretation $A_{II}$:**
The market allocation variables in the Japanese and the US market are negative, but near zero value. Both markets performed better than the overall benchmark return, $(1 + R^B)$, but had been underweighted in the active portfolio. To underweight the British market, on the other hand, was a good strategy because it performed worse than the overall benchmark did. Finally, the overweighting of the Euroland market can be seen positively because its better performance in comparison to the benchmark return, $(1 + R^B)$, but has not had much influence on the portfolio's *EVA*. Currency allocation variables show zero values for Japan, USA, and UK, and a positive value in Euroland. The overweighting of the exchange from Euro to British Pounds in the active portfolio (active portfolio did 100%, benchmark 0%), and the underweighting of the unhedged position "Euro to Euro" (the active portfolio's weight here is zero, the one of the benchmark 100%) contributed positively to the portfolio's EVA.

### Analysis of Portfolio $A_{III}$

Portfolio $A_{III}$ resulted from creating a return-maximized portfolio without currency management (see Tables 7 and 8). It invested in the market with the highest base currency market return (Japan) and kept the generated currency exposure unhedged (Japanese yen). The portfolio developed well, it beat the benchmark portfolio by an added value of +0.39% (see page 10):

$$EVA^{A_{III}} = R^{A_{III}} - R^B = 8.405\% - 8.015\% = +0.390\%. \quad (10)$$

**Summary $A_{III}$:**

| $A_{III}$ | market allocation | market selection | currency allocation | currency selection | Sum |
|---|---|---|---|---|---|
| Japan | +0.2925% | 0% | 0% | 0% | +0.2925% |
| USA | −0.0962 | 0 | 0 | 0 | −0.0962 |
| UK | +0.2075 | 0 | 0 | 0 | +0.2075 |
| Euroland | −0.0138 | 0 | 0 | 0 | −0.0138 |
| Sum | +0.3900% | 0% | 0% | 0% | +0.3900% |

Table 13: Attribution Analysis for Portfolio $A_{III}$

**Interpretation $A_{III}$:**
Besides the market allocation variables, all variables are zero. The zero values occur because the portfolio management did not deviate in market selection or currency management from the passive strategy of the benchmark portfolio. Thus, the positive performance of portfolio $A_{III}$ is due to good market allocation only. Specifically the overweighting of the Japanese and the underweighting of the British market worked well, whereas the effect of underweighting the US and Euroland market is more or less unimportant.

# 6 Conclusion

The intention of the paper was to show the crucial role of performance measurement in performance evaluation. Especially when studying global investment portfolios, one has to be aware of the dependencies between market and currency management which affects the way of measuring the corresponding return variables. Because market and currency returns cannot simply be added – they must be connected via a product –, the overall portfolio performance cannot be simply separated into market returns on the one hand and currency returns on the other. Yet, the interpretation of currency return as return from a forward currency contract, which can be represented as a discounted difference between term interest rates of the corresponding countries, gives at least independency for a part of the investment process, namely the selection process. Market management is then measured through market premiums, currency management through currency premiums. Market premiums represent the discounted difference by which the local market return over- or underperformed the "riskless" return of the same market, reflecting differential interest rates on cash equivalents. Currency premiums, on the other hand, are exactly these interest rates, short-term Eurodeposits, measured in the base currency of the analysed portfolio. Currency management, therefore, becomes simply global cash management. With this new performance measurement appropriate attribution variables are defined which split up the success of active management decisions in comparison to a benchmark portfolio, the (economic) added value, into market allocation, market selection, currency allocation, and currency selection variables.

# Appendix

## A Performance Measures in Examples

- $r_i$: The local market rate of return is the relation of gains or losses in a market compared to the initial investment in this market, both values measured in local currency. Take Germany as a Euroland country to invest in $(i = \text{€})$ and look at the following example:

$$\begin{array}{ccc} \text{Initial Value} & \text{(one period of time)} & \text{Final Value} \\ \hline \text{€}116.94 & \xrightarrow{(1+r_{\text{€}})} & \text{€}123.96 \end{array} \tag{11}$$

The definition of $r_{\text{€}}$ as rate of return in Germany in local currency would yield:

$$r_{\text{€}} = \frac{\text{€}123.96 - \text{€}116.94}{\text{€}116.94} \approx 0.0600 = 6.00\%.$$

- $f_{ij}$: The forward currency rate of return is the rate of return derived exchanging currency $j$ for one unit of currency $i$ with the actual exchange rate and reexchanging after one period of time for currency $j$ with the help of a forward contract fixed at the beginning of the transaction. For the above example (the data is taken from the German newspaper 'Handelsblatt', edition May 25th, 2001):

$$\begin{array}{ccc} \text{Initial Value} & \text{(one period of time)} & \text{Final Value} \\ \hline \pounds 70.83 & \xrightarrow{(1+r_{\text{€}})\cdot\mathbf{(1+f_{\text{€}\pounds})}} & \pounds 75.18 \\ \mathbf{\pounds 1 \simeq \text{€}1.6510}\Big\downarrow & & \Big\uparrow\ \mathbf{\text{€}1 \simeq \pounds 0.6065} \\ \text{€}116.94 & \xrightarrow{(1+r_{\text{€}})} & \text{€}123.96 \end{array} \tag{12}$$

With

$$\begin{aligned} \frac{\pounds 75.18 - \pounds 70.83}{\pounds 70.83} &= \frac{70.83 \cdot 1.651 \cdot (1 + r_{\text{€}}) \cdot 0.6065 - 70.83}{70.83} = 1.651 \cdot (1 + r_{\text{€}}) \cdot 0.6065 - 1 \\ &= (1 + r_{\text{€}}) \cdot (1 + \frac{0.6065 - \frac{1}{1.651}}{\frac{1}{1.651}}) - 1 = (1 + r_{\text{€}}) \cdot (1 + f_{\text{€}\pounds}) - 1 \\ &= 1.06 \cdot (1 + 0.0013) - 1 = 0.0614 = 6.14\%, \end{aligned}$$

this yields: $f_{\text{€}\pounds} = \dfrac{0.6065 - \frac{1}{1.651}}{\frac{1}{1.651}} \approx 0.13\%.$

Note that for exchange rates like $\pounds 1 \simeq \text{€}1.651$, $\text{€}1 \simeq \pounds\frac{1}{1.651} \approx \pounds 0.6057$.

Conversely, to show that equation (2) holds for the forward currency rate of return, consider the Eurodeposit rates of return in Euro and Pounds for a period of three months, which are 4.55% p. a. and 5.1% p. a., respectively $(c_{\text{€}} = 1.0455^{\frac{3}{12}} - 1, c_{\pounds} = 1.051^{\frac{3}{12}} - 1)$:

$$f_{\text{€}\pounds} = \frac{c_{\pounds} - c_{\text{€}}}{1 + c_{\text{€}}} = \frac{1.051^{\frac{1}{4}} - 1.0455^{\frac{1}{4}}}{1.0455^{\frac{1}{4}}} \approx 0.0013 = 0.13\%.$$

- $e_{j\$}$: This is the rate of return derived exchanging US-Dollar for one unit of currency $j$ with the actual exchange rate and reexchanging after one period of time, using the current exchange rate between currency $j$ and US-Dollar. For the above example:

$$\begin{array}{ccc}
\text{Initial Value} & \text{(one period of time)} & \text{Final Value} \\
\hline
\$100 & \xrightarrow{(1+r_€)\cdot(1+f_{€£})\cdot\mathbf{(1+e_{£\$})}} & \$106.17 \\
\mathbf{\$1 \simeq £0.7083} \downarrow & & \uparrow \mathbf{£1 \simeq \$1.4122} \\
£70.83 & & £75.18 \\
£1 \simeq € 1.6510 \downarrow & & \uparrow € 1 \simeq £0.6065 \\
€116.94 & \xrightarrow{(1+r_€)} & €123.96
\end{array} \qquad (13)$$

With

$$\begin{aligned}
\frac{\$106.17 - \$100}{\$100} &= \frac{100 \cdot 0.7083 \cdot 1.651 \cdot (1+r_€) \cdot 0.6065 \cdot 1.4122 - 100}{100} \\
&= 0.7083 \cdot 1.651 \cdot (1+r_€) \cdot 0.6065 \cdot 1.4122 - 1 \\
&= (1+r_€) \cdot (1+f_{€£}) \cdot \Big(1 + \frac{1.4122 - \frac{1}{0.7083}}{\frac{1}{0.7083}}\Big) - 1 \\
&= (1+r_€) \cdot (1+f_{€£}) \cdot (1+e_{£\$}) - 1 \\
&= 1.06 \cdot 1.0013 \cdot (1+0.0003) - 1 = 0.0617 = 6.17\%,
\end{aligned}$$

this yields:

$$e_{£\$} = \frac{1.4122 - \frac{1}{0.7083}}{\frac{1}{0.7083}} \approx 0.003 = 0.03\%.$$

- $w_i$: This is the dollar weighted percentage of the original investment in market $i$. If in the above example the whole portfolio was initialy worth \$ 1000, the fraction invested in Euroland (Germany) would be calculated as follows:

$$w_€ = \frac{\$100}{\$1000} = 0.1.$$

- $w_{ij}$: This number gives the percentage of the final value in market $i$ which was transfered to currency $j$. It is weighted in terms of local currency. In the above example the currency of the investment in Euroland (Germany) was fully cross-hedged to British Pounds, therefore:

$$w_{€£} = \frac{€123.96}{€123.96} = 1.0 = 100\% \quad \Longrightarrow \quad w_{€€} = 0, \quad w_{€\$} = 0, \quad w_{€¥} = 0.$$

# B Covered Interest Rate Parity

Introduce the following notation:

$h_{ij}$ exchange rate between currencies $i$ and $j$ which exchanges one unit of currency $i$ for units of currency $j$ (note: $h_{ij} = \frac{1}{h_{ji}}$),

$t_{ij}$ forward exchange rate between currencies $i$ and $j$ which allows exchanging one unit of currency $i$ for units of currency $j$ at a prespecified time in the future (note: $t_{ij} = \frac{1}{t_{ji}}$).

Now look at the following two strategies:

Strategy 1: Exchange an outlay $V_j$ with the current exchange rate $h_{ji}$ to currency $i$, invest the resulting amount in the Eurodeposit cash market at $c_i$, and hedge the outcome back to currency $j$ with the help of a forward currency contract $t_{ij}$.

Strategy 2: Invest an outlay $V_j$ directly in the Eurodeposit cash market at $c_j$.

If the markets are arbitrage-free, investing in market $i$ and hedging currency risk (strategy 1) and investing directly in market $j$ without currency risk (strategy 2) must lead to the same result, hence:

$$V_j \cdot h_{ji} \cdot (1 + c_i) \cdot t_{ij} = V_j \cdot (1 + c_j). \tag{14}$$

Rearranging gives

$$t_{ij} = h_{ij} \cdot \frac{1 + c_j}{1 + c_i}. \tag{15}$$

Finally, defining the rate of return of a forward currency contract, $f_{ij}$, as the rate of return derived exchanging currency $j$ for one unit of currency $i$ with the actual exchange rate and reexchanging after one period of time for currency $j$ with the help of a forward contract fixed at the beginning of the transaction, yields (see p. 17 for a numerical example)

$$f_{ij} = h_{ji} \cdot t_{ij} - 1 = \frac{t_{ij} - h_{ij}}{h_{ij}} \overset{(15)}{=} \frac{c_j - c_i}{1 + c_i}. \tag{16}$$

This is exactly equation (2).

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